Estimating and Testing Investment-based Asset Pricing Models*

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Abstract

The standard investment-based asset pricing model with homogeneous of degree one operating profit and adjustment cost functions predicts that a firm's stock return should be equal to physical capital investment return, state-by-state. Yet, previous work testing the model typically examines the weaker prediction that stock returns and investment returns should be equal on average, and focuses on simple specifications with one physical capital input and quadratic adjustment costs. We document that by following this approach, the implied time series R^2 of the standard model is negative. We show how to incorporate both the model-implied time series and cross sectional restrictions in the estimation and testing of investment-based models using the generalized method of moments. Our approach uncovers a novel tradeoff between cross sectional and time series fit in the data: the baseline one-capital input investment-based model with quadratic adjustment costs cannot fit both sets of moments simultaneously. Perhaps surprisingly, even when only the time series moments are used in the estimation of the model to maximize its time series fit, the implied time series R^2 of the model remains negative. Our approach can be extended to estimate and test non-homogeneous of degree one models in which the state-by-state equality between investment and stock returns does not hold. By incorporating a larger set of model predictions in the estimation and testing of investment-based models, our methodology can be useful to guide improvements in the specification of this class of models in future research.

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1 Introduction

The neoclassical investment-based asset pricing model links firm characteristics to stock returns. Under constant returns to scale, the model predicts that realized investment returns, which can be measured in the data through a production function from investment and output data, should be equal to stock returns state-by-state at any point in time (Cochrane 1991, and Restoy and Rockinger 1994). Despite this strong prediction, most of the structural work in investment-based asset pricing to date, tests the model by the generalized methods of moments (GMM) using the weaker condition that stock returns and investment returns should be equal on average. Following this procedure, Liu, Whited, and Zhang (2009) (henceforth LWZ) shows, through structural estimation, that the neoclassical investment-based model with one capital input and quadratic adjustment costs matches well the cross-section of average stock returns of a large range of portfolio sorts.

In this paper, we estimate the investment-based model incorporating the theoretical prediction that realized investment and stock returns should be equal at any point in time, not just on average. Methodologically, our analysis augments the set of moments targeted in the GMM estimation of the model, raising the bar for the evaluation of the model. Specifically, if the model is correctly specified, the average sum of squared residuals (difference between investment and stock returns) for each portfolio, should be zero. Thus, as in nonlinear least squares, we select the model parameters to minimize the distance between stock returns and model-implied investment returns at each point in time. We label this set of moments, time series moments. In addition, as in previous work, we also consider cross sectional moments, that is, the model prediction that, investment and stock returns should be equal on average.

In the baseline estimation of the investment-based model with one capital input and quadratic adjustment costs, we follow LWZ and estimate the model by the GMM at the portfolio-level, using ten book-to-market portfolios (we also consider other test assets in the appendix). Estimating the model using portfolios allows to characterize the data in a simple manner as the number of portfolios is significantly smaller than the number of firms in the data, and to reduce the impact of regression noise. The estimation targets twenty moments: ten cross sectional moments as in LWZ, and our novel ten time series moments. To understand the role of the two set of moments on the results, we investigate how the estimation results change when we vary the relative weight of the two set of moments on the GMM weighting matrix. If the model is correct, the set of moments used should not matter much for the results: the same set of model parameters should match both the cross sectional and the time series moments.

Our methodology and analyses focus on the economic evaluation of the fit of investment-based model. In particular, how well the model is able to simultaneously capture the average cross sectional variation and the time series variation of stock returns across portfolios, as opposed to a statistical evaluation or rejection of the model. To that end, we focus our evaluation of the model mostly on measures such as cross sectional R^2, time series R^2, and magnitude of the pricing errors (average residuals). Statistically speaking, as discussed in Cochrane (1991), the investment-based model should be rejected at any level of significance. The model predicts that stock returns and investment returns are equal at each point in time without any error term, which is not possible to achieve in the data. Nevertheless, if the model is a good description of reality, the residuals should be small, and largely random either across portfolios or over time, that is, they should not exhibit a systematic behavior. Hence, as part of the evaluation of the model, we also study the properties of the error terms (residuals) implied by the estimation.

Our empirical results can be summarized as follows. First, consistent with previous work, when we only use the equality of the average stock and investment returns as moments in the GMM estimation, the model matches the data very well, with low pricing errors (about 1.1% per year) and a cross sectional R^2 of a standard plot of the average portfolio-level stock returns against the average portfolio-level estimated investment return of 65%. The inspection of the model residuals in the time series reveals, however, some problems for the model: stock returns and the residuals are strongly positively correlated in the time series (correlation on average 83% across portfolios) which means that most of the time series variation of stock returns is captured by the residuals, not by the predicted investment returns. Thus, in contrast with the model predictions, stock returns and investment returns are far from perfectly correlated: on average across portfolios, the correlation is either negative, or close to zero. We conclude that when estimating the baseline model using the cross sectional moments only, the model fails to capture its time series implications.

Second, when we incorporate the model-implied time series moments in the estimation using GMM to help improve the fit of the model on the time series, we uncover a novel tradeoff between cross sectional fit and time series fit: the baseline investment-based model with one capital input cannot fit both sets of moments simultaneously. As we increase the relative weight of the time series moments in the estimation, the fit on the cross sectional moments deteriorates significantly: the cross sectional R^2 decreases from 65% when only cross sectional moments are used in the estimation, to -78% when only time series moments are used in the estimation. Perhaps surprisingly, even when only the time series moments are used in the estimation, the model fit is poor: the average time series R^2 is on average -27%. That is, the standard investment-based model with one capital input and quadratic adjustment costs is not able to capture the time series behavior of stock returns in the data even when the estimation is designed to maximize its time series fit.

We investigate potential empirical reasons for the poor fit of the model in the time series, despite its success in the cross section. First, we address the possibility that quantities (e.g. investment) and asset prices are misaligned in the real data. For example, stock prices (and hence stock returns) might respond instantaneously to aggregate shocks, whereas investment might take more time to adjust, in which case investment returns lag stock returns (see, for example, Lamont 2000 for a more formal analysis of this issue). To address this concern, we investigate the time series fit of the model using 5-year (instead of annual) compounded returns. If the misalignment in the data is relatively short lived, the misalignment should be less pronounced at longer-horizon returns (we confirm this conjecture using simulated data from a calibrated version of the baseline model). We show that using 5-year horizon compounded returns the average (across portfolios) stock and investment return correlation indeed increases from -19% to 16%. Still, the time series R^2 across portfolios remains very low at -20%. Thus, the data misalignment in asset prices and real quantities does not appear to be the main cause for the inability of the model to explain the time series behavior of stock returns.

Second, we investigate the role of portfolio aggregation for the results. As noted in Belo, Gala, Salomao, and Vitorino (2022), (henceforth BGSV) and Gonçalves, Xue, and Zhang (2020) (henceforth GXZ), the portfolio aggregation procedure in LWZ suffers from an aggregation bias. Specifically, in this approach, the portfolio-level investment return is computed by first computing the portfolio-level characteristics (e.g., the portfolio-level investment rates), and then plugging these aggregate characteristics directly in the investment return formula. Given the nonlinearity of the investment returns, the portfolio-level investment return obtained using this procedure is no longer equal to a value- or equal-weighted portfolio stock return.

We thus investigate if the aggregation bias induced by the LWZ portfolio-level aggregation can explain the poor fit of the investment-based model in the time series. Following BGSV and GXZ, we estimate the model using portfolio-level investment returns properly aggregated from firm-level investment returns. Specifically, for each firm, we first compute the investment return, and then compute the portfolio-level investment return as the value- or equal-weighted average of the firm-level investment returns. We find that the fit of the model in the time series remains poor (R^2 of -26% when only time series moments are targeted). Thus, portfolio-level aggregation issues also do not appear to be the main cause for the inability of the model to explain the time series behavior of stock returns.

Our approach can be extended to models without homogeneous of degree one operating and adjustment cost functions, in which case the state-by-state stock and investment return equality does not hold. As we show here, most investment-based asset pricing models imply a strong relationship between stock returns and firm characteristics, such as, for example, a firm's current marginal product of capital or current and lagged investment rate. This relationship can be assessed in the model and in the data, both in the cross section and in the time series. A successful investment-based model should have a link between stock returns and its characteristics that are consistent with the data (in terms of slope coefficients and goodness of fit). We apply this approach to a more general specification of the investment-based model with decreasing returns to scale, nonconvex adjustment costs, and operating fixed costs. We document that this version of the model also cannot match the time series relationship between stock returns and firm characteristics observed in the data. This approach can be used simply as an external validity test or be incorporated in the estimation of the structural parameters of any investment model as an additional moment condition using the simulated method of moments.

Our findings have implications for future research in investment-based asset pricing. Our methodology helps detect a dimension of the fit of standard specifications of the investment-based model that requires improvement. To help the fit of the investment-based model in the time series, additional capital inputs (intangible capital and physical capital) and labor inputs as in BGSV, intangible capital as in Peters and Taylor (2017), or short-term and long-term assets as in GXZ, can be added to the analysis. In addition, explicitly accounting for firm- or industry-level heterogeneity in the technologies, which are assumed to be similar in the baseline analysis, as well as more general specifications of the adjustment cost functions should be investigated,. Taken together, our methodology, which adds the time series implications of the model explicitly into the testing and estimation of the investment-based model, can thus be useful to help improve the specification of investment-based models in future research.

Related Literature:

Our work is closely related to Liu, Whited, and Zhang (2009) who first estimate the neoclassical investment-based model on the cross section of stock returns. Different from LWZ, and similar to BGSV, our estimation procedure requires the model to match the realized time series of the observed stock returns as close as possible, and not just on average. LWZ document that the implied stock and investment returns have low correlation, which they label a correlation puzzle. Similarly, the correlation puzzle is documented at the aggregate level in Kuehn (2009). Our analysis is broader in that we show how to incorporate the time series implications of the model directly in the estimation and evaluation of the model, and we discuss potential alternative empirical reasons for the poor fit of the model. In robustness analysis, LWZ includes the cross section of portfolio variance moments in the estimation, but, as we show here, matching variance moments does not help with the time series fit of the model.

Li, Ma, Wang, and Yu (2021) estimate an investment-based model with two capital inputs using firm-level data and Bayesian estimation methods. Like our work, their approach also looks at the time-series implications of the model. Different from our approach, and departing from the baseline neoclassical investment-based model with stable technologies, they estimate time-varying technological parameters.

Gonçalves, Xue, and Zhang (2020) documents the aggregation bias in the the original LWZ portfolio-level aggregation approach (see also BGSV and Zhang 2017 for earlier discussions of this aggregation bias in LWZ). Our analysis shows that the aggregation bias alone cannot explain the poor fit of the investment-based model with one-capital input in the time series.

Delikouras and Dittmar (2021) estimate and test standard investment-based models using GMM with cross sectional moments and investment Euler equations, which requires the specification of a stochastic discount factor, and also find that the baseline investment-based model is unable to match both sets of moments jointly. Our paper shares the goal of testing investment-based models across a larger set of model implications, but differs in the approach. Our analysis emphasizes the tension in the model's cross sectional fit versus time series fit. More importantly, our analysis focuses on the properties of the firm's technology and does not require the specification of a stochastic discount factor, thus avoiding the joint hypothesis testing problem. This allows researchers to focus on the properties of the firm's technology, and how the specification of the firm's technology affects the time series and cross sectional fit of the model in a simple manner. Naturally, the ultimate goal is to obtain a specification of firm's technology and of the stochastic discount factor that simultaneously matches the cross sectional, the time series, and the investment Euler equations as in Delikouras and Dittmar (2021). Our paper thus complements their approach by providing the first step towards that goal, that is, how to specify the firm's technology to better match the data.

Finally, this paper is closely related to the strand of production-based asset pricing literature that links firm characteristics to asset returns. See, for example, Zhang (2005), Belo (2010), Belo, Lin, and Bazdresch (2014), İmrohoroğlu and Tüzel (2014), Kogan and Papanikolaou (2014), Kung and Schmid (2015), Croce (2014), and Deng (2021), among many others. We contribute to this literature by improving the econometric methodology for estimating and testing these models.

The rest of the paper is organized as follows. Section 2 presents the model, Section 3 describes econometric methodology. Section 4 reports estimation and tests results. Section 5 proposes a general specification-free test of the model. Section 6 concludes.

2 The Neoclassical Investment-Based Model

We briefly present here the standard neoclassical investment-based model of the firm with one capital input as in LWZ. We use their notation whenever possible. Time is discrete and the horizon infinite. Firms choose costlessly adjustable inputs each period, taking their prices as given, to maximize operating profits (revenues minus expenditures on these inputs).

Taking operating profits as given, firms choose investment and debt to maximize the market equity.

Operating profits for firm i at time t are given by $\Pi(K_{it}, X_{it})$, in which K_{it} is capital and X_{it} is a vector of exogenous aggregate and firm-specific shocks. The firm has a Cobb-Douglas production function with constant returns to scale. As such, $\Pi(K_{it}, X_{it}) = K_{it}\partial\Pi(K_{it}, X_{it})/\partial K_{it}$, and the marginal product of capital, $\partial\Pi(K_{it}, X_{it})/\partial K_{it} = \alpha Y_{it}/K_{it}$, in which α is the capital's share in output and Y_{it} is sales.

Capital depreciates at an exogenous rate of δ_{it}, which is firm-specific and time-varying:

$$K_{it+1} = I_{it} + (1 - \delta_{it}) K_{it}, \tag{1}$$

in which I_{it} is investment. Firms incur adjustment costs when investing. The adjustment costs function, denoted $\Phi(I_{it}, K_{it})$, is increasing and convex in I_{it}, is decreasing in K_{it}, and has constant returns to scale in I_{it} and K_{it}. We use a standard quadratic functional form:

$$\Phi(I_{it}, K_{it}) = \frac{c}{2}\left(\frac{I_{it}}{K_{it}}\right)^2 K_{it}, \tag{2}$$

in which $c > 0$ is the slope parameter.

Firms finance investment with one-period debt. At the beginning of period t, firm i issues an amount of debt, denoted B_{it+1}, that must be repaid at the beginning of $t+1$. Let r_{it}^B denote the gross corporate bond return on B_{it}. We can write taxable corporate profits as operating profits minus depreciation, adjustment costs, and interest expense: $\Pi(K_{it}, X_{it}) - \delta_{it}K_{it} - \Phi(I_{it}, K_{it}) - \left(r_{it}^B - 1\right) B_{it}$. Let τ_t denote the corporate tax rate. We define the payout of firm i as:

$$D_{it} \equiv (1 - \tau_t)\left[\Pi(K_{it}, X_{it}) - \Phi(I_{it}, K_{it})\right] - I_{it} + B_{it+1} - r_{it}^B B_{it} + \tau_t\delta_{it}K_{it} + \tau_t\left(r_{it}^B - 1\right) B_{it}, \tag{3}$$

in which $\tau_t\delta_{it}K_{it}$ is the depreciation tax shield and $\tau_t\left(r_{it}^B - 1\right) B_{it}$ is the interest tax shield.

Let M_{t+1} denote the stochastic discount factor from period t to $t+1$, which is correlated with the aggregate component of the productivity shock X_{it}. The firm chooses optimal capital investment and debt to maximize the cum-dividend market value of equity:

$$V_{it} \equiv \underset{\{I_{it+s}, K_{it+s+1}, B_{it+s+1}\}_{s=0}^{\infty}}{max} E_t \left[\sum_{s=0}^{\infty} M_{t+s} D_{it+s} \right], \tag{4}$$

subject to a transversality condition given by $lim_{T \to \infty} E_t \left[M_{t+T} B_{it+T+1} \right] = 0$.

Firms' equity value maximization implies that $E_t \left[M_{t+1} r_{it+1}^I \right] = 1$, in which r_{it+1}^I is the investment return, defined as

$$r_{it+1}^I \equiv \frac{(1-\tau_{t+1}) \left[\alpha \frac{Y_{it+1}}{K_{it+1}} + \frac{c}{2} \left(\frac{I_{it+1}}{K_{it+1}} \right)^2 \right] + \tau_{t+1} \delta_{it+1} + (1-\delta_{it+1}) \left[1 + (1-\tau_{t+1})\, c \left(\frac{I_{it+1}}{K_{it+1}} \right) \right]}{1 + (1-\tau_t)\, c \left(\frac{I_{it}}{K_{it}} \right)}. \tag{5}$$

The investment return is the ratio of the marginal benefits of investment at period $t+1$ to the marginal costs of investment at t.

The first-order condition of maximizing Equation (4) with respect to B_{it+1} implies that $E_t \left[M_{t+1} r_{it+1}^{Ba} \right] = 1$, in which $r_{it+1}^{Ba} \equiv r_{it+1}^{B} - \left(r_{it+1}^{B} - 1 \right) \tau_{t+1}$ is the after-tax corporate bond return. Define $P_{it} \equiv V_{it} - D_{it}$ as the ex-dividend equity value, $r_{it+1}^S \equiv \left(P_{it+1} + D_{it+1} \right) / P_{it}$ as the stock return, and $w_{it} \equiv B_{it+1} / \left(P_{it} + B_{it+1} \right)$ as the market leverage. Under constant returns to scale, the investment return equals the weighted average of the stock return and the after-tax corporate bond return:

$$r_{it+1}^I = w_{it} r_{it+1}^{Ba} + (1 - w_{it})\, r_{it+1}^S. \tag{6}$$

Equivalently, the stock return equals the levered investment return, denoted r_{it+1}^{Iw}:

$$r_{it+1}^S = r_{it+1}^{Iw} \equiv \frac{r_{it+1}^I - w_{it} r_{it+1}^{Ba}}{1 - w_{it}}. \tag{7}$$

We let $P_{it} \equiv V_{it} - D_{it}$ be the ex-dividend equity value. The first-order condition of maximizing Equation (4) with respect to I_{it} implies that the market value of the firm is given by:

$$P_{it} + B_{it+1} = \left[1 + (1-\tau_t)\, c\left(\frac{I_{it}}{K_{it}}\right)\right] K_{it+1}. \quad (8)$$

3 Econometric Methodology

Section 3.1 describes moment conditions in existing studies and presents our new moment conditions. Section 3.2 illustrates our GMM estimation methodology. Section 3.3 proposes both economic and statistical tests to evaluate the model.

3.1 Moment conditions

To test the investment-based asset pricing model, existing studies as in LWZ and Belo, Xue, and Zhang (2013) (henceforth BXZ) test a weaker set of moment conditions, i.e. cross sectional moments, implied by Equation (7): expected stock returns equal expected levered investment returns on average,

$$g^{XS} = E\left[r^S_{it+1} - r^{Iw}_{it+1}\right] = 0. \quad (9)$$

Specifically, we define the model errors from the moment conditions as:

$$e^{XS}_i \equiv E_T\left[r^S_{it+1} - r^{Iw}_{it+1}\right], \quad (10)$$

in which $E_T[\cdot]$ is the sample mean of the series in brackets. We call e^{XS}_i the cross sectional error.

The q-theory model has rich implications on both the time series and cross section dimensions, as implied by Equation (7). Unlike existing studies, which estimate the model parameters by matching the time series means of stock and levered investment returns, we require the estimation to match the time series of stock and levered investment returns as closely as possible. This estimation procedure utilizes valuable time-series information when estimating the structural parameters and evaluating the fit of the underlying structural model.

We specify the time series moment condition for each portfolio as the average of the squared differences between stock returns and levered investment returns as in the nonlinear least squares estimation (henceforth NLLS):

$$g^{TS} = E\left(r^{S}_{it+1} - r^{Iw}_{it+1}\right)^2 = 0. \tag{11}$$

We define the model errors from above moment conditions by removing the cross-sectional mean and focusing only on the time-series fit. We call it the time series error:

$$e_i^{TS} \equiv \left(r^{S}_{it+1} - E_T\left[r^{S}_{it+1}\right]\right) - \left(r^{Iw}_{it+1} - E_T\left[r^{Iw}_{it+1}\right]\right). \tag{12}$$

We note that we use the nonlinear least squares objective function as the target time series moment for each portfolio. Under the null that the model is correctly specified, this moment should be zero for each portfolio, and hence it is a valid moment condition. Naturally, if there is some noise in the data, the moment will deviate from zero (stock returns will deviate from investment returns), but it should not deviate too much.

An alternative approach to implement NLLS with GMM is to use the first order conditions of the NLLS optimization problem which minimizes the sum of the squared differences between stock returns and levered investment returns across N portfolios:

$$g^{TS-FOC} = \frac{1}{N}\frac{1}{T}\sum_{i=1}^{N}\sum_{t=1}^{T}\left[\left(r^{S}_{it+1} - r^{Iw}_{it+1}\right)\frac{\partial r^{Iw}_{it+1}}{\partial \theta}\right] = 0, \tag{13}$$

in which $\theta \equiv (\alpha, c)$. The first order conditions by definition should be zero at the minimum, and can directly be used as the moment conditions. Indeed, that is the standard approach used to map NLLS into GMM (see, for example, Cochrane (2009), Chapter 11).

Following this procedure, if the estimation is just identified (using 2 first order conditions as moments to estimate 2 parameters), the weighting matrix does not matter for the results because all moment conditions can be zero. Hence, the GMM estimates using the NLLS first order conditions yield exactly identical estimates to the NLLS estimation. But when the model is over-identified, as is the case here (since we also use cross sectional moments in the estimation and several portfolios), the choice of different weights on the NLLS first order conditions in the GMM estimation affects the results, because it is not possible to match all the NLLS first order conditions at the same time. This makes the interpretation of the results more cumbersome because the size of the deviation of each first order condition of the NLLS moments does not have a natural interpretation. Hence, we use the NLLS objective function, not the NLLS first order conditions, as target moments in the estimation, because the magnitude of the errors have a natural interpretation as the average sum of squared residuals of each portfolio.[1]

An alternative way of incorporating the time series restrictions of the model in the estimation is to augment the set of moment conditions by adding instruments (variables that should be orthogonal to the error terms) in a manner that is analogous to the estimation of conditional asset pricing moments. This approach is well suited for model implied moment conditions in which the errors in each point in time are expectation errors (such as, for example, in the moment conditions implied by investment Euler equations). This is because when the error term is an expectation error, this error term should be orthogonal to any variable (called instrument) in the agent's information set available at each point in time. In turn, this orthogonality gives rise to a set of unconditional moment conditions that incorporate

[1]One potential issue with this approach is that, as noted, in the presence of measurement error or other noise in the data, the residuals will not be zero, in which case the NLLS objective function (and hence the corresponding moment conditions) will not be zero even if the model is correct due to data issues. In turn, this might affect the interpretation of the chi-square tests assessing the validity of the model. This is not a concern for our analysis given that our focus is on the economic interpretation of the model fit, and less on the statistical tests of the model. Indeed, for the same reason, most of our analyses are based on first stage GMM (which minimizes the residuals), not on optimal GMM. Nevertheless, our statistical tests are still valid under the standard null that the model is valid. Also, in robustness checks, we confirm that the main results reported here are similar to those obtained when we use the NLLS first order conditions as test moments (see online appendix).

the time series implications of the model and can be estimated using standard GMM. In our approach, however, the error term inside each moment conditions are not expectation errors because the model predicts that stock and (levered) investment returns should be equal state-by-state without any error. Hence, the error terms in our approach arise due to, for example, measurement or misspecification errors, and hence the theory does do not imply the orthogonality conditions in the same way that the expectation errors do. Therefore, we do not follow this approach to incorporate time series restrictions here.

Together with the cross sectional moments, LWZ also investigate if the model can match the cross section of stock return variances, which should be equal to the cross section of levered investment return variances. Thus, to compare these variance moments with our time series moments, we also include the variance moments in some of our analyses. Specifically, following LWZ, we define the variance moment condition as:

$$g^{Var} = E\left[\left(r^S_{it+1} - E\left[r^S_{it+1}\right]\right)^2 - \left(r^{Iw}_{it+1} - E\left[r^{Iw}_{it+1}\right]\right)^2\right] = 0. \tag{14}$$

Accordingly, we define the model errors as below and call it the variance error:

$$e^{Var}_i \equiv E\left[\left(r^S_{it+1} - E\left[r^S_{it+1}\right]\right)^2 - \left(r^{Iw}_{it+1} - E\left[r^{Iw}_{it+1}\right]\right)^2\right]. \tag{15}$$

A simple inspection of the previous equations reveals that minimizing the variance errors is not equivalent to minimizing the sum of squared residuals, hence it does not maximize the model fit in the time series. Indeed, as we confirm in the estimation below, the model fit in the time series actually deteriorates once these variance moments are included in the estimation.

3.2 GMM estimation

We estimate the parameters α and c, using one-step GMM to minimize a weighted average of e_i^{XS}, a weighted average of e_i^{TS}, or a weighted average of e_i^{XS} and e_i^{TS}. We view that q-theory model implications should be present in both the cross section and the time series dimension. Therefore, the objective of the estimation is to minimize a combination of the cross sectional errors and the time series errors.

Specifically, we stack the cross sectional moment conditions and the time series moment conditions in the matrix $g \equiv \left[g^{XS}; g^{TS}\right]$. We then estimate the parameters $\theta \equiv (\alpha, c)$, by minimizing a weighted combination of the sample moments, denoted by g_T:

$$\min_{\theta}\ g_T' W g_T, \tag{16}$$

in which W is the prespecified weighting matrix. As suggested in Cochrane (2009), a prespecified weighting matrix can force the estimation and evaluation to pay attention to economically interesting moments, in contrast to an optimal (or other) weighting matrix. In our context, we use several different specifications of the weighting matrix, which vary on the relative weights of the cross sectional and time series moments in the estimation. Accordingly, $W = [I, 0]$ indicates that we put identity weights on g^{XS} and zero weights on g^{TS} (we label this as Only XS in the tables below). This is a special case of our methodology identical to the GMM estimation in existing studies as in LWZ/BXZ. When $W = [0, I]$, we assign identity weights on g^{TS} and no weights on g^{XS} (we label this as Only TS in the tables below). More generally, when the weights on g^{XS} and g^{TS} are both positive, we can specify relatively more or less weights on g^{XS} over g^{TS}. It allows the estimation to match relatively more or less the cross-section or time-series features of the data.

3.3 Model evaluation

As noted, our goal is to evaluate the fit of the investment-based model on economic grounds, in particular, how well the model is able to capture the average cross sectional variation and the time series variation of stock returns across portfolios. Accordingly, most of our analyses focuses on first stage GMM estimates, and the evaluation of the model is based on the properties of easy to interpret measures such as cross sectional R^2 of predicted vs realized average returns of the portfolios, the model-implied time series R^2 for each portfolio, and the magnitude of the cross sectional and time series mean absolute pricing errors. (In the online appendix we show that our conclusions are similar if we use second stage GMM.)

Statistically speaking, as discussed in Cochrane (1991), the investment-based model should be rejected at any level of significance. The model predicts that stock returns and investment returns are equal at each point in time without any error term, which is not possible to achieve in the data. Nevertheless, we also perform standard statistical tests of the model. We assume that stock return and investment returns are observed with an error. The general distribution theory applies to GMM with prespecified weighting matrices (Cochrane 2009). Let $D = \partial g_T / \partial \theta$. We estimate S, a consistent estimate of the variance-covariance matrix of the sample errors g_T, with a standard Bartlett kernel with a lag length of two. The estimate of θ, denoted $\hat{\theta}$, is asymptotically normal with the variance-covariance matrix:

$$var\left(\hat{\theta}\right) = \frac{1}{T}\left(D'WD\right)^{-1} D'WSWD\left(D'WD\right)^{-1}. \tag{17}$$

To construct standard errors for individual model errors, we use:

$$var\left(g_T\right) = \frac{1}{T}\left[I - D\left(D'WD\right)^{-1} D'W\right] S \left[I - D\left(D'WD\right)^{-1} D'W\right]', \tag{18}$$

which is the variance-covariance matrix for g_T. We follow Hansen (1982) to form a χ^2 test on the null hypothesis that all of the model errors are jointly zero:

$$g_T' \left[var\left(g_T\right)\right]^{+} g_T \sim \chi^2 \left(\#moments - \#paras\right), \tag{19}$$

in which χ^2 denotes the chi-square distribution, and the superscript + denotes pseudo-inversion.[2]

4 Estimation and Tests Results

Section 4.1 describes the data. Section 4.2 reports the estimation and tests results in matching the cross sectional moments. Section 4.3 and 4.4 report the joint estimation and tests results of the cross sectional moments and the time-series moments as well as the joint estimation of the cross sectional moments and the variance moments, respectively. Section 4.5 explores and tests potential reasons for the poor time-series fit of the model.

4.1 Data

To facilitate the comparison with prior studies, we make minimal changes on the sample construction from LWZ. We download the publicly available data from LWZ and make only one adjustment. We measure the capital stock, K_{it}, as net property, plant, and equipment (PPE), as opposed to gross PPE, following more recent studies as in BXZ and many others. Net PPE is more consistent with the capital accumulation Equation (1), in which K_{it+1} is defined as net of capital depreciation, $\delta_{it}K_{it}$. LWZ use 30 testing portfolios: 10 book-to-market (BM) portfolios, 10 standardized unexpected earnings portfolios, and 10 corporate investment portfolios. To save space, we report our results with 10 BM portfolios. Our basic conclusions are similar if other testing portfolios are used (see online appendix).

[2]We also develop an external validity specification test of the model. In particular, we ask the estimation to match the cross sectional moments as closely as possible and evaluate how the fitted model matches the time series moments. Similar conclusions are obtained as from the economic fit of the model. More details on the statistical test and results are reported in the online appendix.

The estimation and tests are conducted at the portfolio level, to facilitate the analysis and to reduce the impact of noise in the data. We consider two alternative portfolio aggregation methods. The first aggregation method follows LWZ. Here, portfolio-level characteristics are computed as $I_{it+1}/K_{it+1} = \sum_{j=1}^{N} I_{ijt+1}/ \sum_{j=1}^{N} K_{ijt+1}$, $Y_{it}/K_{it} = \sum_{j=1}^{N} Y_{ijt}/ \sum_{j=1}^{N} K_{ijt}$, etc, where portfolio is indexed by i, and firm is indexed by j. We then plug in these characteristics in the formulas of the investment return to compute the portfolio-level investment return. The second aggregation method is based on a proper aggregation of the firm-level investment returns (following BGSV and GXZ). In particular, portfolio-level investment return is computed as an equal-weighted average of the firm-level investment return, $r_{it}^{I} = \frac{1}{N} \sum_{j=1}^{N} r_{ijt}^{I}$, to be consistent with the fact that portfolio-level stock return is also computed as an equal-weighted average of the firm level stock returns. To construct the firm-level investment return we need additional firm-level data items that are not provided in the LWZ sample (which only provides portfolio-level data). In the online appendix we describe in detail the firm-level data used to construct the firm-level investment return.

4.2 Matching the cross sectional moments

We first estimate the investment-based model using only the cross sectional moments given by Equation (9), as in LWZ. Column (1) in Table 2 reports the point estimates and overall performance of the model. The parameter estimates, capital share, α, and the adjustment cost parameter, c, are similar to those reported in LWZ. In addition, consistent with LWZ, the model does a good job in explaining the cross sectional variation in average returns of the 10 BM portfolios (value premium). The mean absolute cross sectional error, $\left|e_i^{XS}\right|$, given by Equation (10) is only 2.47% per annum. The mean absolute high-minus-low cross sectional error, $\left|e_{HML}^{XS}\right|$, is 1.13% per annum which is quite small. The $XS - R^2$ is the cross sectional R^2 of the average stock and investment returns of 10 portfolios. The $XS - R^2$ of 65% shows that the model is able to explain the value spread in the cross section. Finally, the χ^2 test examines the joint errors of 10 cross sectional moments, and the model is not rejected statistically with a p-value of 0.42.

In contrast with the good cross sectional fit, the model is unable to explain the time series variation of the stock returns of the 10 BM portfolios. Panel A in Table 1 shows that, although the mean and standard deviation of stock and investment returns match reasonably well, the stock and investment returns are negatively correlated for every portfolio, with an average time-series correlation of -0.19. This is in sharp contrast with the model prediction that stock returns and investment returns should be equal at each point in time without any error term, in which case the correlation between stock and investment return should be one. As a result, the stock returns and the residuals are strongly positively correlated in the time series (correlation is on average 0.83 across portfolios) which means that most of the time series variation of stock returns is captured by the residuals, not by the predicted investment returns.

Column (1) in Table 2 provides additional evidence for the poor time-series fit of the model. The mean absolute time series error, $\left|e_i^{TS}\right|$, given by Equation (12) is about 24% per annum. The $TS-R^2$ is the time series R^2, computed using the pooled stock and investment returns of 10 portfolios across time, and indicates a bad fit with a value of -86%.

We also perform a principal components analysis of the residuals. If the model is a good description of reality, the residuals should be small, largely random across portfolios and over time, and not exhibit a systematic behavior either across portfolios or over time. The results in the last column of Table 1 reveal that the residuals have a strong systematic component. Over 80% of the time series variation is due to the first principal component. This large systematic component in the residuals suggests that the time series fit is unlikely due to random noise in the data.

Figure 1 plots the stock returns of 10 BM portfolios at each point in time against the levered investment returns and the error terms, respectively. This figure illustrates in a clear manner the model's overall poor time series fit. If the model performs perfectly, all the observations should lie on the 45-degree line. However, the top left figure shows that the scattered points of stock and investment returns are largely random with a pooled correlation of -0.12. In sharp contrast with this pattern, the scattered points of stock returns and error terms on the top right are very well aligned along the 45-degree line, with a pooled correlation of 0.80. Thus, almost all of the time series variation in stock returns of each portfolio is explained by the model residual, not by the model-implied fitted investment return.

[Table 1 here]

[Figure 1 here]

4.3 Matching the cross sectional and the time series moments jointly

To improve the time series fit of the model, we now evaluate the fit of the model after adding the time series moments in the estimation using GMM.

Table 2 reports the results. We uncover a novel tradeoff between cross sectional fit and time series fit: the baseline investment-based model with one capital input cannot fit both sets of moments simultaneously. Figure 2 illustrates this tradeoff.

[Table 2 here]

[Figure 2 here]

As we increase the relative weight of the time series moments in the estimation from column (1) (zero weight on TS moments, all weight on XS moments) to column (8) (all weight on TS moments, zero weight on XS moments), the fit on the cross sectional moments deteriorates significantly: the mean absolute cross sectional error monotonically increases from 2.47% per annum when only cross sectional moments are used in the estimation (column 1), to 5.45% when only time series moments are used in the estimation (column 8), the mean absolute high minus low cross sectional error increases from 1.13% per annum (column 1) to 17.36% (column 8), and the cross sectional R^2 decreases from 65% (column 1) to -78%

(column 8). There is, as expected, an improvement in the fit on the time series: the mean absolute time series error monotonically decreases from 23.97% per annum (column 1) to 19.47% (column 8) and the time series R^2 increases from -86% (column 1) to -27% (column 8). Perhaps surprisingly, even when only the time series moments are used in the estimation (column 8), the model fit on the time series is still poor. Thus, the standard investment-based model with one capital input and quadratic adjustment costs is not able to capture the time series behavior of stock returns in the data even when the estimation is designed to maximize its time series fit.

4.4 Matching the cross sectional and the variance moments jointly

To compare our time series moments with the variance moments proposed in LWZ, we combine cross sectional moments and variance moments in the estimation using GMM.

Panel C in Table 3 reports the results. Unlike our time series moments, including the variance moments in the estimation do not improve the time series fit of the model. As we increase the weights of the variance moments in the estimation from column (11) (zero weights on variance moments) to column (15) (all weights on variance moments), the mean absolute time series error and the time series R^2 get slightly worse. The lack of improvement in the time series fit is not surprising because, as discussed in section 3.1, the variance moments in LWZ are not designed to improve the time series fit. Specifically, matching variance does not take into account the time-series correlation between stock returns and investment returns. As an extreme example, take two time series with the same variance but that move in opposite direction to each other. Thus, the variance moments are not a powerful test of the model prediction that stock and investment returns should be equated at each point in time.

[Table 3 here]

4.5 Potential empirical reasons for the poor time series fit

In this subsection, we investigate two potential empirical reasons for the poor fit of the baseline model in the time series, despite its success in the cross section. Here, we focus on empirical reasons, and not on theoretical reasons, given that we only tested one version of the investment-based model: one physical capital input and quadratic adjustment costs. The set of alternative specification of the investment-based models is potentially infinite. The previous results should be interpreted as documenting the poor time series fit of the baseline specification of the model, and not a general test and rejection of this class of models. Nevertheless, the methodology we propose here can be applied to any specification of the investment-based model, as we discuss in more detail below.

4.5.1 Aggregation bias

First, we investigate the role of portfolio-level aggregation for the results. As noted in BGSV and GXZ, the portfolio aggregation procedure in LWZ suffers from an aggregation bias. Specifically, in this approach, the portfolio-level investment return is computed by first computing the portfolio-level characteristics (e.g., the portfolio-level investment rates), and then plugging these aggregate characteristics directly in the investment return formula. Given the nonlinearity of the investment returns, the portfolio-level investment return obtained using this procedure is no longer equal to a value- or equal-weighted portfolio stock return. As a result, the estimatio does not recover the structural parameters. For a proper aggregation, the investment return should be first computed for each firm, and the portfolio-level investment return should be then computed as the value- or equal-weighted average of the firm-level investment returns (and matched with the corresponding value- or equal-weighted portfolio stock return).

To investigate if the aggregation bias induced by the LWZ portfolio-level aggregation can explain the poor fit of the investment-based model in the time series we perform two sets of analysis. First, we use simulated data in which the model holds at the firm-level (the firm-level stock return and investment return are equal), and evaluate the impact of portfolio-aggregation error. Second, we do a proper portfolio-level aggregation in the real data and investigate the results.

The impact of aggregation bias using artificial data

We generate artificial data from a model economy in which the assumptions of the baseline investment-based model hold (and hence the stock and investment return equality holds). To generate the data in a simple manner, we use the real data on firm-level characteristics (investment rate, sales-to-capital, etc) as inputs to construct model-implied investment return. We set the true model parameter values for capital share as $\alpha = 0.05$, and the adjustment cost parameter as $c = 5$, and compute the artificial model-implied firm-level investment return using the real data as in equation (5). To generate firm-level stock return data in this economy, we then use the stock and investment return equality equation implied by the neoclassical model as in Equation (7). Thus, by construction, the observed stock return and the model-implied investment return are equal at each point in time. As in the empirical analysis, we create 10 BM portfolios which we use to replicate the GMM estimation in the empirical analysis. The parameter values used generate a value premium in the artificial data that is similar to that in the real data, as reported in Panel A in Table 4.[3]

To examine the impact of aggregation on the evaluation of the model, we replicate the LWZ portfolio aggregation method using the artificial data. Panel A in Table 5 confirms the aggregation bias in the parameter estimates, consistent with BGSV: the estimation fails to recover the true parameter values. Another evidence of the aggregation bias is that the parameter estimates vary with the set of moments used (column 1 to column 5). The aggregation bias is also able to break the perfect correlation between stock return and investment return: Panel A in Table 5 shows that the maximum TS-R2 that the model can achieve is

[3] In untabulated results, we confirm that GMM with a correctly specified model (homogeneous of degree one) and with proper firm-level aggregation recovers the true model parameters. In addition, the model fit is perfect, and using cross sectional moments or time series moments in the estimation makes no difference in the estimation and evaluation of the model.

0.14. While this number is not negative as in the data, it is substantially smaller than one, giving some hope that the aggregation bias alone might be an important contributor for the poor empirical time series fit of model. But when we look at correlations, the evidence is less supportive. Panel A in Table 4 reports that the average time series correlation between stock returns and investment returns across portfolios remains significantly higher than in the data, around 0.50 here versus -0.19 in the real data.

The results using the artificial data also show that our time series moments are more powerful to detect aggregation error. When estimating and testing only cross sectional moments as in column (1) Table 5, the mean absolute cross sectional error and mean absolute high-minus-low cross sectional error are both low, the XS R^2 is very high, 98%, and the model is not rejected with a p-value on testing the joint errors of 10 cross sectional moments of 0.99. In contrast, when estimating and testing with time series moments as in column (5) Table 5, both the cross sectional fit and time series fit are poor, and the p-value on testing the joint errors of 10 time series moments is only 0.13, indicating a poor model fit stemming from, in this case, aggregation bias.

[Table 4 here]

[Table 5 here]

The impact of aggregation bias using real data

We now investigate the model fit in the real data using a proper portfolio-aggregation and hence avoiding the aggregation bias. Panel C in Table 1 and Panel B in Table 3 report the estimation results from this analysis. Although the time series fit of the model improves significantly with the correct aggregation, it remains poor (e.g., time series R^2 of -26% when only time series moments are targeted and average time series correlation of -0.06).

Taken together, the results from these analyses suggest that portfolio-level aggregation issues do not appear to be the main cause for the inability of the model to explain the time series behavior of stock returns.

4.5.2 Misalignment between asset price data and quantities

As a second possible empirical cause for the poor time series fit of the baseline investment-based model, we address the possibility that quantities (e.g. investment) and asset prices are misaligned in the real data. For example, stock prices (and hence stock returns) might respond instantaneously to aggregate shocks, whereas investment might take more time to adjust, in which case investment returns lag stock returns (see, for example, Lamont 2000, for a more formal analysis of this issue). However, if the misalignment in the data is relatively short lived, the misalignment should be less pronounced at longer-horizon returns. Thus, we conjecture that compounded investment and stock returns should be significantly less affected by data misalignment issues than one year horizon returns.

The impact of data misalignment using artificial data

We use artificial data to formally test the conjecture that compounded (here, using a five-year horizon) investment and stock returns should be significantly less affected by the data misalignment issues than annual return data.

As in the previous section, we first generate artificial data from a specification of the model where the equality between investment and stock returns holds (using the same parameter values as in the previous section), and introduce misalignment in estimation. To evaluate the impact of data misalignment, we estimate the model using annual returns and compounded returns, and compare the results. To properly identify the impact of misalignment on the results, we estimate the model at the portfolio-level using the proper aggregation method. In this case, the only reason for the imperfect time series fit of the model is due to data misalignment, and not by the portfolio-level aggregation bias.

We introduce misalignment in the artificial data as follows. If investment return lags stock returns, in the real data, we should match the more timely stock return data with stale investment return. Thus, in artificial data, we purposely estimate and test the model by matching stock return at $t+1$ (r^S_{it+1}) with lagged investment return at t (r^{Iw}_{it}) in both the cross section and in the time series.

Panel B in Table 4 and Table 5 shows that the data misalignment alone breaks significantly the perfect time series fit of the model. The average time series correlation between stock and investment return is 0.17, the mean absolute time series error becomes sizable, and the time series R^2 turns negative, consistent with the data. This suggest that data misalignment is potentially a good explanation for the empirical poor time series fit of the model using annual returns.

As noted, in the presence of data misalignment, using compounded returns in the estimation should mitigate the impact of the misalignment and improve the time series fit (i.e. increase the correlation between compounded stock returns and investment returns). To verify this conjecture, we now perform the estimation of the model using 5-year annualized compounded stock and investment returns in the time series moments. Panel C in Table 4 and Table 5 report the results. The average time series correlation between stock and investment return increases to 0.83, the mean absolute time series error drops significantly, and the time series R^2 is above 80%. Taken together, this result suggests that the issues introduced by data misalignment should be significantly mitigated if we estimate the model using compounded returns, that is, if we focus on the model-implied relationship between stock and investment returns at longer horizons.

The impact of data misalignment using real data

In light of the previous analysis using artificial data, we now investigate the time series fit of the investment-based model when we use 5-year compounded (instead of annual) returns in the real data. Table 1 Panel B reports the results. When we use long horizon compounded returns in the estimation, the average time series correlation between stock and investment returns indeed increases from -0.19 when annual returns are used to 0.16. Similarly, Table 3 Panel A shows that the mean absolute time series error and time series R^2 improve compared with annual returns. Despite this significant improvement, the time series fit of the model is far from satisfactory. The (average) time series R^2 remains very low at -20 as reported in Table 3 column (5) when only the time series moments are used in the estimation. Taken together, the results from this analysis suggest the misalignment between price and investment data (at least the type of misalignment examined here) also does not appear to be the main cause for the inability of the model to explain the time series behavior of stock returns.

5 Specification-free Tests

The analysis so far has been based on the specification of the investment-based model in which the Hayashi (1982) conditions hold (i.e. homogeneous of degree one operating profit and adjustment cost functions), and hence, stock returns and levered investment returns should be equal (Equation (7)). This result underlies the moment conditions in Section 3.1 used to estimate and test the investment-based model. Although the Hayashi conditions are strong assumptions and should be more properly interpreted as an approximation of the reality, it is natural to question the validity of this assumption, especially in light of the poor time series fit of the baseline investment-based model reported in the previous sections.

Here, we propose a general way to evaluate the performance of investment-based asset pricing models that can be used in practice even when the Hayashi conditions do not hold. We label this more general method as a specification-free approach to evaluate investment-based models because it does not rely so heavily on functional form assumptions. As a result, this approach is useful for a broader set of specifications of the investment-based model.

The Hayashi conditions might not hold due to several reasons such as, for example, decreasing returns to scale technology, nonconvex adjustment cost, or fixed operating costs. In this case, the stock return and investment return equality does not hold, and the moment conditions used in the previous sections are not valid for the estimation and evaluation of the model.

Most investment-based asset pricing models, however, imply a relationship between stock returns and firm characteristics X_{it+1} of the firm. For example, we can investigate this relationship using a simple linear regression of the form:

$$r^S_{it+1} = \beta X_{it+1}, \tag{20}$$

which we can estimate in both the time series (using the time series of portfolio returns and portfolio-level characteristics to reduce the noise in firm-level data) and in the cross section (using the average portfolio returns against the average portfolio-level characteristics). For example, in the baseline neoclassical model in which the Hayashi conditions hold, the key firm characteristics X_{it+1} in the investment return (Equation (5)) include current profitability (marginal product of capital), and current and lagged investment rate. In other investment-based models, the key relevant firm-level characteristics might be different and model specific, but as long as the firm characteristics can be measured in both the real data and in the model, we can evaluate if the model generates a relationship between stock returns and firm characteristics that is consistent with the data. A successful investment-based model should have estimation results inside the model that are consistent with the real data both in terms of slope coefficients in the regression, and in terms of goodness of fit of the regression.

Similar arguments have been made in the investment-q literature. Eberly, Rebelo, and Vincent (2008) show that when Hayashi conditions do not hold thus q no longer serves as a sufficient statistic for investment, optimal investments from a model featuring decreasing return to scale and a fixed cost can still be very closely approximated by a log-linear function of q. Gala, Gomes, and Liu (2020) show that even under very general assumptions about the nature of markets, production and investment technologies, optimal investments are functions of and well captured by the relevant state variables such as firm size and productivity.

To illustrate how this approach can be used to evaluate investment-based models, we estimate the time series and cross sectional relationships between stock returns and firm characteristics defined in equation 20 in the the real data, and also using simulated data from two different calibrated versions of the investment-based models. Thus, here, we are using these stock return-firm characteristics regressions as a way to evaluate the calibrated investment-based models. More generally, however, the stock return-firm characteristics relationship can also be used to estimate the model parameters as well. Given that in general we do not have closed form expressions for the equilibrium stock returns in the specifications of the investment-based model in which the Hayashi conditions do not hold, the estimation can be done using the Simulated Method of Moments (SMM), using the stock return-firm characteristics regression estimated in the data as a set of target moments. This estimation is outside the scope of our paper, however. Our goal here is to show how future research can incorporate the time series and cross sectional implications of investment-based models in order to do a proper evaluation and testing of these models.

We consider two specifications of the investment-based model. The first specification is the standard neoclassical investment-based model where, as in the previous sections, we assume that the Hayashi conditions hold, in which case stock returns and investment returns are equal. We generate data from this model in the same way as in the previous Section 4.5. This specification of the model is useful here because we already know that in this model stock returns and firm characteristics are closely linked by Equation (7), and this strong relationship should be preserved in the stock return-firm-characteristics regressions.

The second model is an off-the-shelf investment-model based on Lin and Zhang (2013) where the Hayashi conditions do not hold. Specifically, a firm i's operating profit function features decreasing returns to scale and a positive fixed cost:

$$\Pi_{it} = X_t Z_{it} K_{it}^{\alpha} - f, \tag{21}$$

in which $0 < \alpha < 1$ is the curvature parameter, and $f > 0$ is a positive fixed cost, capturing the existence of fixed outside opportunity costs each period. X_t and Z_{it} are aggregate and idiosyncratic productivity shocks respectively. Capital investment entails the following adjustment costs:

$$\Phi\left(I_{it},K_{it}\right)=\begin{cases}a^{+}K_{it}+\frac{c^{+}}{2}\left(\frac{I_{it}}{K_{it}}\right)^{2}K_{it} & I_{it}>0\\ 0 & I_{it}=0\\ a^{-}K_{it}+\frac{c^{-}}{2}\left(\frac{I_{it}}{K_{it}}\right)^{2}K_{it} & I_{it}<0\end{cases}, \tag{22}$$

where $a^{-} > a^{+} > 0$ and $c^{-} > c^{+} > 0$ capture nonconvex and asymmetric adjustment costs. Nonconvex part captures the cost independent of the size of investment. Convex part captures higher cost for more rapid changes. Asymmetric part captures costly reversibility. Firms face higher costs in contracting than in expanding.

Because this model does not satisfy the Hayashi conditions, the model-implied stock returns do not equal investment returns. We label it as non-homogeneous of degree one model, or non-HD1 model. To generate simulated data from this model, we calibrate the model as in Lin and Zhang (2013) to match average quantities and asset prices moments both in the aggregate and cross section, including the value premium.[4]

Table 6 reports the regression results in the real data, and using simulated data from the two investment-based models. We assess the relationship between stock returns and the following firm characteristics: profitability ($YK_{it+1} \equiv \frac{Y_{it+1}}{K_{it+1}}$), investment growth ($\triangle IK_{it+1} \equiv \frac{I_{it+1}}{K_{it+1}}/\frac{I_{it}}{K_{it}} - 1$), size ($K_{it} \equiv logK_{it}$), and lagged investment rate ($IK_{it} \equiv \frac{I_{it}}{K_{it}}$) over time and in the cross section, in which case we average the characteristics over time for each portfolio. We normalize both the dependent and the independent variables (using pooled data). Consistent with the previous analysis, we run the regressions at the portfolio-level (using 10 BM portfolios) to reduce noise in the firm-level data.

[Table 6 here]

Columns (1) and (2) in Table 6 report the results using the simulated data from the neoclassical investment-based model in which the Hayashi conditions hold (homogeneous of degree 1, HD1). As discussed before, this model predicts that stock returns and investment

[4]In the online appendix we provide a detailed description of the model, and its calibration.

returns are equal state-by-state, and hence the model implies a tight relationship between stock returns and firm-characteristics, both in the cross section and in the time series. This tight relationship is confirmed in these regressions by the high cross sectional R^2 (98%), and time series R^2 (86%). So, the simple linear functional form relationship preserves the model implied strong link between stock returns and characteristics.

Columns (3) and (4) report the results using simulated data from the non-homogeneous of degree one Lin and Zhang (2013) model.[5] Interestingly, even though this model does not predict the equality between stock returns and investment returns, this model also implies that stock returns and firm-characteristics are highly correlated, both in the cross sectional (R^2 of 93%), and in the time series (R^2 of also 93%). This result suggests that the strong link between stock returns and firm characteristics appears to be a more general feature of investment-based models, and it is not specific to the specification of the model in which the Hayashi condition holds.

The next step in the evaluation of the models is to compare the stock return-firm-characteristic regression results in the model, with those in the real data. Columns (9) and (10) in Table 6 report the regression results in the real data. Consistent with both the baseline neoclassical investment-based model and the non-homogeneous of degree one Lin and Zhang (2013) model, the cross sectional fit in the real data is quite high, about 98. But the time series fit of the stock return-firm characteristics regression in the real data is very poor, with a time series R^2 of about 3%, and the coefficients on the firm-characteristics are mostly insignificant. Thus, this regression shows that once again none of the two specifications of the investment-based model considered here can match the observed time series relationship between stock returns and firm characteristics.

We also investigate the role of data misalignment between price and investment data, on the time series relationship between stock returns and firm characteristics. Analogous to the analysis in Section 4.5, Column (5) and (6) show the regression results when we introduce

[5]Lagged investment rate is not included due to its high correlation with profitability in the simulated data.

misalignment in simulated data from the baseline investment-based model in which the Hayashi conditions hold. Indeed, the time series R^2 drops from 86% to 31%, thus getting closer to data, which has a time series R^2 of 3% as reported in column (10). We then replicate the regressions using long-horizon relationship to mitigate the impact of data misalignment. Column (7) and (8) show that when we use annualized 5-year compounded returns and 5-year average characteristics, the time series R^2 improves significantly to 84%. However, when we replicate the same regression in the real data, Column (11) and (12) show that the time series R^2 in the long horizon data is still very low, about 23%. Again, these results are consistent with the analysis in Section 4.5 (the long horizon), and show that the linear stock return-firm characteristics specification proposed here captures in a simple and consistent way the relationship between these variables in both the model and the real data.

Taken together, the results here confirm the main finding of the paper that the standard investment-based model fails on the time series prediction. In contrast to the previous newly proposed method in this paper, the specification-free method does not need stock and investment return equality as moment condition to estimate and evaluate the model. Therefore, this method can be used for a broader set of investment-based models in at least two ways.

First, it can be simply used as an external validity test as we do here. For any investment model, researchers can do the standard calibration matching the first and second moments and achieve a good fit on them. As a way to evaluate the model on the time series fit, researchers can implement this method to assess the relationship between stock returns and characteristics over time, and the model should match the empirical relationship estimated in the data.

Second, the empirical relationship between stock returns and characteristics can be incorporated in estimating the structural parameters of any investment model as a moment condition with simulated method of moments. Thus, the method picks the model parameters that make the actual and simulated moments as close to each other as possible.

6 Conclusion

This paper proposes a new set of moments for the estimation and testing of the standard investment-based model. As in nonlinear least squares, these moments are based on the average squared sum of the residuals of each portfolio, and hence capture the time series implications of the model. Our results show that the standard investment-based model with one-capital input and quadratic adjustment costs that is very successful at capturing the cross sectional variation in average stock returns across several portfolio sorts, is unable to capture the time series variation in stock returns: the model generates a time series R^2 that is negative or close to zero. We also show how our approach can be extended to specifications of investment-based models in which the Hayashi conditions do not hold, and show that the poor time series fit is also present in a specification of the investment-based model with decreasing returns to scale, non-convex adjustment costs, and fixed cost.

Our findings have implications for future research. Because we only test two specifications of the investment-based model, our findings do not mean that the investment-based paradigm cannot match the time series data well. Our findings do mean, however, that a different specification of the model is needed to capture the time series dimension of the stock returns in the real data, which we argue is an important dimension to match. To help the fit of the class of investment-based models in the time series, additional capital inputs (intangible capital and physical capital) and labor inputs as in BGSV, or short-term and long-term assets as in GXZ, can be added to the analysis. In addition, different functional forms of adjustment costs, or explicitly accounting for firm- or industry-level heterogeneity in the technologies, which are assumed to be similar across firms in the baseline model, should be investigated. Taken together, by incorporating the time series and cross sectional implications of the model explicitly into the structural estimation of the investment-based model, our methodology can be useful to detect model misspecifications and hence help improve the specification of investment-based models in future research.

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Table 1. Description of stock returns, investment returns, and errors

This table reports the mean, standard deviation (Std), time-series correlation for stock returns (r^S_{it}), investment returns (r^{Iw}_{it}), and errors (ϵ_{it}). Errors are computed as $\epsilon_{it} = r^S_{it} - r^{Iw}_{it}$. Investment returns are based on estimation results from one-step GMM on the cross sectional moments given by Equation (9), using BM deciles as the testing portfolios. In Panels A and B, aggregation is at the portfolio level as in LWZ, and in Panel C, aggregation is based on firm-level investment return. In Panel B, stock and investment returns are annualized compounded 5-year returns. Weighting matrix is an identity matrix. Last column shows the percent variability explained by each one of the first three principal components of the errors. Out of 10 portfolios, we report results for 1, 5, 10, and the average of all portfolios to save space.

	Mean			Std			TS Correlation			PCA
Portfolio	r^S_{it}	r^{Iw}_{it}	ϵ_{it}	r^S_{it}	r^{Iw}_{it}	ϵ_{it}	(r^S_{it}, r^{Iw}_{it})	$(r^S_{it}, \epsilon_{it})$	$(r^{Iw}_{it}, \epsilon_{it})$	ϵ_{it}
	Panel A: Annual returns									
1	0.09	0.13	-0.04	0.28	0.15	0.34	-0.21	0.91	-0.60	80.02
5	0.18	0.15	0.03	0.25	0.15	0.32	-0.20	0.89	-0.63	7.68
10	0.26	0.29	-0.03	0.27	0.33	0.44	-0.05	0.65	-0.79	3.79
Avg	0.18	0.18	0.00	0.25	0.19	0.34	-0.19	0.83	-0.69	-
	Panel B: Compounded 5-year returns (annualized)									
1	0.05	0.09	-0.05	0.08	0.06	0.10	0.05	0.78	-0.58	66.08
5	0.15	0.13	0.02	0.09	0.05	0.09	0.36	0.81	-0.26	19.38
10	0.23	0.26	-0.04	0.10	0.25	0.24	0.27	0.13	-0.92	6.77
Avg	0.15	0.15	0.00	0.09	0.10	0.13	0.16	0.61	-0.63	-
	Panel C: Firm-level aggregation									
1	0.10	0.13	-0.03	0.25	0.05	0.27	-0.17	0.98	-0.36	83.69
5	0.16	0.15	0.02	0.24	0.08	0.26	-0.16	0.95	-0.45	7.49
10	0.25	0.27	-0.02	0.27	0.24	0.35	0.09	0.72	-0.62	3.30
Avg	0.17	0.16	0.00	0.24	0.09	0.26	-0.06	0.92	-0.40	-

Table 2. GMM estimation and tests of the investment-based model

This table reports the one-step GMM results from estimating jointly the cross sectional moments and the time series moments given by Equation (9) and (11) respectively, using BM deciles as the testing portfolios. Each column differs in the prespecified weighting matrix, in which the first component refers to the weights on the cross sectional moments and the second component refers to the weights on the time series moments. α is the capital share and c is the adjustment cost parameter. The t-statistics, denoted $[t]$, test that a given parameter equals zero. $\left|e_i^{XS}\right|$ is the mean absolute cross sectional errors given by Equation (10). $\left|e_{H-L}^{XS}\right|$ is the mean absolute high-minus-low cross sectional errors. $\left|e_i^{TS}\right|$ is the mean absolute time series errors given by Equation (12). $XS - R^2$ is the cross sectional R^2. $TS - R^2$ is the time series R^2. χ^2, d.f., and p are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero. $\left|e_i^{XS}\right|$, $\left|e_{H-L}^{XS}\right|$, and $\left|e_i^{TS}\right|$ are expressed as a percentage per annum. Aggregation is at the portfolio level.

	Only XS	Both XS and TS Moments						Only TS
Column:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Weights:	$[I\ 0]$	$[I\ 0.1]$	$[I\ 0.3]$	$[I\ 0.5]$	$[I\ I]$	$[I\ 2]$	$[I\ 10]$	$[0\ I]$
	Parameter estimates							
α	0.23	0.19	0.16	0.15	0.14	0.13	0.12	0.11
$[t]$	2.74	4.57	6.04	6.59	6.93	6.90	6.55	4.13
c	8.43	4.49	2.35	1.50	0.60	0.01	-0.55	-0.71
$[t]$	1.16	1.28	1.02	0.77	0.36	0.01	-0.42	-0.42
	Goodness of fit							
$\left\|e_i^{XS}\right\|$	2.47	2.49	2.94	3.31	3.91	4.43	5.14	5.45
$\left\|e_{H-L}^{XS}\right\|$	1.13	4.72	7.92	9.66	11.98	13.93	16.41	17.36
$\left\|e_i^{TS}\right\|$	23.97	22.15	20.97	20.50	20.05	19.79	19.55	19.47
$XS - R^2$	0.65	0.58	0.40	0.26	0.04	-0.20	-0.57	-0.78
$TS - R^2$	-0.86	-0.61	-0.45	-0.39	-0.32	-0.29	-0.27	-0.27
χ^2	8.15	13.53	13.33	13.22	13.24	13.41	13.70	8.86
d.f.	8.00	18.00	18.00	18.00	18.00	18.00	18.00	8.00
p	0.42	0.76	0.77	0.78	0.78	0.77	0.75	0.35

Table 3. GMM estimation and tests of the investment-based model across alternative specifications

This table reports the one-step GMM results across alternative specifications from estimating jointly the cross sectional moments and the time series moments given by Equation (9) and (11) respectively in Panel A and B, and estimating jointly the cross sectional moments and the variance moments given by Equation (9) and (14) respectively in Panel C, using BM deciles as the testing portfolios. Each column differs in the prespecified weighting matrix, in which the first component refers to the weights on the cross sectional moments and the second component refers to the weights on the time series moments. α is the capital share and c is the adjustment cost parameter. The t-statistics, denoted $[t]$, test that a given parameter equals zero. $\left|e_i^{XS}\right|$ is the mean absolute cross sectional errors given by Equation (10). $\left|e_{H-L}^{XS}\right|$ is the mean absolute high-minus-low cross sectional errors. $\left|e_i^{TS}\right|$ is the mean absolute time series errors given by Equation (12). $XS-R^2$ is the cross sectional R^2. $TS-R^2$ is the time series R^2. χ^2, d.f., and p are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero. $\left|e_i^{XS}\right|$, $\left|e_{H-L}^{XS}\right|$, and $\left|e_i^{TS}\right|$ are expressed as a percentage per annum. In Panels A and C, aggregation is at the portfolio level as in LWZ, and in Panel B, aggregation is based on firm-level investment return. In Panel A, stock and investment returns are annualized compounded 5-year returns.

	Panel A: Compounded 5-year returns					Panel B: Firm-level aggregation					Panel C: Variance moments				
	Only XS	Both XS and TS			Only TS	Only XS	Both XS and TS			Only TS	Only XS	Both XS and Variance			Only Variance
Column:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Weights:	$[I\ 0]$	$[I\ 0.1]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$	$[I\ 0]$	$[I\ 1]$	$[I\ 10]$	$[I\ 40]$	$[0\ I]$	$[I\ 0]$	$[I\ 100]$	$[I\ 500]$	$[I\ 1000]$	$[0\ I]$
	Parameter estimates														
α	0.27	0.26	0.21	0.16	0.12	0.06	0.06	0.06	0.06	0.06	0.23	0.22	0.21	0.21	0.21
$[t]$	2.00	2.20	3.35	7.00	10.72	10.70	10.75	12.40	13.21	5.49	2.74	3.69	3.21	2.91	2.33
c	11.89	11.00	7.46	2.91	0.56	-0.37	-0.08	0.30	0.38	0.41	8.43	8.68	9.08	9.20	9.35
$[t]$	1.14	1.21	1.51	1.66	0.65	-0.97	-0.14	0.50	0.79	0.63	1.16	2.15	2.24	2.27	2.28
	Goodness of fit														
$\left\|e_i^{XS}\right\|$	2.83	2.84	2.89	3.36	4.60	1.13	1.42	2.60	3.10	3.35	2.47	2.73	3.86	4.23	4.84
$\left\|e_{H-L}^{XS}\right\|$	1.14	1.55	3.71	8.84	14.29	0.75	1.68	2.54	2.24	2.03	1.13	2.52	3.96	4.41	5.03
$\left\|e_i^{TS}\right\|$	9.74	9.61	9.00	7.95	7.40	20.11	19.69	19.31	19.22	19.18	23.97	23.99	24.06	24.09	24.12
$XS-R^2$	0.60	0.60	0.58	0.35	-0.12	0.90	0.84	0.53	0.32	0.20	0.65	0.52	0.07	-0.12	-0.44
$TS-R^2$	-0.85	-0.80	-0.58	-0.27	-0.20	-0.26	-0.23	-0.22	-0.22	-0.22	-0.86	-0.87	-0.89	-0.89	-0.91
χ^2	12.14	13.16	13.29	13.30	12.11	4.30	12.82	12.68	12.76	10.38	8.15	11.93	11.89	11.87	8.62
d.f.	8.00	18.00	18.00	18.00	8.00	8.00	18.00	18.00	18.00	8.00	8.00	18.00	18.00	18.00	8.00
p	0.14	0.78	0.77	0.77	0.15	0.83	0.80	0.81	0.81	0.24	0.42	0.85	0.85	0.85	0.37

Table 4. Simulation: Description of stock returns, investment returns, and errors

This table reports the mean, standard deviation (Std), time-series correlation for stock returns (r_{it}^{S}), investment returns (r_{it}^{Iw}), and errors (ϵ_{it}) from simulated data. Errors are computed as $\epsilon_{it} = r_{it}^{S} - r_{it}^{Iw}$. Investment returns are based on estimation results from one-step GMM on the cross sectional moments given by Equation (9), using BM deciles as the testing portfolios. In Panels A, aggregation is at the portfolio level as in LWZ. In Panel B and C, aggregation is based on firm-level investment return, and stock returns are matched with lagged investment returns. In Panel C, stock and investment returns are annualized compounded 5-year returns. Weighting matrix is an identity matrix. Last column shows the percent variability explained by each one of the first three principal components of the errors. Out of 10 portfolios, we report results for 1, 5, 10, and the average of all portfolios to save space.

	Mean			Std			TS Correlation			PCA
Portfolio	r_{it}^{S}	r_{it}^{Iw}	ϵ_{it}	r_{it}^{S}	r_{it}^{Iw}	ϵ_{it}	$(r_{it}^{S}, r_{it}^{Iw})$	$(r_{it}^{S}, \epsilon_{it})$	$(r_{it}^{Iw}, \epsilon_{it})$	ϵ_{it}
	Panel A: Aggregation bias									
1	0.07	0.06	0.01	0.12	0.19	0.15	0.63	0.01	-0.77	49.29
5	0.09	0.08	0.00	0.15	0.19	0.13	0.71	0.12	-0.62	17.56
10	0.28	0.27	0.00	0.30	0.55	0.52	0.38	0.16	-0.85	8.46
Avg	0.12	0.12	0.00	0.16	0.27	0.24	0.50	0.12	-0.80	-
	Panel B: Misalignment									
1	0.07	0.07	0.00	0.13	0.12	0.13	0.40	0.59	-0.50	53.60
5	0.09	0.08	0.00	0.15	0.13	0.18	0.17	0.68	-0.60	15.50
10	0.28	0.28	-0.01	0.30	0.29	0.41	0.02	0.71	-0.69	9.36
Avg	0.12	0.12	0.00	0.16	0.15	0.20	0.17	0.67	-0.61	-
	Panel C: Compounded returns									
1	0.06	0.06	0.00	0.08	0.08	0.04	0.91	0.19	-0.24	57.00
5	0.07	0.07	0.00	0.07	0.07	0.04	0.79	0.31	-0.34	13.94
10	0.26	0.25	0.00	0.17	0.17	0.08	0.90	0.24	-0.21	9.04
Avg	0.11	0.11	0.00	0.08	0.08	0.05	0.83	0.28	-0.29	-

Table 5. Simulation: GMM estimation and tests of the investment-based model

This table reports the one-step GMM results across alternative specifications using simulated data from estimating jointly the cross sectional moments and the time series moments given by Equation (9) and (11) respectively, using BM deciles as the testing portfolios. Each column differs in the prespecified weighting matrix, in which the first component refers to the weights on the cross sectional moments and the second component refers to the weights on the time series moments. α is the capital share and c is the adjustment cost parameter. The t-statistics, denoted $[t]$, test that a given parameter equals zero. $\left|e_i^{XS}\right|$ is the mean absolute cross sectional errors given by Equation (10). $\left|e_{H-L}^{XS}\right|$ is the mean absolute high-minus-low cross sectional errors. $\left|e_i^{TS}\right|$ is the mean absolute time series errors given by Equation (12). $XS-R^2$ is the cross sectional R^2. $TS-R^2$ is the time series R^2. χ^2, d.f., and p are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero. $\left|e_i^{XS}\right|$, $\left|e_{H-L}^{XS}\right|$, and $\left|e_i^{TS}\right|$ are expressed as a percentage per annum. In Panels A, aggregation is at the portfolio level as in LWZ. In Panel B and C, aggregation is based on firm-level investment return, and stock returns are matched with lagged investment returns. In Panel C, stock and investment returns are annualized compounded 5-year returns.

	Panel A: Aggregation bias					Panel B: Misalignment					Panel C: Compounded returns				
	Only XS	Both XS and TS			Only TS	Only XS	Both XS and TS			Only TS	Only XS	Both XS and TS			Only TS
Column:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Weights:	$[I\ 0]$	$[I\ 0.1]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$	$[I\ 0]$	$[I\ 0.1]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$	$[I\ 0]$	$[I\ 0.1]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$
	Parameter estimates														
α	0.68	0.29	0.16	0.13	0.13	0.05	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05
$[t]$	0.55	2.19	5.83	6.24	3.70	8.26	20.85	18.62	13.76	5.46	31.46	31.65	32.83	34.79	11.48
c	72.12	21.18	6.18	2.42	1.84	4.40	2.14	0.59	-0.07	-0.22	5.22	5.21	5.05	4.37	3.51
$[t]$	0.47	1.48	2.24	0.98	0.42	2.15	1.49	0.82	-0.14	-0.26	8.62	8.61	8.45	6.92	2.62
	Goodness of fit														
$\left\|e_i^{XS}\right\|$	0.77	1.20	2.63	3.64	3.88	0.26	0.57	1.27	1.92	2.22	0.08	0.08	0.07	0.15	0.30
$\left\|e_{H-L}^{XS}\right\|$	0.66	2.87	8.02	11.43	12.09	1.02	0.31	2.47	5.60	6.54	0.19	0.18	0.12	0.08	0.32
$\left\|e_i^{TS}\right\|$	17.99	15.19	12.17	11.26	11.25	15.21	13.73	12.90	12.96	13.06	3.67	3.66	3.63	3.51	3.45
$XS-R^2$	0.98	0.94	0.75	0.53	0.47	1.00	0.99	0.94	0.86	0.82	1.00	1.00	1.00	1.00	1.00
$TS-R^2$	-1.11	-0.49	0.04	0.15	0.14	-0.43	-0.17	-0.04	-0.03	-0.04	0.80	0.80	0.81	0.82	0.82
χ^2	1.59	12.26	13.86	14.37	12.52	0.30	13.38	13.45	13.35	12.14	0.29	12.08	12.09	12.01	11.14
d.f.	8.00	18.00	18.00	18.00	8.00	8.00	18.00	18.00	18.00	8.00	8.00	18.00	18.00	18.00	8.00
p	0.99	0.83	0.74	0.70	0.13	1.00	0.77	0.76	0.77	0.15	1.00	0.84	0.84	0.85	0.19

Table 6. Specification-free test

This table reports the specification-free test results by estimating the relationship between stock returns and characteristics given by Equation (20) both in the cross section and time series, using BM deciles as the testing portfolios. Column (1) and (2) use simulated data from a standard homogeneous of degree one (HD1) investment model. Column (3) and (4) use simulated data from a non-homogeneous of degree one model. Column (5) and (6) introduce misalignment in simulated data from the HD1 model. Column (9) and (10) use real data. Column (7) and (8), (11) and (12) use annualized compounded 5-year returns and 5-year average characteristics in simulated data from the HD1 model and real data respectively. Aggregation is based on correct firm-level aggregation.

	Model								Data			
	HD1 Annual		non-HD1 Annual		HD1 Misalignment		HD1 Long-horizon		Annual		Long-horizon	
	XS	TS	XS	TS	XS	TS	XS	TS	XS	TS	XS	TS
Column:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
YK_{it+1}	-0.26	0.42	0.39	0.42	0.30	0.55	0.72	0.63	0.52	0.15	1.47	0.28
$[t]$	-1.02	17.25	3.89	22.80	0.38	9.98	1.88	23.03	5.75	2.31	1.82	4.35
$\triangle IK_{it+1}$	0.88	0.72	0.93	0.97	0.77	-0.02	0.60	0.49	-0.33	-0.09	-0.22	0.23
$[t]$	11.98	21.94	3.99	33.75	5.46	-0.35	8.75	13.15	-8.80	-0.92	-3.84	3.19
K_{it}	0.14	0.08	-0.01	-0.01	0.16	-0.01	0.19	0.06	-0.14	-0.03	-0.06	-0.01
$[t]$	5.22	3.76	-0.41	-0.67	2.23	-0.27	2.78	2.34	-8.77	-0.69	-0.84	-0.12
IK_{it}	0.04	-0.01	-	-	-0.05	-0.32	-0.17	-0.21	-0.33	-0.18	-0.67	-0.23
$[t]$	0.97	-0.38	-	-	-0.49	-5.41	-4.62	-6.26	-22.27	-2.10	-9.18	-3.08
R^2	0.98	0.86	0.93	0.93	0.98	0.31	0.98	0.84	0.98	0.03	0.99	0.23

Figure 1. Description of stock returns, investment returns, and errors

This figure scatter plots stock returns against investment returns, and stock returns against error terms, based on estimation results from one-step GMM on the cross sectional moments given by Equation (9), using BM deciles as the testing portfolios. Aggregation is at the portfolio level (top) and at the firm level (bottom).

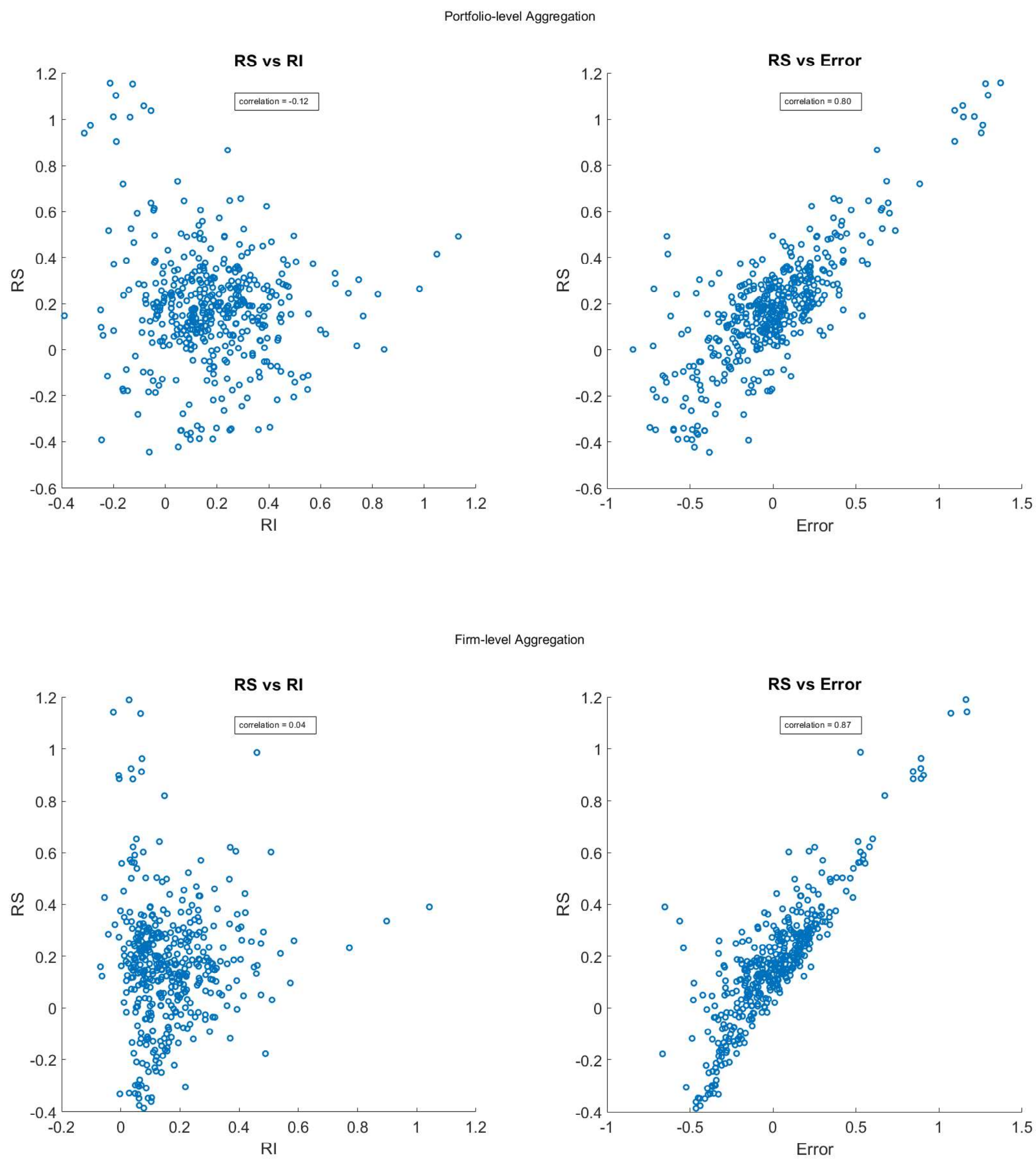
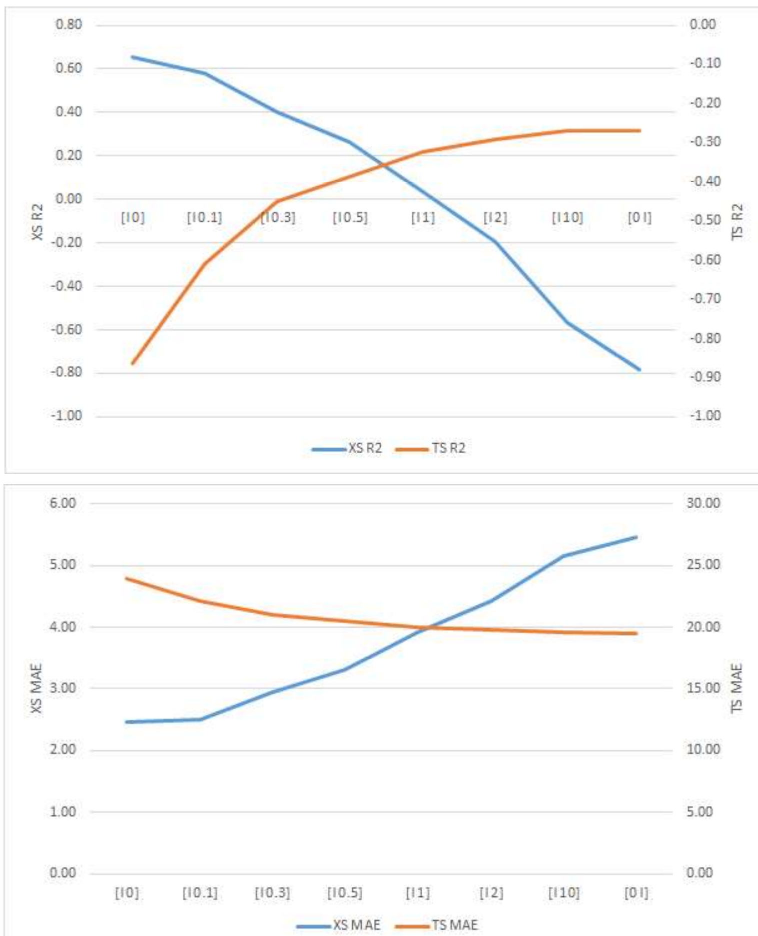


Figure 2. Cross-sectional fit versus time-series fit

This figure plots the cross sectional R^2 and the time series R^2 (top) and the mean absolute cross sectional errors (in percentage per annum) given by Equation (10) and the mean absolute time series errors (in percentage per annum) given by Equation (12) (bottom). Horizontal axis shows the prespecified weighting matrix, in which the first component refers to the weights on the cross sectional moments and the second component refers to the weights on the time series moments. Aggregation is at the portfolio level.



Appendix A: Additional Analyses and Robustness Checks

Table A.1: Euler equation errors.

Table A.2: GMM estimation and tests of the investment-based model, second stage.

Table A.3: GMM estimation and tests of the investment-based model using alternative testing portfolios.

Table A.4: GMM estimation and tests of the investment-based model using alternative time series moments (NLLS first order conditions).

Table A.1. Euler equation errors

This table reports the Euler equation errors in percentage terms. Results are from one-step GMM from estimating jointly the cross sectional moments and the time series moments given by Equation (9) and (11) respectively, using BM deciles as the testing portfolios. Each column differs in the prespecified weighting matrix, in which the first component refers to the weights on the cross sectional moments and the second component refers to the weights on the time series moments. Aggregation is at the portfolio level. We report results for only three (1, 5, 10) out of the 10 portfolios to save space.

Weights:	$[I\ 0]$	$[I\ 0.1]$	$[I\ 0.3]$	$[I\ 0.5]$	$[I\ I]$	$[I\ 2]$	$[I\ 10]$	$[0\ I]$
g^{XS}-1	-3.86	-5.40	-6.77	-7.50	-8.43	-9.11	-9.45	-9.15
g^{XS}-5	2.52	2.06	1.83	1.78	1.83	2.02	2.72	3.31
g^{XS}-10	-2.73	-0.68	1.15	2.16	3.55	4.83	6.96	8.21
g^{TS}-1	11.53	10.73	10.08	9.79	9.48	9.30	9.16	9.07
g^{TS}-5	9.77	9.14	8.73	8.58	8.43	8.35	8.30	8.29
g^{TS}-10	18.80	15.05	12.93	12.17	11.51	11.23	11.20	11.30

Table A.2. GMM estimation and tests of the investment-based model, second stage

This table reports the second stage GMM results from estimating jointly the cross sectional moments and the time series moments given by Equation (9) and (11) respectively, using BM deciles as the testing portfolios. Each column differs in the first stage prespecified weighting matrix, in which the first component refers to the weights on the cross sectional moments and the second component refers to the weights on the time series moments. α is the capital share and c is the adjustment cost parameter. The t-statistics, denoted $[t]$, test that a given parameter equals zero. $\left|e_i^{XS}\right|$ is the mean absolute cross sectional errors given by Equation (10). $\left|e_{H-L}^{XS}\right|$ is the mean absolute high-minus-low cross sectional errors. $\left|e_i^{TS}\right|$ is the mean absolute time series errors given by Equation (12). $XS-R^2$ is the cross sectional R^2. $TS-R^2$ is the time series R^2. χ^2, d.f., and p are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero. $\left|e_i^{XS}\right|$, $\left|e_{H-L}^{XS}\right|$, and $\left|e_i^{TS}\right|$ are expressed as a percentage per annum. Aggregation is at the portfolio level.

	Only XS	Both XS and TS Moments						Only TS
Column:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Weights:	$[I\ 0]$	$[I\ 0.1]$	$[I\ 0.3]$	$[I\ 0.5]$	$[I\ I]$	$[I\ 2]$	$[I\ 10]$	$[0\ I]$
	Parameter estimates							
α	0.22	0.18	0.16	0.15	0.14	0.13	0.12	0.11
$[t]$	12.21	17.52	25.23	29.28	28.58	26.85	24.91	23.97
c	8.03	4.54	2.63	1.81	0.82	0.15	-0.45	-0.60
$[t]$	7.75	7.26	4.72	3.38	1.83	0.40	-1.40	-1.95
	Goodness of fit							
$\left\|e_i^{XS}\right\|$	2.53	2.85	3.30	3.53	3.87	4.35	5.03	5.32
$\left\|e_{H-L}^{XS}\right\|$	2.42	5.54	8.17	9.57	11.64	13.57	15.97	16.87
$\left\|e_i^{TS}\right\|$	23.75	22.10	21.05	20.61	20.11	19.83	19.58	19.50
$XS-R^2$	0.59	0.52	0.37	0.27	0.08	-0.15	-0.50	-0.69
$TS-R^2$	-0.83	-0.61	-0.47	-0.40	-0.34	-0.30	-0.27	-0.27
χ^2	8.15	13.53	13.34	13.23	13.24	13.42	13.71	9.65
d.f.	8.00	18.00	18.00	18.00	18.00	18.00	18.00	8.00
p	0.42	0.76	0.77	0.78	0.78	0.77	0.75	0.29

Table A.3. GMM estimation and tests of the investment-based model using alternative testing portfolios

This table reports the one-step GMM results from estimating jointly the cross sectional moments and the time series moments given by Equation (9) and (11) respectively, using standardized unexpected earnings (SUE) deciles (Panel A) and corporate investment (CI) deciles (Panel B). Each column differs in the prespecified weighting matrix, in which the first component refers to the weights on the cross sectional moments and the second component refers to the weights on the time series moments. α is the capital share and c is the adjustment cost parameter. The t-statistics, denoted $[t]$, test that a given parameter equals zero. $\left|e_i^{XS}\right|$ is the mean absolute cross sectional errors given by Equation (10). $\left|e_{H-L}^{XS}\right|$ is the mean absolute high-minus-low cross sectional errors. $\left|e_i^{TS}\right|$ is the mean absolute time series errors given by Equation (12). $XS-R^2$ is the cross sectional R^2. $TS-R^2$ is the time series R^2. χ^2, d.f., and p are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero. $\left|e_i^{XS}\right|$, $\left|e_{H-L}^{XS}\right|$, and $\left|e_i^{TS}\right|$ are expressed as a percentage per annum. Aggregation is at the portfolio level.

	Panel A: SUE					Panel B: CI				
	Only XS	Both XS and TS			Only TS	Only XS	Both XS and TS			Only TS
Column:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Weights:	$[I\ 0]$	$[I\ I]$	$[I\ 5]$	$[I\ 10]$	$[0\ I]$	$[I\ 0]$	$[I\ I]$	$[I\ 5]$	$[I\ 10]$	$[0\ I]$
	Parameter estimates									
α	0.18	0.14	0.12	0.12	0.11	0.12	0.12	0.11	0.11	0.11
$[t]$	10.11	10.25	8.36	8.12	6.95	11.69	12.15	12.69	12.85	6.86
c	3.67	0.89	-0.59	-0.91	-1.28	0.41	0.22	-0.02	-0.10	-0.22
$[t]$	3.09	0.79	-0.53	-0.88	-1.93	1.85	0.97	-0.08	-0.34	-0.24
	Goodness of fit									
$\left\|e_i^{XS}\right\|$	0.69	1.70	3.07	3.45	3.98	1.59	1.77	2.05	2.21	2.48
$\left\|e_{H-L}^{XS}\right\|$	0.34	4.90	9.58	10.90	12.64	1.45	3.35	5.90	6.80	8.16
$\left\|e_i^{TS}\right\|$	18.01	16.94	16.33	16.24	16.17	18.74	18.69	18.62	18.59	18.55
$XS-R^2$	0.96	0.75	0.24	0.04	-0.26	-0.42	-0.53	-1.04	-1.36	-2.07
$TS-R^2$	-0.34	-0.18	-0.11	-0.10	-0.09	-0.23	-0.22	-0.21	-0.21	-0.21
χ^2	4.61	11.13	11.59	11.69	6.86	10.41	12.60	12.51	12.49	6.16
d.f.	8.00	18.00	18.00	18.00	8.00	8.00	18.00	18.00	18.00	8.00
p	0.80	0.89	0.87	0.86	0.55	0.24	0.82	0.82	0.82	0.63

Table A.4. GMM estimation and tests of the investment-based model using alternative time series moments

This table reports the one-step GMM results from estimating jointly the cross sectional moments and the alternative time series moments given by Equation (9) and (13) respectively, using BM deciles as the testing portfolios. Each column differs in the prespecified weighting matrix, in which the first component refers to the weights on the cross sectional moments and the second component refers to the weights on the time series moments. α is the capital share and c is the adjustment cost parameter. The t-statistics, denoted $[t]$, test that a given parameter equals zero. $\left|e_i^{XS}\right|$ is the mean absolute cross sectional errors given by Equation (10). $\left|e_{H-L}^{XS}\right|$ is the mean absolute high-minus-low cross sectional errors. $\left|e_i^{TS}\right|$ is the mean absolute time series errors given by Equation (12). $XS-R^2$ is the cross sectional R^2. $TS-R^2$ is the time series R^2. χ^2, d.f., and p are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero. $\left|e_i^{XS}\right|$, $\left|e_{H-L}^{XS}\right|$, and $\left|e_i^{TS}\right|$ are expressed as a percentage per annum. Aggregation is at the portfolio level.

	Only XS	Both XS and TS Moments						Only TS
Column:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Weights:	$[I\ 0]$	$[I\ 10]$	$[I\ 20]$	$[I\ 30]$	$[I\ 40]$	$[I\ 50]$	$[I\ 100]$	$[0\ I]$
	Parameter estimates							
α	0.23	0.21	0.21	0.20	0.20	0.20	0.20	0.11
$[t]$	2.74	2.80	3.01	3.11	3.17	3.20	3.28	10.27
c	8.43	8.20	7.52	7.27	7.16	7.10	7.06	-0.75
$[t]$	1.16	1.15	1.15	1.16	1.16	1.17	1.19	-1.11
	Goodness of fit							
$\left\|e_i^{XS}\right\|$	2.47	2.92	3.15	3.24	3.28	3.31	3.36	5.51
$\left\|e_{H-L}^{XS}\right\|$	1.13	3.10	3.84	4.12	4.25	4.33	4.43	17.55
$\left\|e_i^{TS}\right\|$	23.97	23.78	23.47	23.36	23.31	23.28	23.26	19.46
$XS-R^2$	0.65	0.47	0.40	0.38	0.36	0.35	0.33	-0.82
$TS-R^2$	-0.86	-0.84	-0.80	-0.78	-0.78	-0.77	-0.77	-0.27
χ^2	8.15	10.46	10.70	10.79	10.83	10.86	10.90	-
d.f.	8.00	10.00	10.00	10.00	10.00	10.00	10.00	0.00
p	0.42	0.40	0.38	0.37	0.37	0.37	0.37	-

Appendix B: Investment Model with Frictions

The model is closely related to Lin and Zhang (2013). Production only takes one input, capital K, with decreasing return to scale. Firm i's operating profit function is given by

$$\Pi_{it} = X_t Z_{it} K_{it}^{\alpha} - f, \tag{23}$$

in which $0 < \alpha < 1$ is the curvature parameter, and $f > 0$ is a positive fixed cost, capturing the existence of fixed outside opportunity costs each period. Production is subject to both aggregate and idiosyncratic productivity shocks. The aggregate productivity X_t, has a stationary Markov transition function. Let $x_t = log X_t$, the transition function follows

$$x_{t+1} = \rho_x x_t + \sigma_x \mu_{t+1}, \tag{24}$$

in which μ_{t+1} is an i.i.d. standard normal shock. Firm i's productivity Z_{it} has a transition function follows

$$z_{it+1} = \bar{z}\left(1 - \rho_z\right) + \rho_z z_{it} + \sigma_z \nu_{it+1}, \tag{25}$$

in which $z_{it} = log Z_{it}$, and ν_{it+1} is an i.i.d. standard normal shock. Two shocks are uncorrelated.

Firm i's capital accumulates as

$$K_{it+1} = I_{it} + \left(1 - \delta\right) K_{it}, \tag{26}$$

in which δ is the rate of depreciation. Capital investment entails adjustment costs

$$\Phi\left(I_{it}, K_{it}\right) = \begin{cases} a^{+} K_{it} + \frac{c^{+}}{2}\left(\frac{I_{it}}{K_{it}}\right)^2 K_{it} & I_{it} > 0 \\ 0 & I_{it} = 0 \\ a^{-} K_{it} + \frac{c^{-}}{2}\left(\frac{I_{it}}{K_{it}}\right)^2 K_{it} & I_{it} < 0 \end{cases}, \tag{27}$$

where $a^- > a^+ > 0$ and $c^- > c^+ > 0$ capture nonconvex and asymmetric adjustment costs. Nonconvex part captures the cost independent of the size of investment. Convex part captures higher cost for more rapid changes. Asymmetric part captures costly reversibility. Firms face higher costs in contracting than in expanding.

The stochastic discount factor is exogenously given, denoted by M_{t+1}

$$M_{t+1} = \beta \frac{e^{\gamma(x_t - x_{t+1})}}{E_t\left[e^{\gamma(x_t - x_{t+1})}\right]}, \tag{28}$$

in which $0 < \beta < 1$, $\gamma > 0$ are constants. The risk-free rate is set to be constant.

Upon observing shocks, firms optimally choose investment to maximize the market value of equity, given by

$$V_{it} \equiv V\left(K_{it}, X_t, Z_{it}\right) = \max_{I_{it}}\left[\Pi_{it} - I_{it} - \Phi\left(I_{it}, K_{it}\right) + E_t\left[M_{t+1} V\left(K_{it+1}, X_{t+1}, Z_{it+1}\right)\right]\right]. \tag{29}$$

At the optimum, $V_{it} = D_{it} + E_t\left[M_{t+1} V_{it+1}\right]$, with $D_{it} \equiv \Pi_{it} - I_{it} - \Phi\left(I_{it}, K_{it}\right)$. Equivalently, $E_t\left[M_{t+1} r^S_{it+1}\right] = 1$ in which $r^S_{it+1} = V_{it+1}/\left(V_{it} - D_{it}\right)$ is the stock return. Similarly, $E_t\left[M_{t+1} r^I_{it+1}\right] = 1$, in which r^I_{it+1} is the investment return. However, in this investment model with frictions, Hayashi (1982) conditions do not hold, thus investment returns do not equal to stock returns.

The model is calibrated at annual frequency. The time discount factor, $\beta = 0.9718$, is set to match the real risk-free rate of 2.9% per annum. The price of risk parameter, $\gamma = 6$, is set to match the average Sharpe ratio. The persistence of aggregate corporate profits ρ_x is set to be 0.90 and conditional volatility $\sigma_x = 0.06$. For the adjustment cost parameters: $a^+ = 0.01$, $a^- = 0.1$, $c^+ = 10$, and $c^- = 200$; for the remaining parameters, $\rho_z = 0.90$, $\sigma_z = 0.10$, $\bar{z} = -0.98$, $\alpha = 0.65$, $\delta = 0.10$, and $f = 0.115$.

The model is solved with value function iterations on discrete state space. In total 1000 artificial samples are simulated from the model, each with 3000 firms and 500 years. The first 450 years are dropped to neutralize the impact of the initial condition. The remaining

50 years of simulated data are treated as from the model's stationary distribution. Empirical tests are performed on each artificial sample and cross-simulation median results are reported as model moments to compare with those in the real data. With the calibrated parameters, the model produces a value premium of 4% per annum.

Appendix C: External Validity Specification Test

When evaluating the investment-based asset pricing model, existing studies conduct the χ^2 test using the same set of moment conditions as in the estimation. We argue that this procedure has low power to reject the model when presented with model misspecification. Thus, we develop a Wald test for model errors that are not used for estimation. It holds the model to a higher standard than a simple test of overidentifying restriction and thus accomplishes a purpose similar to that of an out-of-sample test.

Specifically, we ask the estimation to match the cross sectional moments as closely as possible and evaluate how the fitted model matches the time series moments. Following the procedure described in Cochrane (2009), we start to estimate the parameters by only using g^{XS} and obtain the distribution of all moments $var\left(g_T\right)$. Denote $var\left(g_T\right)^{TS}$ as the block of time series moments in $var\left(g_T\right)$, and we use it to compute the joint error for g^{TS} to incorporate sampling uncertainty about the parameters from their estimation stage and correlation between the estimation moments and the evaluation moments. We want to test the null hypothesis that $g^{TS} = 0$. This hypothesis constitutes a test of the external validity of the model, as it assesses the model's ability to explain patterns in the data that are not used to estimate its parameters. Under the null hypothesis that the model is correctly specified, these moments should equal zero. Formally, the χ^2 test is:

$$g_T^{TS'} \left[var\left(g_T\right)^{TS}\right]^{+} g_T^{TS} \sim \chi^2\left(\#moments - \#paras\right). \tag{30}$$

We compare standard overidentifying tests with our proposed external validity specification tests. Although the standard test has some difficulties in rejecting the model, the external validity specification test increases the power of the tests and hence can be useful in practice to detect possible model misspecifications.

Table A.5 reports the results. Columns (1) and (2) report the standard overidentifying tests results in which the same set of moment conditions, the cross sectional moments, are used in the estimation and tests. Columns (3) and (4) report the specification tests results in which cross sectional moments are used in the estimation and time series moments are used in the tests. Columns (1) and (3) report the results based on an identity weighting matrix in the estimation, whereas columns (2) and (4) report the results based on the optimal weighting matrix. Panel A, column (1) shows that the p-value on testing the joint errors of 10 cross sectional moments is 0.42 (0.44 with an optimal weighting matrix), far from rejecting the model, despite the fact that the time series fit is very poor. In comparison, the p-value on evaluating the joint errors of the 10 time series moments is 0.23 in column (3) (0.18 with an optimal weighting matrix), getting the model much closer to rejection based on its time series fit.

The poor time series fit is more prominent with the correct portfolio aggregation as reported in Panel B in Table A.5. Column (1) shows that the p-value on testing the joint errors of 10 cross sectional moments is 0.83 (0.82 with an optimal weighting matrix), indicating a close to perfect model fit, despite the fact that the time series fit is very poor. In comparison, the p-value on evaluating the joint errors of the 10 time series moments is 0.16 in column (3) (0.19 with an optimal weighting matrix), much more likely leading to a rejection of the model based on its time series fit.

[Table A.5 here]

Table A.5. External validity specification test

This table reports the external validity specification test results based on different sets of moments used in estimation and tests. The cross sectional moments, denoted g^{XS}, are given by Equation (9). The time series moments, denoted g^{TS}, are given by Equation (11). Weighting matrix is either an identity matrix or an optimal weighting matrix. In Panels A, aggregation is at the portfolio level as in LWZ, and in Panel B, aggregation is based on firm-level investment return. χ^2, d.f., and p are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero.

Estimation	g^{XS}		g^{XS}	
Tests	g^{XS}		g^{TS}	
Column:	(1)	(2)	(3)	(4)
Weights:	$[I\ 0]$	$[S^{-1}\ 0]$	$[I\ 0]$	$[S^{-1}\ 0]$
	Panel A: Portfolio-level aggregation			
χ^2	8.15	7.96	10.55	11.36
d.f.	8.00	8.00	8.00	8.00
p	0.42	0.44	0.23	0.18
	Panel B: Firm-level aggregation			
χ^2	4.30	4.42	11.73	11.28
d.f.	8.00	8.00	8.00	8.00
p	0.83	0.82	0.16	0.19