Voluntary Disclosure with Evolving News

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Abstract

We study a dynamic voluntary disclosure setting where the manager's information and the firm's value evolve over time. The manager is not limited in her disclosure opportunities but disclosure is costly. The results show that the manager discloses even if this leads to a price *decrease* in the current period. The manager absorbs this price drop in order to increase her option value of withholding disclosure in the future. That is, by disclosing today the manager can improve her continuation value. The results provide a number of novel empirical predictions regarding asset prices and disclosure patterns over time. These include, among others, that disclosures are negatively correlated in time, and stock return skewness is negatively correlated with lagged returns for firms with low idiosyncratic risk, in more competitive industries, and in industries with greater systematic uncertainty.

1 Introduction

A firm's informational environment is generally characterized by continuous inflows of new information. For example, advances made through research and development could lead to patents and eventual product launches. Similarly, the firm's direction or strategy may change based on current or projected industry conditions. Firm managers must continuously decide whether to release such new information to investors or the public, even if there is no legal obligation to do so. Accordingly, the process of price discovery for the firm typically involves voluntary information disclosures by firm executives regarding the firm's present situation.

Casual observation and findings in the empirical literature further motivate us to study voluntary disclosure in the presence of evolving news. A few studies have documented that firms' disclosure decisions vary with their performance (e.g., Miller (2002), Sletten (2012)). Moreover, while the extant theoretical literature has shown that firms release information to improve their market valuations, voluntary corporate disclosures which lead to price decreases or a *negative* market reaction are pervasive in practice. Indeed, numerous studies

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have documented that firms often voluntarily release information which is met with a negative market reaction (e.g., Skinner (1994), Soffer et al. (2000), Matsumoto (2002), Baik and Jiang (2006), Anilowski et al. (2007), and Kross et al. (2011), among others). The goal of this paper is to investigate the theoretical underpinnings of firm disclosure behavior in the presence of evolving information, and to find an endogenous explanation for this anomalous yet enduring empirical regularity.

Our setting is one where the manager privately observes the firm's fundamental value in each of two periods. The manager may choose to disclose, at a cost (such as a proprietary or certification cost), her private information of the firm's value in each period. The model has two key components. The first component, which is the central nuance of this paper, is that the firm's value between periods evolves according to a simple, correlated process. This allows disclosure in the present period to influence market beliefs in the future. Moreover, the manager must take into consideration potential changes in the firm value when deciding her disclosure policy in the present period. The second feature of our model is that, at the beginning of the second period, the firm distributes its cash flows as dividends. This ultimately serves as a signal of the underlying firm value given non-disclosure in the previous period.

Our main result shows that first-period disclosure by the manager whose value is at the disclosure threshold *always results in a price decrease* relative to non-disclosure (Theorem 1). Stated differently, the threshold-type manager receives a higher price by keeping quiet in the first period than from disclosing. This occurs because early disclosure increases the option value of withholding information in the future. In this sense, early disclosure *generates* a real option for the manager. Specifically, by disclosing in the first period, this raises the second-period disclosure threshold and helps to protect the manager if firm value declines in the future. However, if it turns out to be the case that firm value has improved in the second period, the manager can simply disclose this value to the market. This leads the manager to disclose *excessively* in equilibrium.

We note that the economic forces driving the main result are in contrast to those in extant dynamic voluntary disclosure models. Previous models of dynamic disclosure generally involve a manager who can generate a real option from *concealing* information in the present period (e.g., Acharya et al. (2011), Guttman et al. (2014)). These models are dynamic but entail a constant firm value. In contrast, in our setting we find that the manager can improve her option value of withholding disclosure in the future by *revealing* information in the current period. Hence, we find that allowing firm value to change over time leads to significantly different disclosure incentives and behavior. We note that this improved option value from early disclosure prevails even when the manager has a countervailing incentive to withhold information, such as in the form of exogenous positive news which may overstate the firm's value (as in Acharya et al. (2011)).

The manager faces two conflicting real options when making her disclosure decision in the first period. On one hand, withholding disclosure allows for the possibility that realized cash flows may overstate the firm's profitability, thus resulting in a more favorable price. On the other hand, the firm value may decline in the future. As we show, early disclosure gives the manager more flexibility to conceal future bad news. The evolving nature of the firm leads the option value generated from disclosure to dominate the real option from keeping quiet. Consequently, the manager is inclined to disclose even if this hurts the first-period price. The result follows from three key equilibrium properties.

These properties all concern the equilibrium price following non-disclosure in the second period, given that the manager did not disclose in the previous period. First, we find that there is limited upside of the impact from strong dividends (and thus public news) on the second-period price following non-disclosure in that period. While positive news always improves the second-period non-disclosure price, it is still the case that the manager would have disclosed in the first period if her private information was sufficiently high. The upside of strong positive news is thus mitigated by the manager's non-disclosure in the first period. Likewise, as the second equilibrium property, we find that the second-period non-disclosure price increases in the first-period disclosure threshold at a rate less than the autocorrelation of the firm's value. While a higher first-period threshold implies that the second-period firm value must also be high, increases in the first-period threshold do not fully "carry over" to the second period. The reason is that, upon non-disclosure in the first period, the market updates its beliefs regarding the evolved second-period value using the conditional expectation for *the set of all non-disclosing types.* The market thus determines the *average* evolved firm value, which leads the second-period non-disclosure price to increase in the first-period threshold at a slower rate.

Third, we find that, for the threshold-type manager, the second-period non-disclosure price is *always lower* if the manager had concealed information in the first period than if she had disclosed. This implies that the threshold-type manager's option to withhold information in the second period is strictly higher if she had disclosed her private information in the first period. This occurs since the manager is pooled with the other first-period non-disclosing firms, and since it is unlikely that the signal from realized dividends will push the market expectation of firm value to be as high as the first-period disclosure threshold level. Conversely, by disclosing, the second period non-disclosure price increases in the disclosed value at a rate *equal* to the autocorrelation (in contrast to the second property above). Hence, by disclosing in the present period, the manager can positively influence market beliefs in the following period by raising that period's non-disclosure price. In other words, disclosure in the present period increases the option value of withholding disclosure in the *following* period. These three equilibrium properties lead the manager to reveal her information in the first-period, even if she endures a strictly lower market valuation by doing so (relative to concealing information).

The results of the model provide a rich set of empirical predictions regarding asset prices and the pattern of disclosures over time. Our main result implies that disclosure in the present period leads to less disclosure in the future, while non-disclosure in the present often leads to higher disclosure in the future. Moreover, the reduction in future disclosure probability holds for *all* types of disclosing managers, including very high types. This occurs because disclosure by the manager in the first period shifts market beliefs of the conditional mean of second-period profitability, and hence the disclosure threshold in the second period is adjusted for the disclosed value accordingly. Consequently, all disclosing managers have a lower probability of disclosure in the second period, even those that disclosed a very high value in the first period. Hence, voluntary disclosures are *negatively* correlated in time. We additionally find that this negative correlation is stronger for firms in less competitive industries and in industries with greater systematic uncertainty.

We also investigate asset pricing implications of the results. We find that evolving private information influences return skewness. Conditional on non-disclosure in the first period, the market pays close attention to the public signal (dividend announcement). If the public signal releases bad news due to a negative systematic shock, the manager is often compelled to disclose bad news as well due to the lower threshold level, resulting in negative return skewness. Conversely, the public signal may release good news due to a positive systematic shock. Under an unchanging environment, the firm manager would typically be inclined to withhold disclosure following elevated market beliefs due to the positive public signal. However, in our stochastic setting, firm value evolves contemporaneously with the release of news about the previous period's underlying value. In particular, the innovation in underlying profitability can exceed the positive systematic shock to the public signal, leading the evolved value to surpass the threshold level. In this case, the manager discloses good news following the release of good news by the public signal, resulting in positive skewness conditional on non-disclosure in the first period. This also helps to explain the results of Miller (2002), who finds that firms tend to increase their voluntarily disclosures and provide forward-looking information following positive earnings shocks.

Furthermore, an equilibrium property that we exploit in our setting is that disclosing firms have a different average return than non-disclosing firms. Indeed, due to the price drop from disclosure, the average return for disclosing firms can be lower than the corresponding non-disclosure return, and the comparative level of these returns depends on fundamental parameters of the model. This allows us to determine the average first-period return based on the expected disclosure behavior, and thus tie return moments and disclosure patterns to past (first-period) returns.

One set of predictions which emerges with this analysis is the relation between return skewness and past returns. For example, we find that stock return skewness is negatively correlated with lagged returns among firms with low idiosyncratic risk. The reasoning is as follows. An equilibrium property of the analysis is that firms with low idiosyncratic risk disclose more frequently in the first period, which we find leads to greater and more positive subsequent return skewness. Moreover, disclosure by these low risk firms results in an average return that is lower than the return from non-disclosure. As a result, disclosing firms endure a low return but are met with high subsequent skewness, whereas non-disclosing firms generate high returns in the current period and face low, and often negative, future return skewness. This implies that lagged returns and return skewness are negatively correlated for firms with low idiosyncratic risk. This helps to explain and provides additional texture to the empirical results of Harvey and Siddique (2000) and Chen et al. (2001), where firms with high current returns experience negative future return skewness. They additionally document that returns are more negatively skewed for larger firms, which typically have lower idiosyncratic risk (e.g., Fu (2009)), consistent with the implications of our model. Along similar lines, we find that lagged returns and return skewness are negatively (positively) correlated for firms in more (less) competitive industries and in industries with greater (lower) systematic uncertainty.

Relatedly, we examine implications for the relation between current returns and the firm's future voluntary disclosure behavior. Specifically, our findings imply that higher returns predict lower future voluntary disclosure for firms with high idiosyncratic risk, high persistence in cash flows, low sensitivity to systematic risk, and in less competitive industries. These findings thus provide guidance for the connection between managerial voluntary disclosure and the firm's past performance. We discuss these predictions, as well as implications related to return variance, price autocorrelation, and price informativeness, more thoroughly in Section 5.

1.1 Related Literature

Grossman (1981) and Milgrom (1981) first studied static voluntary disclosure and showed that, in the absence of disclosure costs, the agent always reveals her private information in equilibrium.[1] Jovanovic (1982) and Verrecchia (1983) extend this result by examining a static disclosure setting where information release is costly. We build from these studies and incorporate disclosure costs as the basic friction which prevents unraveling.

Our model is related to the literature on dynamic voluntary disclosure. Einhorn and Ziv (2008) and Marinovic and Varas (2016) also consider settings in which the firm value evolves over time. Einhorn and Ziv (2008) examine a repeated game in which disclosures made in the present affect the market's perception that a future-period manager has received material information. Importantly, Einhorn and Ziv (2008) assume that the manager's private information (cash flows) is always made common knowledge at the end of each period, which removes strategic considerations regarding future market beliefs of firm value. Moreover, Einhorn and Ziv (2008) assume that the manager is purely myopic (or short-lived) in the sense that she only seeks to maximize the firm's price in the current period, whereas we assume the manager prefers to maximize both short and long-term prices (though we analyze the purely myopic case to establish a benchmark result).

Marinovic and Varas (2016) investigate a continuous-time, binary disclosure model where the firm's value fluctuates according to a Markov process. They assume that the firm faces a risk of litigation when bad news is withheld, and thus *not* disclosing is costly. The model here differs from Marinovic and Varas (2016) primarily in that litigation risk is a fundamental feature of their setting. In contrast, we investigate dynamic disclosure without imposing an exogenous cost of withholding disclosure.

Our setting is also related to a stream of literature in dynamic disclosure where the manager may choose the timing of her disclosure, but the underlying value of the firm does not change. Acharya et al. (2011) investigate a model where an exogenous correlated signal is publicly revealed at a known time. Their results show clustering of announcements in bad times, where the manager discloses immediately if the public signal is sufficiently low.

[1] This is commonly referred to as the "unraveling result." Grossman (1981) and Milgrom (1981) show that, if disclosure is costless, then another friction, such as lack of common knowledge that the agent received information, must be present in order to prevent unraveling. This latter friction was first explored by Dye (1985) and Jung and Kwon (1988). Voluntary disclosure models typically include either disclosure costs or uncertainty regarding the agent's information endowment to prevent unraveling.

Relatedly, Guttman et al. (2014) consider a two-period model where the manager may receive two independent signals of the firm value in each period. They show that the market value of the firm is higher if one signal is disclosed in the second period rather than if one signal is disclosed in the first period. Dye and Sridhar (1995) examine a two-period setting with multiple firms where the probability of receiving information is correlated across firms. They find that the second-period non-disclosure price is decreasing in the number of first-period disclosures, which can thus generate disclosure herds. The main difference in our setting and Dye and Sridhar (1995), Acharya et al. (2011), and Guttman et al. (2014) is that we assume that firm value changes over time. Moreover, a driving force in these studies is that the manager can improve her option value by concealing information, whereas we find the opposite force.

Shin (2003, 2006) considers disclosure in a binomial setting where projects may either succeed or fail. The equilibrium constructed is one where the manager follows a "sanitation strategy" where only project successes are disclosed in the interim period. In a similar vein, Goto et al. (2008) extend Shin's (2003) framework to include risk-averse investors. The present setting varies from Shin (2003, 2006) and Goto et al. (2008) in that we are more focused on intertemporal considerations of voluntary disclosure.

Another stream in the disclosure literature considers voluntary disclosure in settings where the manager has additional private information concerning her type. This allows disclosure to entail an additional signaling value. Teoh and Hwang (1991) consider a binary disclosure setting where firms, in addition to value, have private type information that cannot be revealed. They find that high-type firms may disclose bad news, whereas low-type firms do not. Beyer and Dye (2012) examine a setting where the manager is either a forthcoming or strategic type, and find that the strategic manager may disclose bad news in order to build a reputation for being forthcoming. Our setting differs from these models as the value structure is interdependent between periods and the manager does not have additional private information. This paper is also related to models where disclosure is not verifiable. In particular, Stocken (2000) considers a repeated game of unverifiable disclosure and shows that the equilibrium entails truthful disclosure by the sender. This implies that the sender discloses bad news in order to build credibility with investors (the receiver). In contrast, our model features verifiable disclosure and the private signal realizations of the sender are correlated over time.

The paper proceeds as follows. Section 2 outlines the model, while Section 3 presents the main results. Section 4 considers comparative statics and Section 5 presents empirical implications. In Section 6, we consider discounting and a three-period extension of our baseline model. The latter analysis provides additional insights concerning the long-term consequences of disclosure. The final section concludes. All proofs are relegated to the Appendix.

2 Model of Dynamic Disclosure

Our baseline setting is a discrete, two-period model. This parsimonious setting captures the main insights and clearly illustrates the economic forces driving the results. The firm generates a cash flow s_t in each period ($t = 0, 1$). We assume that a risk-neutral firm manager privately observes the mean of cash flows, or underlying profitability, y_0, in time 0 and that (s_0, y_0) is a bivariate normal variable with zero mean and correlation $\rho > 0$.[2] Specifically, we assume that $\sigma_s = \sigma_y/\rho$, where σ_s and $\sigma_y > 0$ are volatility parameters of s_0 and y_0, respectively. We note that the results of the model are not qualitatively affected if $\sigma_s \neq \sigma_y/\rho$. We assume this for ease of exposition so that the mean of s_0 can simply be represented by y_0. Thus, conditional on y_0, the cash flow s_0 is given by

$$s_0 = y_0 + w_0,$$

where w_0 is normally distributed with mean zero and standard deviation $\sigma_w \equiv \sigma_s\sqrt{1-\rho^2}$.[3] This may be interpreted such that y_0 is the profitability of the underlying fundamental and w_0 is an industry or macroeconomic shock to cash flows.

Upon learning y_0, the manager may disclose the information to the market, in which case it becomes public information. We assume that disclosure is verifiable in the sense that the manager cannot manipulate the disclosed value. Disclosure is also assumed to be costly for the firm, where $c > 0$ is incurred upon disclosure. The disclosure cost can be interpreted, for instance, as a certification cost, whereby the manager must hire an auditor to certify that the information disclosed is factual. Alternatively, the disclosure may be relevant to proprietary information that could be adopted by competitor firms. Indeed, a wide-scale survey of executives at large public firms finds evidence consistent with this view: "Nearly three-fifths of survey respondents agree or strongly agree that giving away company secrets is an important barrier to more voluntary disclosure" (Graham et al. (2005, p. 62)).[4]

[2] The zero-mean assumption on (s_0, y_0) is without loss of generality.

[3] Including noise in the cash flow prevents the market from filtering out the mean cash flow perfectly upon observing dividends in the event that the manager does not disclose.

[4] Empirical evidence of proprietary costs of disclosure has been documented by Ellis et al. (2012) and

After the manager makes her disclosure decision at time 0, the market, composed of risk-neutral investors, determines the date 0 price of the firm. Then, s_0 is realized and the cash flow net of the disclosure cost (if the manager had disclosed) is distributed to shareholders.

We allow the mean of cash flows to evolve in the sense that new developments may have occurred between time 0 and time 1 such that the underlying firm profitability improves or declines. This is captured by the time 1 mean cash flow, given by:

$$y_1 = \kappa y_0 + \eta,$$

where $\kappa \in (0, 1]$ denotes autocorrelation of the mean cash flow, and η is a normal variable with mean zero and standard deviation $\sigma_\eta > 0$. We assume that η and (s_0, y_0) are independent. Regardless of the time 0 disclosure decision, the manager privately observes y_1. The distribution of η is common knowledge. We assume that the second-period cash flow s_1 is given simply by $s_1 = y_1$.[5] At time 1, after observing y_1 the manager may disclose y_1 to the market. The market then determines the time 1 price of the firm after observing the manager's disclosure decisions at time 0 and at time 1, and the cash flow in the first period. Finally, the cash flow s_1 net of the disclosure cost (if the manager had disclosed) is distributed to shareholders and the game ends. A timeline of the model is presented in Figure 1.

The cum dividend price in each period satisfies:

$$\begin{aligned} p_0 &= E[s_0 - cd_0 + s_1 - cd_1 | \Omega_0] \\ p_1 &= E[s_1 - cd_1 | \Omega_1], \end{aligned}$$

where d_t is an indicator equal to one if the manager discloses in time t and zero otherwise. Ω_t denotes the *market's* information set at time t; Ω_0 includes d_0 and the manager's disclosure strategy, and Ω_1 includes s_0, d_0, d_1, and the manager's disclosure strategy.

The manager is risk neutral and thus her objective is to maximize the sum of the current market price and the expected market price conditional on y_0:

$$\max_{d_0, d_1} p_0 + E[p_1 | y_0].$$

The manager is concerned with the market price at all times as it is often the case that

Aobdia (2018). Other costs of disclosure—arranging press releases, conference calls, and meetings with analysts—are nontrivial and impose time costs on the manager and monetary costs on the firm.

[5] Allowing (s_1, y_1) to be bivariate normal would not qualitatively affect the results.

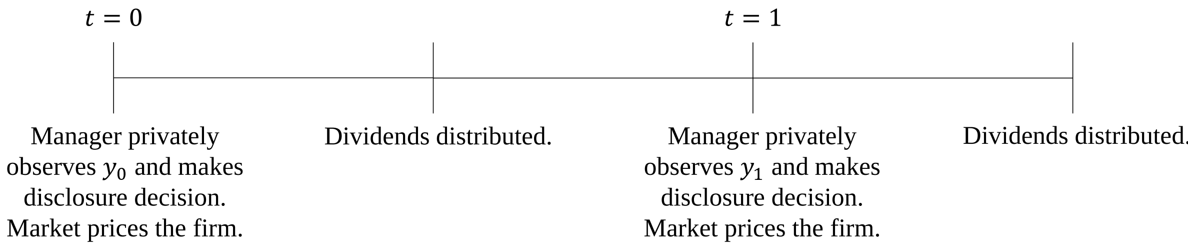


Fig. 1. Timeline.

an executive's compensation includes bonuses which are determined in part by share price.[6] For simplicity, we assume that there is no discounting by the manager or the market. We discuss the quantitative effects of discounting in Section 6.1.

3 Equilibrium

In this section, we characterize the equilibrium of our baseline setting. Before we begin the analysis of the dynamic model, we first analyze the myopic benchmark, which will be helpful in the ensuing analysis.

3.1 Myopic benchmark

In this special case, we assume that the manager is myopic and simply aims to maximize the price of the current period. This is a variant of the static costly disclosure model studied by Jovanovic (1982) and Verrecchia (1983). The main difference is that the non-myopic market must still take into account the expected cash flow of the second period when pricing the firm in the first period. This setting provides a point of comparison with the fully dynamic main model and also allows us to more precisely convey how evolving news affects the non-myopic manager's disclosure strategy.

As the game ends after the second period, the manager's disclosure strategy in the second period is identical in both the myopic and non-myopic setting. Therefore, in this benchmark case we focus on the manager's disclosure strategy in the first period.

Since the price, and thus the manager's payoff, from disclosure is increasing in her private information y_0, any equilibrium strategy must be a *disclosure threshold strategy.* We let x^* denote the equilibrium myopic disclosure threshold in the first period, defined whereby the manager discloses if and only if $y_0 \geq x^*$. For ease of the analysis, we introduce the function

[6] A similar assumption regarding the manager's utility function is made in previous dynamic voluntary disclosure models, such as Acharya et al. (2011) and Guttman et al. (2014).

$\delta(x)$, which is the negative expectation of a standard normal variable conditional on being truncated above at x:

$$\delta(x) = -E[\xi|\xi < x] = \phi(x)\Phi(x)^{-1}, \tag{1}$$

where ξ is a standard normal variable, and $\phi(\cdot)$ and $\Phi(\cdot)$ are the density function and distribution function of the standard normal distribution, respectively.

In the myopic case, the manager is concerned only with the current period's price. The manager thus balances the cost of disclosure with potential undervaluation by the market from non-disclosure. Although the manager is myopic here, she indirectly cares about the market's belief in the second period, as the market continues to be non-myopic and factors this belief into the first-period price. If the manager discloses (i.e., $y_0 \geq x^*$), then the time 0 price upon disclosure, denoted by $p^d(y_0)$, is given by:

$$p^d(y_0) = E[s_0 - c + s_1 - cd_1|\Omega^d] = (1+\kappa)y_0 - c(1 + E[\alpha_d|\Omega^d]), \tag{2}$$

where Ω^d is the information available to the market when the manager discloses at time 0, and $\alpha_d = \Pr(d_1 = 1|\Omega^d, s_0)$ is the market's belief of the probability of disclosure at time 1, given the manager's disclosure and cash flows at time 0. Cash flows s_0 have not yet been observed when the market sets the first-period price p^d.[7] Hence, the market's belief of future disclosure is determined by taking the expectation of α_d with respect to a realization of s_0. In the next section, we show that α_d is independent of cash flows, $E[\alpha_d|\Omega^d] = \alpha_d$.

If the manager does not disclose (i.e., $y_0 < x^*$), the time 0 price upon non-disclosure, p^n, is given by

$$\begin{aligned} p^n &= E[s_0 + s_1 - cd_1|\Omega^n] \\ &= (1+\kappa)E[y_0|y_0 < x^*] - cE[\alpha_n|\Omega^n] = -(1+\kappa)\sigma_y\delta\left(\frac{x^*}{\sigma_y}\right) - cE[\alpha_n|\Omega^n], \end{aligned} \tag{3}$$

where Ω^n is the information available to the market in time 0 when the manager does not disclose, and $\alpha_n = \Pr(d_1 = 1|\Omega^n, s_0)$ is the market's belief of the probability of disclosure at time 1, given non-disclosure and cash flows s_0 at time 0. In the Appendix, we show that α_n depends on the observed cash flows, and thus we have $E[\alpha_n|\Omega^n] \neq \alpha_n$. Furthermore, as we see in equation (3), when determining the non-disclosure price, the market adjusts for the fact that profitability y_0 must be below the threshold, given that the manager did not disclose.

[7] The price upon disclosure depends on the disclosed value y_0. Thus, p^d should be a function with an argument y_0. For ease of exposition, we omit the argument from both the first- and second-period disclosure prices unless it is necessary.

This conditional expectation of y_0 is also used when determining the expectation of the following period's cash flow. Consequently, one straightforward but important implication of non-disclosure is that this allows managers with very low realizations of y_0 to pool with non-disclosing intermediate types.

The threshold x^* is determined such that the manager is indifferent between disclosure and non-disclosure when her private information y_0 is at the threshold level, $y_0 = x^*$. This implies that $p^n = p^d$ for the threshold-type manager. Hence, x^* is given by the following condition:

$$c(1+\alpha_d) = (1+\kappa)\sigma_y v\left(\frac{x^*}{\sigma_y}\right) + cE[\alpha_n|\Omega_n], \tag{4}$$

where $v(x) = x + \delta(x)$. The indifference condition above has an intuitive economic interpretation. The left-hand side is the total expected disclosure cost when the manager discloses at time 0. The right-hand side is the size of undervaluation plus the market's belief of the expected disclosure cost at time 1 when the threshold-type manager conceals information at time 0. The myopic manager at the threshold level is thus indifferent between incurring the direct cost from disclosure and the undervaluation endured from non-disclosure. Importantly, we observe that, for the manager whose private information is at the threshold, $y_0 = x^*$, the disclosure price and non-disclosure price are *equal*; that is, disclosure does not entail a price jump relative to non-disclosure for the threshold-type manager. The following proposition establishes existence and uniqueness of this threshold:

Proposition 1 *There exists a unique myopic disclosure threshold x^* such that the manager discloses if and only if $y_0 \geq x^*$.*

The myopic disclosure threshold x^* provides a useful benchmark for comparison, and is also frequently used in the analysis of the dynamic non-myopic case examined in the following section. The non-myopic setting naturally captures the basic friction of the myopic case, and also incorporates the feature that the manager must account for the fact that her second-period disclosure decision depends on her action in the first period. In essence, this allows the manager to control her ability to pool with higher non-disclosing types and shield herself in the second period from low realizations of profitability, as we show in the next section.

3.2 Second-Period Disclosure

We now turn to our main setting where the manager considers both period's prices in the first period. In solving the equilibrium strategy for the dynamic setting, we begin with the

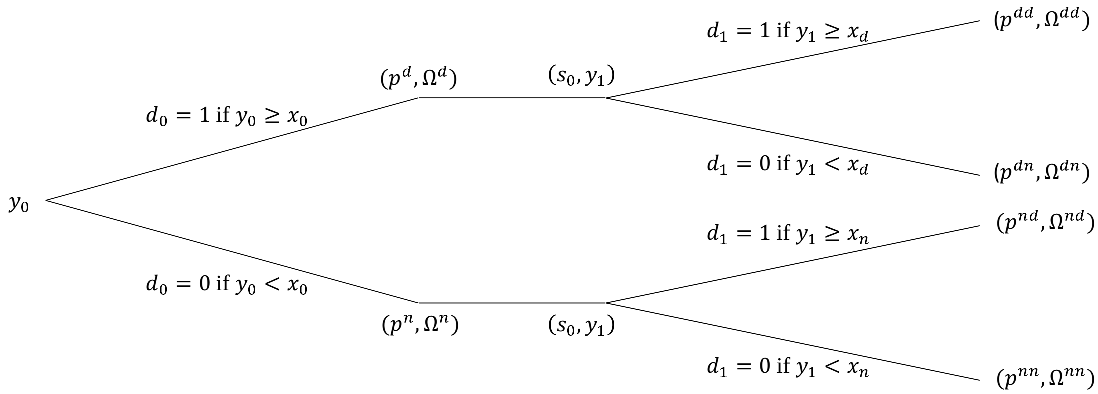


Fig. 2. Game tree of the dynamic model.

manager's decision at time 1 after she has learned y_1. There are two possible paths the manager could have taken prior to time 1: disclosure or non-disclosure in time 0. Below, we analyze each case separately. A tree representation of the dynamic model is presented in Figure 2. Note that the superscript on the price and the market's information set denotes the path of disclosure decisions. For example, p^{dn} represents the market price at time 1 when the manager discloses at time 0 and conceals information at time 1.

Suppose that the time 0 disclosure decision can be characterized by some threshold x_0, such that the manager discloses her private information only if $y_0 \geq x_0$. For now, we keep the time 0 disclosure threshold exogenous and fixed as we analyze the second-period disclosure decision (we endogenize the time 0 decision in the following section). At date 1, the manager will choose to disclose her private information if and only if the market price at time 1 with disclosure exceeds the market price absent disclosure.

Time 1 disclosure decision upon initial disclosure

First, we consider the case where the manager had disclosed her private information at time 0, i.e., $d_0 = 1$. As in the myopic setting, the manager follows a threshold strategy, where she discloses only when her private information is (weakly) above a cutoff level. To determine this threshold level, we again examine the market price following disclosure and non-disclosure. The manager chooses to disclose at time 1 if her payoff from disclosure exceeds that from remaining quiet:

$$p^{dd} = y_1 - c > p^{dn} = E[y_1|\Omega^{dn}],$$

where p^{dd} is the disclosure price at time 1, p^{dn} is the non-disclosure price at time 1 upon initial disclosure, $\Omega^{dn} = \{s_0, y_0, y_1 < x_d\}$ is the information available to the market when the manager disclosed at time 0 but is not disclosing currently, and x_d denotes the disclosure threshold at date 1. Recall that the manager whose private information is equal to the threshold level (i.e., $y_1 = x_d$) must be indifferent between disclosure and non-disclosure. This implies that $p^{dd} = p^{dn}$ for the manager with $y_1 = x_d$. The equilibrium threshold thus satisfies:

$$x_d = c + E[y_1|s_0, y_0, y_1 < x_d] = \kappa y_0 + \eta^*,$$

where η^* solves

$$c = \eta^* - E[\eta|\eta < \eta^*] = \sigma_\eta v\left(\frac{\eta^*}{\sigma_\eta}\right), \tag{5}$$

where $v(\cdot)$ is defined as in the previous section. In equation (5), we see that the manager's indifference is satisfied when the size of undervaluation from non-disclosure just balances the disclosure cost. Moreover, the innovation in firm profitability must be sufficiently strong to induce the manager to disclose. We show that the threshold x_d uniquely exists:

Proposition 2 *There exists a unique equilibrium disclosure threshold satisfying equation (5).*

While the equilibrium is defined in terms of the threshold strategy, it is helpful to consider the equilibrium in terms of the non-disclosure price to better understand the economic implications. Due to indifference at the threshold level, the non-disclosure price in the second period is given as $p^{dn} = x_d - c$. Hereafter, we use the disclosure threshold and non-disclosure price interchangeably. We also define the *true* probability of disclosing in the second period given cash flows, the privately observed profitability, and the manager's first-period disclosure as $\theta_d = \Pr(d_1 = 1|s_0, y_0, d_0 = 1)$. We have the following useful properties of the non-disclosure price and the probability of disclosure.

Lemma 1 *The non-disclosure price at time 1 upon initial disclosure, p^{dn}, is increasing in y_0 at a rate equal to the autocorrelation κ and is independent of s_0 and x_0. The probability of disclosure at time 1 upon first-period disclosure θ_d is independent of y_0, s_0, and x_0, and is given by $\theta_d = Pr(\eta > \eta^*) = \Phi(-\eta^*/\sigma_\eta)$. The market's belief of future disclosure given disclosure in $t = 0$ and cash flows is the same as the true probability: $\alpha_d = \theta_d$.*

As the disclosed value at time 0 increases by one, the non-disclosure price at time 1 increases at a rate of κ. This property is straightforward, as the firm's fundamental value follows a mean-reverting process with autocorrelation κ. This implies that disclosure at time 0 has full impact on the non-disclosure price at time 1. As we will see in the following section, this property becomes a salient factor that influences the time 0 disclosure decision. Moreover, since the true profitability y_0 is revealed in time 0, the realization of cash flows s_0 and the disclosure threshold x_0 do not deliver additional information to the market that is relevant to y_1. Hence, the non-disclosure price and the probability of disclosure at time 1 are independent of s_0 and x_0.

Finally, since the disclosed value at time 0 fully carries over to the disclosure threshold at time 1, the probability of disclosure at time 1 is independent of the disclosed value y_0, which also implies that the market's belief coincides with the true probability of disclosure. Put differently, disclosure of y_0 in time 0 shifts the conditional mean of market beliefs regarding y_1. The second-period disclosure threshold, x_d, is then determined under this updated distribution. As a result, disclosure in $t = 1$ is independent of the actual disclosed value y_0, as this information is factored into the second-period threshold x_d.

However, as we show later, the probability of disclosure in the second period *depends on the disclosure decision in the first period*. This property becomes more apparent further in the analysis.

Time 1 disclosure decision upon initial non-disclosure

We now consider the case where the manager did not disclose at date 0, i.e., $d_0 = 0$. In this case, the announcement and distribution of realized cash flows, $s_0 = y_0 + w_0$, at the beginning of the second period provides relevant information to the market regarding y_0 and thus y_1. As above, the manager will disclose at date 1 if and only if the market price from disclosure exceeds that of non-disclosure:

$$p^{nd} = y_1 - c > p^{nn} = E[y_1|\Omega^{nn}], \tag{6}$$

where p^{nd} is the disclosure price at time 1, p^{nn} is the non-disclosure price at time 1 upon initial non-disclosure, $\Omega^{nn} = \{s_0, y_0 < x_0, y_1 < x_n\}$ is the information available to the market when the manager has not disclosed in both periods, and x_n denotes the disclosure threshold at date 1 given initial non-disclosure, the first-period threshold x_0, and realized cash flows s_0 at date 0.

We first discuss the determination of the manager's threshold strategy and then investigate salient economic properties of this period's non-disclosure price. In the previous section, following disclosure in $t = 0$, the manager's disclosure decision was based on whether the innovation to profitability was sufficiently strong, i.e., $\eta \geq \eta^*$. In contrast, following non-disclosure in $t = 0$, the manager must base her disclosure decision on both the public realization of cash flows s_0 and private realization of the innovation η. We seek a threshold that is analogous in form to x_d, presented in the previous section. That is, we wish to express x_n as the decomposition of the public information release and an upper bound to the manager's private information, which here is the privately observed shock to profitability and potential inaccuracy from the public signal with respect to underlying profitability.

Upon observing the dividend announcement s_0 and without taking into consideration the manager's time 0 disclosure decision (i.e., the naive belief), the market's posterior is that y_0 is normally distributed with mean $\rho^2 s_0$ and variance $(1-\rho^2)\sigma_y^2$.[8] From the perspective of the *manager*, we can thus determine the residual from the market's belief as the difference between the realized y_0 and the projection of y_0 on s_0:

$$z = y_0 - E[y_0|s_0] = y_0 - \rho^2 s_0. \tag{7}$$

This parameter z essentially captures the inaccuracy of the public signal relative to the underlying value y_0, and is the private information of the manager. From the perspective of the *market*, since they do not observe y_0, parameter z is thus treated as a random variable. Moreover, the market updates on the fact that the manager had not previously disclosed in time 0, which means that the random variable y_0 is truncated above at x_0. Consequently, this implies that the random variable z must also be bounded above; specifically, this upper bound is determined as $x_0 - \rho^2 s_0$.

Similarly, upon observing the announcement s_0 and without taking into consideration the disclosure decisions in both time 0 and time 1, the market's (naive) belief of y_1 is $\kappa\rho^2 s_0$. Thus, the manager's private information becomes the sum of two variables—the inaccuracy of the public signal (with some adjustment for autocorrelation) and the innovation to profitability:

$$y_1 - E[y_1|s_0] = \kappa y_0 + \eta - \kappa\rho^2 s_0 = \kappa z + \eta. \tag{8}$$

As above, from the perspective of the market, since they do not observe y_1 if the manager withholds, parameter $\kappa z + \eta$ is treated as the sum of two random variables, with z bounded above by $x_0 - \rho^2 s_0$ and η unbounded. Moreover, the market updates on the fact that the manager does not disclose in time 1, which means that the random variable y_1 is truncated above at x_n. Consequently, this implies that the random variable $\kappa z + \eta$ must also be bounded

[8]This is due to the property that y_0 and s_0 are bivariate normal and $\sigma_s = \sigma_y/\rho$. The mean of y_0 conditional on s_0 is thus $\rho\sigma_y s_0/\sigma_s = \rho^2 s_0$.

above at $x_n - \kappa\rho^2 s_0$.

In sum, given non-disclosure in time 0 and time 1, the market cannot distinguish whether non-disclosure in the current period is due to a poor innovation or because the cash flow realization was inaccurately strong. Hence, the time 1 non-disclosure price is given by

$$\begin{aligned} p^{nn} &= \underbrace{E[y_1|s_0]}_{\substack{\text{Naive expectation} \\ \text{given } s_0}} + \underbrace{E[\kappa z + \eta|\Omega^{nn}]}_{\substack{\text{Expectation of manager's} \\ \text{private information} \\ \text{given } s_0,\, d_0 = d_1 = 0}} \\ &= \kappa\rho^2 s_0 + E[\kappa z + \eta | z < x_0 - \rho^2 s_0, \kappa z + \eta < x_n - \kappa\rho^2 s_0]. \end{aligned}$$

At the threshold, the manager is indifferent between disclosure and non-disclosure. This implies that the threshold-type manager's disclosure price in the second period equates the non-disclosure price:

$$x_n - c = p^{nn}.$$

This allows us to represent the manager's threshold as the decomposition of the public information and a cutoff level for her private information:

$$x_n = \kappa\rho^2 s_0 + \epsilon^*, \tag{9}$$

where ϵ^* solves

$$c = \epsilon^* - E[\kappa z + \eta | z < x_0 - \rho^2 s_0, \kappa z + \eta < \epsilon^*]. \tag{10}$$

Analogous to η^* in x_d, the cutoff level ϵ^* represents the mean-adjusted disclosure threshold for the manager, which accounts for both pieces of the manager's private information: potential inaccuracy from the realized cash flows and the innovation to profitability. Accordingly, the right-hand side of condition (10) represents the difference between the manager's private information at the cutoff level and the market's expectation of this information. Hence, the threshold-type manager is just indifferent between absorbing the direct cost of disclosure and the market's undervaluation of her private information. The following result establishes existence and uniqueness of ϵ^*:

Proposition 3 *There exists a unique fixed point satisfying condition (10).*

We next investigate important properties of the non-disclosure price $p^{nn} = x_n - c$. The equilibrium conditions (9) and (10) imply that s_0 has mixed effects on the manager's disclosure behavior. A strong signal improves the first component of the threshold x_n, i.e., the market's naive expectation increases. However, a high s_0 also implies that it is more likely that the manager's value is being overstated, as the market's expectation of the manager's private information decreases in s_0. Correspondingly, the manager must lower the equilibrium cutoff level ϵ^* to equate the size of undervaluation to the disclosure cost. Hence, the overall impact of s_0 on the disclosure threshold and the non-disclosure price is unclear. Another salient implication is that a high time 0 threshold x_0 is advantageous to the non-disclosing manager, as this should favorably influence market beliefs, and thus the non-disclosure price, in time 1. The following lemma allows us to more precisely see the effects of s_0 and x_0 on the price following non-disclosure.

Lemma 2 *The non-disclosure price at time 1 following non-disclosure in time 0, p^{nn}, is increasing in x_0 at a rate less than κ, and is increasing in s_0 at a rate less than $\kappa\rho^2$.*

As we see in Lemma 2, the non-disclosure price is indeed increasing in s_0, and hence the manager's non-disclosure price in the second period is more favorable when higher cash flows are observed. As discussed above and captured in equation (9), s_0 affects the market's naive expectation of y_1 just conditional on cash flows. Naturally, this effect is stronger when s_0 is more informative (high ρ) or when y_t has greater autocorrelation (high κ). However, the effect of strong cash flows is somewhat mitigated by the fact that the manager did not disclose in the first period. In particular, even if first-period cash flows are very high, it is still the case that the manager's information at time 0 was not sufficiently strong to induce disclosure. Stated differently, since the set of non-disclosing firms in time 0 is bounded above by x_0, it is more likely, for example, that a large industry shock is driving strong cash flows. In turn, the market partially "discounts" the information and the non-disclosure price increases in s_0 at a rate *less* than $\kappa\rho^2$. This is also captured by condition (10), whereby increases in s_0 lower the market's upper bound of the residual z, which leads the cutoff ϵ^* in condition (9) to be decreasing in s_0 (shown in the Appendix). Hence, a high first-period cash flow is always beneficial, but this benefit is somewhat mitigated by the manager's non-disclosure in the first period.

We additionally see from Lemma 2 that the non-disclosure price is increasing in the first-period disclosure threshold x_0. This property is straightforward, as a higher disclosure threshold at time 0 means that, for the same value of cash flows, this is likely to be an indication of a high fundamental at time 0 and thus of high profitability at time 1 as well. This implies that the non-disclosure price at time 1 will be higher as the manager increases the time 0 disclosure threshold.

However, what is striking is that an increase in the first-period disclosure threshold by one results in an increase of the time 1 non-disclosure price (and thus the time 1 threshold) by *less* than the autocorrelation κ. This implies that the disclosure threshold in the first period does not fully "carry over" to the second period. To see this, note that the market is determining the *average* evolved firm value in time 1 based on its information set. As the threshold increases in time 0, this increases the upper bound of the residual z, i.e., for the same strong cash flow realization, the market believes that it is less likely due to a large industry shock. Therefore, the market discounts the information less and updates its beliefs positively of the manager's private information. However, due to the truncation, beliefs regarding z, and thus of $\kappa z + \eta$, do not increase at the same rate as increases in the upper bound. Since the market's expectation determines the equilibrium condition for disclosure at time 1, this implies that the time 1 threshold, and thus the time 1 non-disclosure price, correspondingly increase at a rate lower than increases in x_0. Hence, a high-type manager that withholds disclosure in time 0 may tolerate pooling by low types in that period, but the consequences from pooling are intensified in the following period, as this affects market beliefs of the evolved value.[9]

This equilibrium property shows that there is some limitation to the benefits of non-disclosure in the first period, as the threshold level does not fully carry over to the second period and increases in s_0 are mitigated by non-disclosure. While the non-disclosure price p^{nn} increases in x_0 at a rate strictly less than κ, the non-disclosure price upon initial disclosure increases in the disclosed value x_0 at a rate equal to κ (Lemma 1). This difference (together with the fact that $0 < \partial p^{nn}/\partial s_0 < \kappa\rho^2$) is a significant driving force of the main result, as we show later.

Related to the above, a second important property, which will prove useful when examining the time 0 decision, concerns the probability of disclosure in time 1 following non-disclosure in time 0. Analogous to θ_d in the previous section, we define the true probability

[9] To see this in terms of the equilibrium threshold conditions, suppose that the manager increases the equilibrium threshold x_n by increasing the cutoff of her private information, ϵ^*, to $\tilde{\epsilon}^* = \epsilon^* + \kappa\Delta$, in response to an increase in the first-period threshold: $\tilde{x}_0 = x_0 + \Delta$, where $\Delta > 0$. We can see that market beliefs of the manager's private information given non-disclosure in both periods under the increased threshold levels ($\tilde{x}_0$ and $\tilde{\epsilon}^*$) does not increase in line with an increase in the manager's cutoff level. As discussed above, this occurs due to the fact that the other non-disclosing types weigh down market beliefs of the evolved firm value. Consequently, under the increased threshold levels, the size of undervaluation is greater than the disclosure cost, and thus the manager should raise the cutoff level by less than $\kappa\Delta$:

$$
\begin{aligned}
&\tilde{\epsilon}^* - E[\kappa z + \eta | z < \tilde{x}_0 - \rho^2 s_0, \kappa z + \eta < \tilde{\epsilon}^*] \\
>&\epsilon^* + \kappa\Delta - E[\kappa z + \eta | z < x_0 - \rho^2 s_0, \kappa z + \eta < \epsilon^*] - \kappa\Delta = c.
\end{aligned}
$$

of disclosure at time 1 given cash flows s_0, the privately observed profitability y_0, and initial non-disclosure: $\theta_n = \Pr(d_1 = 1|s_0, y_0, d_0 = 0)$.

Lemma 3 *The probability of disclosure at time 1 upon initial non-disclosure, θ_n, is increasing in y_0, decreasing in s_0, and decreasing in x_0. The probability of disclosure for the threshold-type manager ($y_0 = x_0$) is greater than the market's belief of the disclosure probability conditional on initial non-disclosure and cash flows, $\alpha_n < \theta_n$.*

To see this, first note that the true probability of disclosure at time 1 is equal to the probability of the event that $y_1 = \kappa y_0 + \eta \geq x_n$. Strong cash flows s_0 or a high time 0 threshold x_0 lead to a higher price absent disclosure in the second period (Lemma 2). Consequently, a higher realization of the shock to profitability η is required to induce the manager to disclose in the second period, which implies a relatively lower disclosure probability. Conversely, if the concealed profitability at time 0 is high, but still below the time 0 threshold, then the manager's time 1 disclosure price is higher on average, while the expected time 1 non-disclosure price remains the same. Thus, the manager with high undisclosed profitability at time 0 is more inclined to disclose in the next period.

Finally, the market's belief of the probability of disclosure is for the average of the set of non-disclosing managers (i.e., the market's belief differs from the true probability of disclosure). Given that the disclosure probability is increasing in y_0, the threshold-type manager will always have a higher probability of disclosing than the average of the non-disclosing types. Interestingly, this implies that the market's assessment of the expected future disclosure cost for the threshold-type manager, as computed in time 0, will always be less than the true expected future disclosure cost. In this sense, the threshold-type manager has an additional benefit from withholding disclosure in time 0, as the price upon non-disclosure understates the future cost of disclosure.

3.3 First-Period Disclosure

We now analyze the manager's time 0 disclosure decision. If the threshold-type manager ($y_0 = x_0$) discloses at time 0 ($d_0 = 1$), the price p^d in that period is given by equation (2). At date 1, depending on the evolved profitability, the payoff to the manager is equal to either $p^{dd} = y_1 - c$ if $y_1 \geq x_d$, or $p^{dn} = x_d - c$ if $y_1 < x_d$. Thus, the expected utility of the threshold-type manager from disclosing in the first period is given by:

$$p^d + E[\max(p^{dd}, p^{dn})|y_0 = x_0] = p^d + E[p^{dd}|y_0 = x_0] + u_d. \tag{11}$$

The first term in the right-hand side of equation (11) is the manager's first-period payoff from disclosure, which is simply the time 0 market price. The second and third term constitute the manager's expected second-period payoff, which includes the expected second-period disclosure price and the option value of non-disclosure, given by:

$$u_d = E[\max(p^{dn} - p^{dd}, 0)|y_0 = x_0]. \tag{12}$$

Observe that equation (12) is similar to that of an American put option, where the manager can exercise the option to withhold information in the second period when the price with disclosure (i.e., the underlying asset) is below the price without disclosure (i.e., the strike price).

Conversely, if the threshold-type manager does not disclose at time 0, the market price in that period, p^n, is given by equation (3). At time 1, the market price is either $p^{nd} = y_1 - c$ from disclosure or $p^{nn} = x_n - c$ from non-disclosure. Thus, the expected utility of the manager upon non-disclosure in the first period is given by:

$$p^n + E[\max(p^{nd}, p^{nn})|y_0 = x_0] = p^n + E[p^{nd}|y_0 = x_0] + u_n,$$

where the option value upon non-disclosure in the first period, denoted by u_n, is given as:

$$u_n = E[\max(p^{nn} - p^{nd}, 0)|y_0 = x_0]. \tag{13}$$

Similar to equation (12), the above equation also resembles an American put option, where the manager can exercise the option to conceal information when the disclosure price is lower than the non-disclosure price. Moreover, as distinct from u_d and typical put options, the equivalent of the strike price in u_n (i.e., p^{nn}) is itself a random variable, as it depends on the public signal s_0 realized at the beginning of time 1.

We see in the above analysis that the manager must weigh two different real options. The first stems from the fact that underlying profitability evolves over time—even after disclosing today, the manager has the option to conceal in the second period. This option enhances the incentive for disclosure in the first period. The second real option arises from both time-varying profitability and the correlated public signal (cash flows). The manager can keep quiet in the first period in order to take advantage of either a high realization of cash flows s_0, which increases the non-disclosure price (i.e., the strike price), or a low realization of y_1, which decreases the disclosure price (i.e., the underlying asset). Conversely, this option strengthens the incentive for non-disclosure in the first period.

The threshold-type manager $y_0 = x_0$ must be indifferent between initial disclosure and non-disclosure. The equilibrium first-period disclosure threshold thus satisfies:

$$p^d = p^n + u_n - u_d. \tag{14}$$

We have two possible cases:

- Case 1: $p^n < p^d$. In this case, the market price upon disclosure at the first-period disclosure threshold is higher than the non-disclosure price. In order for this to be the case, the value of the put option upon non-disclosure in time 0 must be *higher* than the value of the put option upon disclosure, i.e., $u_n > u_d$. Hence, the option value upon initial non-disclosure is sufficiently high such that the manager withholds disclosure comparatively more often in the first period. As a result, the price *increases* upon disclosure, as the manager bears additional undervaluation due to the put option from non-disclosure in time 0. This is similar to the excessive delay result presented in Proposition 4 of Acharya et al. (2011).

- Case 2: $p^n > p^d$. Here, the market price upon disclosure is below the non-disclosure market price in the first period. This occurs when the value of the put option upon non-disclosure is *lower* than the value of the put option upon initial disclosure, i.e., $u_n < u_d$. Hence, by disclosing at time 0, the manager can *increase* the option value in the second period. Interestingly, in this case, the market price at time 0 *decreases* upon disclosure by the manager. This implies that the manager is disclosing excessively in time 0, and does so even in cases in which the market price drops after disclosure (relative to non-disclosure). In other words, to improve the option value in the second period, the manager delays less and even sacrifices a higher market price in the first period. This is in contrast to the result in Acharya et al. (2011), as the manager's ex ante disclosure can only improve the market price in their setting.

We now present an important equilibrium property which describes the difference in the threshold-type manager's behavior at time 1 depending on the disclosure history.

Proposition 4 *For the threshold-type manager (i.e., $y_0 = x_0$), the second-period non-disclosure price will be lower, the probability of disclosure in the second period will be higher, and the value of the option to conceal in the second period will be lower if the manager had not disclosed at time zero than if she had disclosed, i.e., $p^{nn} < p^{dn}$, $\theta_n > \theta_d$, and $u_n < u_d$.*

Proposition 4 indicates that, upon non-disclosure in $t = 0$, the threshold-type manager *always* begins disclosure at a lower realization of y_1 than if she had disclosed in $t = 0$. This implies that the threshold-type manager's second-period non-disclosure price is always *lower*, the probability of disclosing in the second period is always *higher*, and the value of the option to conceal information in the second period is always *lower* if she had kept quiet in the first period rather than if she had disclosed y_0. This is perhaps surprising, as the result holds independent of the time 0 threshold x_0 and of realized cash flows s_0.

The equilibrium property $p^{nn} < p^{dn}$ occurs due to the evolving nature of firm value. To more clearly see the intuition, first consider the case where observed cash flows are sufficiently high (while keeping x_0 fixed) such that the market assigns the highest possible value following non-disclosure in $t = 0$. As initial non-disclosure implies that $y_0 \leq x_0$, the market's belief of y_0 upon observing sufficiently high cash flows becomes x_0. That is, under the best situation that the threshold-type manager can imagine, the market will assign a value that is identical to the threshold-type's y_0. Thus, a sufficiently high s_0 implies that the fact that the manager withheld information in time 0 becomes irrelevant and does not deliver any additional information to the market (i.e., the resulting market belief is the same as if she had disclosed). Consequently, in this extreme case, the second-period non-disclosure price upon initial non-disclosure approaches the non-disclosure price upon initial disclosure, i.e., $p^{nn} \to p^{dn}$.

Now, as the value of the observed cash flows decreases, the market's belief of y_0 upon observing s_0 deviates further from the threshold-type's y_0, since the market places greater weight on the possibility that $y_0 < x_0$ upon observing an intermediate value of s_0. This implies that the threshold-type manager ($y_0 = x_0$) becomes relatively more undervalued by the market as s_0 decreases. At the same time, the non-disclosure price in the second period decreases (given non-disclosure in $t = 0$): $p^{nn} < p^{dn}$.

Next, consider the case where the manager uses a sufficiently low time 0 threshold, $x_0 \to -\infty$, while keeping cash flows s_0 fixed. In this extreme case, the second-period non-disclosure price for the threshold-type manager will be low regardless of whether or not she discloses at time 0, i.e., $p^{dn} = p^{nn} \to -\infty$. As the threshold x_0 increases, the time 1 non-disclosure price upon initial disclosure, p^{dn}, increases at a rate κ, while the time 1 non-disclosure price upon initial non-disclosure, p^{nn}, increases at a rate less than κ (Lemmas 1 and 2). Consequently, this implies that the non-disclosure price is lower when the threshold-type manager conceals at time 0 than when she discloses.

In this sense, non-disclosure by the threshold-type manager in the present period always negatively affects the market's belief of the future value. This is "costly" in the sense that the threshold-type manager may be leaving money on the table in *future* periods by not disclosing today. The manager can thus positively influence the market's future beliefs, and thus the non-disclosure price and the option value in the subsequent period, by disclosing today. We note that this is a key distinction between the present framework and the unchanging environment of Acharya et al. (2011), as early disclosure in the latter setting eliminates the option value.

The next two equilibrium properties stated in Proposition 4 build from the above. By not disclosing in $t = 0$, the threshold-type manager gains the put option u_n with strike price p^{nn}. Conversely, by disclosing, the manager gains the put option u_d with strike price p^{dn}. Since the value (i.e., the price) of a put option is increasing in its strike price, we have that the option value upon non-disclosure is always lower than the option value upon disclosure, i.e, $u_n < u_d$, as $p^{nn} < p^{dn}$. Finally, $p^{nn} < p^{dn}$ implies that the likelihood of disclosure for the threshold-type manager is higher in the second period following initial non-disclosure than disclosure.

We thus have the following characterization of the first-period threshold and price:

Theorem 1 *There exists a unique fixed point satisfying equation (14). Moreover, Case 2 always occurs. Also, the first-period dynamic disclosure threshold is lower than the myopic disclosure threshold:* $x_0 < x^*$.

Theorem 1 states that, when firm value evolves over time, the threshold-type manager discloses even though this results in a lower first-period price: $p^d < p^n$. In other words, we find that disclosure by the threshold-type results in a *decrease* in the time 0 market price. Intuitively, the benefit of disclosing in the first period is the possibility that the fundamental value drops in the future. In that case, the manager can hide the reduced value and accept the non-disclosure price in the second period. On the other hand, if it turns out to be the case that the fundamental value has improved, the manager can simply disclose this value to the market.

The main economic force driving the result is that the manager can generate an option value from *revealing* information in the first period, thus inducing excessive disclosure. This is in contrast to extant dynamic disclosure models, which generally feature an option value that is generated from *concealing* information, and results in excessive delay of disclosure. Note that the evolution of the firm value is essential for this result; under the unchanging environment, the option value upon disclosure is always zero. Hence, we have identified the key mechanism—time-varying firm value—which *endogenously* generates excessive disclosure, or, in other words, disclosure which results in a price drop (relative to non-disclosure).

Theorem 1 also helps to explain a pervasive finding in the empirical literature, whereby firms voluntarily release information even though this is met with a negative market reaction (Skinner (1994, 1997), Soffer et al. (2000), Matsumoto (2002), Baik and Jiang (2006), Anilowski et al. (2007), Kross et al. (2011)). We next investigate properties of the equilibrium and discuss several empirical implications that arise from this setting.

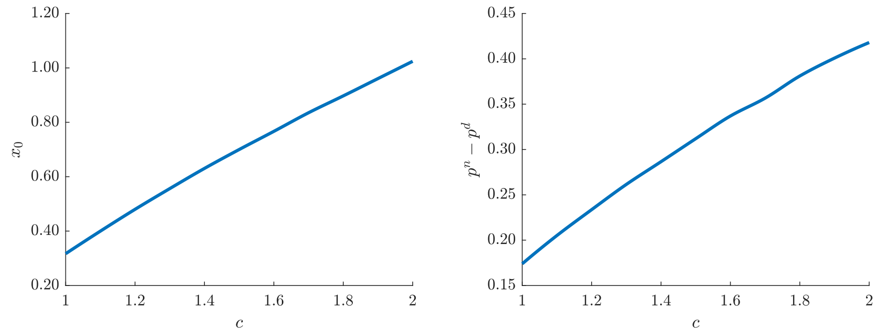
4 Equilibrium Properties

In this section we present a number of equilibrium properties. These properties will be helpful when discussing the empirical implications in Section 5. Below, we examine the disclosure threshold, the price difference from disclosure for the threshold-type manager, and the average return. We first provide a preliminary property regarding the limiting behavior of the first-period threshold.

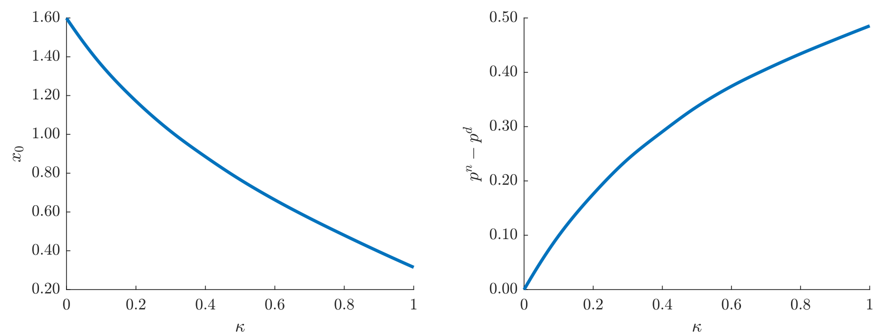
Corollary 1 *We have the following limiting behavior of the first-period disclosure threshold: as $\rho \to 1$ (equivalently, as $\sigma_w \to 0$), or as $\kappa \to 0$, then the equilibrium threshold x_0 approaches the myopic threshold x^*.*

We see that the first-period disclosure threshold is equal to the myopic threshold as ρ approaches one, or equivalently, as σ_w approaches zero. As the signal and the true value become perfectly correlated, the manager no longer considers second-period beliefs in the first-period disclosure decision, since the market learns y_0 in the beginning of $t = 1$ regardless. Accordingly, the manager is only concerned with first-period price, as in the myopic case. In a similar vein, when underlying profitability is independent between the two periods, i.e., $\kappa = 0$, the first-period disclosure decision is irrelevant for market beliefs regarding second-period profitability. As a result, the manager only cares about the first-period price when determining the disclosure decision.

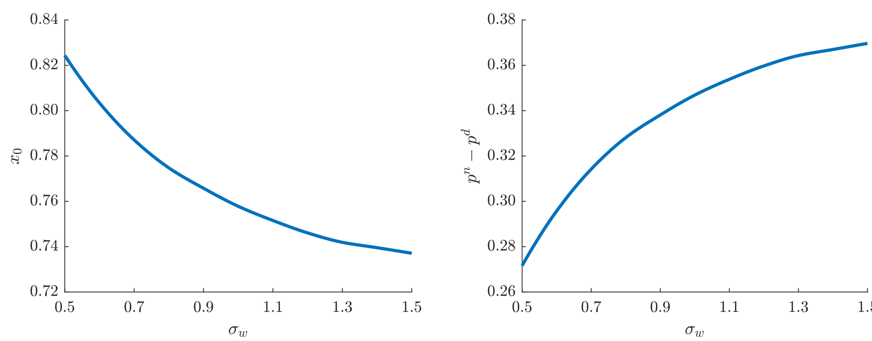
We next briefly discuss how the first-period threshold x_0 changes in the exogenous parameters. First note that, in the static, single-period costly voluntary disclosure models of Jovanovic (1982) and Verrecchia (1983), increasing the volatility of firm value, σ_y, and reducing the disclosure cost are equivalent. This continues to hold in the dynamic setting as well. As σ_y increases, the market's prior information regarding first-period profitability y_0 becomes less informative. Upon non-disclosure in $t = 0$, greater weight is placed on more



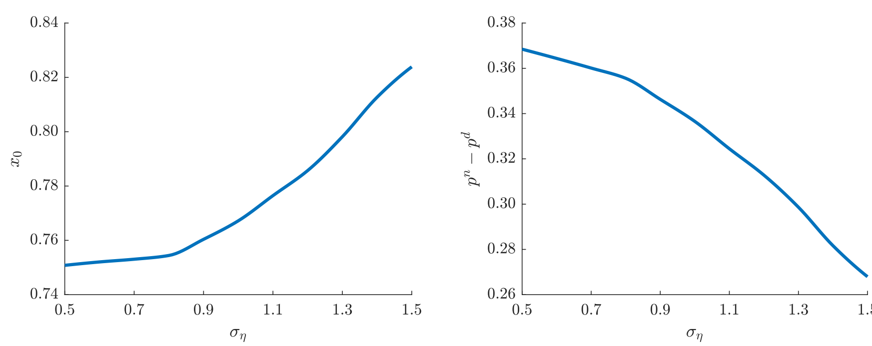
(a) Disclosure cost: c



(b) Autocorrelation: κ



(c) Systematic volatility: σ_w



(d) Idiosyncratic volatility: σ_η

Fig. 3. The left panel shows the first-period threshold, x_0, and the right panel shows the difference between the first-period non-disclosure price and disclosure price for the threshold-type manager, $p^n - p^d(x_0)$. Changes in each are shown for the disclosure cost c, autocorrelation κ, systematic volatility σ_w, and idiosyncratic volatility σ_η. The baseline parameters are: $\sigma_\eta = 1,\ \sigma_y = 0.5,\ c = 1.6$, $\rho = 0.5$, and $\kappa = 0.5$.

extreme values of y_0, which reduces the incentive to withhold information, similar to the static setting. Moreover, the observed dividends s_0 and the disclosure threshold x_0 become relatively more informative for the market given non-disclosure, and therefore have a greater impact on market beliefs as σ_y increases. These dynamic effects additionally amplify the incentive to disclose in the first period under high σ_y.[10]

Our dynamic setting also incorporates volatility regarding the innovation in firm profitability, σ_η. An increase in σ_η leads to an *increase* in the first-period threshold (Figure 3), in contrast to the effect of σ_y above. As σ_η increases, disclosure of y_0 becomes less informative of y_1, thus lowering the value of the option generated from disclosure. At the same time, a higher σ_η implies that the dividend announcement is also less informative of y_1, thus decreasing the option value from non-disclosure. We find that the first effect dominates—u_d decreases at a faster rate than u_n—so that the first-period threshold increases in σ_η. As a result, the manager is more inclined to withhold information at $t = 0$ as σ_η increases.

Next, an increase in autocorrelation κ leads to more disclosure in the first period. This occurs because disclosure of y_0 becomes more informative about y_1 as κ increases. This leads early disclosure to be more valuable to the manager and lowers the disclosure threshold. Likewise, as the volatility σ_w increases, the market puts less weight on the dividend announcement when forming beliefs of y_1 in the absence of disclosure in $t = 0$. This leads disclosure to become more informative and thus more valuable for the manager in the first period, resulting in a lower threshold level.

Finally, an increase in the disclosure cost has two countervailing effects. On one hand, a higher cost *raises* the disclosure option u_d. This occurs because disclosure in the second period becomes less likely as c increases, which implies that disclosure in $t = 0$ is more likely to shape the price in $t = 1$. On the other hand, an increased cost also makes non-disclosure more appealing in the first period. We see in the left panel of Figure 3 that the increase in the option value from disclosure is not sufficient to overtake the direct increase in the disclosure cost, thus resulting in a higher threshold.

A closely related property to the disclosure threshold is the difference between the non-disclosure price and disclosure price for the threshold-type, $p^n - p^d$. This difference signifies the *belief impact* of disclosure in $t = 0$. In particular, early disclosure is more valuable for the manager when the disclosure more strongly influences market beliefs of future profitability. As a consequence, the manager is more inclined to disclose in $t = 0$, and the price drop from disclosure relative to keeping quiet, $p^n - p^d$, is greater. This follows from the fact that the

[10] As the effects of increasing σ_y are similar to those as a decrease in c, we do not report the comparative statics results from changes in σ_y.

value of disclosure is embedded in the option u_d, which inversely affects the disclosure price (equation (14)).

The right panel of Figure 3 shows that the price difference is generally inversely related to the threshold level. This is natural as the manager is more inclined to disclose when the impact of her disclosure will be high. We see that the belief impact from disclosure of y_0 is decreasing in the innovation volatility σ_η, as a higher volatility leads the market to put less weight on y_0. Hence, the price difference $p^n - p^d$ shrinks with heightened uncertainty in y_1. Likewise, as the autocorrelation κ increases, disclosure in the first-period has a greater impact on future beliefs concerning y_1. Correspondingly, we observe a greater price difference as κ increases. A similar explanation holds for the change in σ_w. Finally, an increase in the disclosure cost leads both the price difference and the disclosure threshold to increase. This occurs as increases in c outweigh the increase in the option value u_d, leading p^d to decline more relative to p^n.

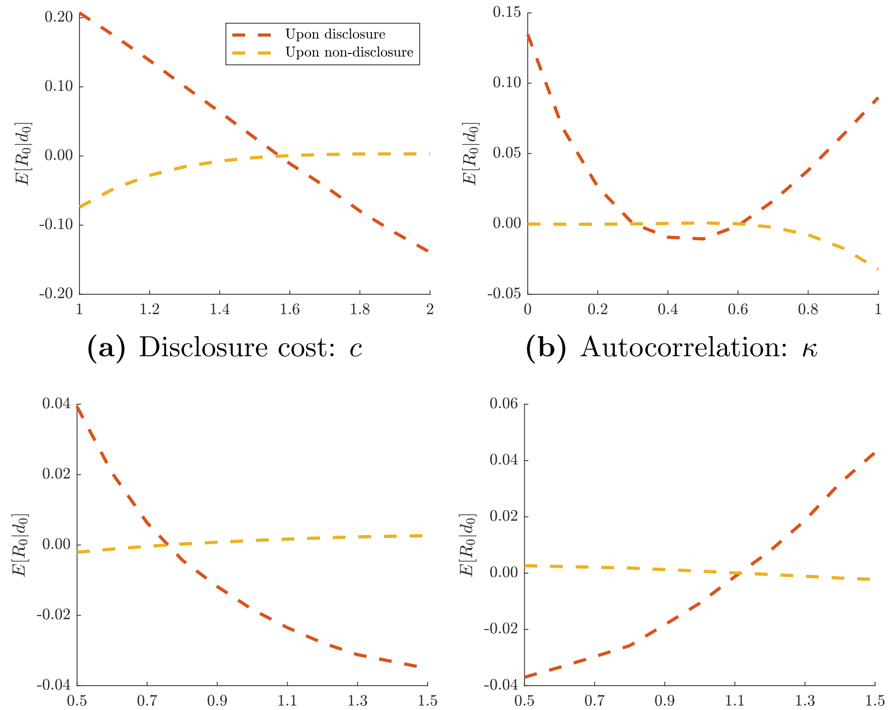
We next discuss a related property concerning the first-period that will also prove helpful when discussing empirical implications. Before proceeding, we first define returns in our setting. The second-period, time 1 cum-dividend return is defined as

$$R_1 = p_1 + s_0 - cd_0 - p_0.$$

Similarly, we define the time 0 return as $R_0 = p_0 - E[p_0]$, where $E[p_0]$ can be considered as the ex ante market price.[11] The return upon disclosure of y_0 in time 0 is denoted as $R^d(y_0)$. We often use the average return for disclosing firms in time 0 and denote this as $\overline{R}^d \equiv E[R^d(y_0)|y_0 \geq x_0]$. The return upon non-disclosure in time 0 is similarly denoted as R^n. We note that all non-disclosing firms in $t = 0$ have a return equal to R^n, and hence R^n coincides with the average return in the first period conditional on non-disclosure.

From Theorem 1, we know that the price always decreases upon disclosure by the threshold-type manager relative to non-disclosure, i.e., $p^n > p^d$ for $y_0 = x_0$. However, all types with first-period profitability above the threshold level disclose in $t = 0$. Hence, as we increase the disclosed value y_0 relative to the threshold x_0, the disclosure price for these higher types continues to rise and the price difference $p^n - p^d$ decreases. As we increase y_0, we can eventually find the *critical type* y_0^* at which the non-disclosure price and disclosure price are equal, i.e., $p^n = p^d(y_0^*)$. Consequently, for types $y_0 > y_0^*$, the first-period return from disclosure is higher than upon non-disclosure, $R^d(y_0) > R^n$. Conversely, for disclosing types below the critical type, i.e., $x_0 < y_0 < y_0^*$, these firms encounter a price decrease upon

[11] That is, we may define a prior period $t = -1$, whereby the corresponding price is $p_{-1} = E[p_0]$.



(a) Disclosure cost: c **(b)** Autocorrelation: κ

(c) Systematic volatility: σ_w **(d)** Idiosyncratic volatility: σ_η

Fig. 4. The average first-period return conditional on the initial disclosure decision: $\overline{R}^d$ and R^n, with changes in the disclosure cost c, autocorrelation κ, systematic volatility σ_w, and idiosyncratic volatility σ_η. The baseline parameters are: $\sigma_\eta = 1,\ \sigma_y = 0.5,\ c = 1.6,\ \rho = 0.5$, and $\kappa = 0.5$.

disclosure and thus a lower return relative to non-disclosure, $R^d(y_0) < R^n$.

A useful property is whether the disclosure return, on average for disclosing types, or the non-disclosure return is higher. We find that this depends on the location of x_0 and y_0^*. In particular, the gap $y_0^* - x_0$ is related to the price difference $p^n - p^d(x_0)$; the greater the price drop from disclosure by the threshold-type, the greater the distance between y_0^* and x_0.

Corollary 2 *The distance between the critical type y_0^*, at which $p^n = p^d(y_0^*)$, and the disclosure threshold x_0 is increasing in the price drop $p^n - p^d(x_0)$ for the threshold type $y_0 = x_0$:*

$$y_0^* - x_0 = \frac{1}{1+\kappa}(p^n - p^d(x_0)).$$

Corollary 2 claims that the distance between y_0^* and x_0 linearly increases in the price difference $p^n - p^d(x_0)$. This helps to identify when we expect the average return from disclosure, $\overline{R}^d$, to be lower than the return from non-disclosure, R^n. We examine this further in Figure 4, which illustrates the average return upon disclosure and non-disclosure under various parameter levels. We see that $\overline{R}^d < R^n$ occurs for low σ_η, high disclosure cost, high σ_w, and for intermediate levels of autocorrelation. In these cases, the critical type y_0^* exceeds the mean of the truncated distribution on the support $[x_0, \infty)$, which leads the average return for disclosing firms to be lower than that of non-disclosing firms. These properties, as well as those presented above, will be helpful when discussing the empirical implications in the following section.

5 Empirical Predictions

We now explore a number of empirical implications of the model. Among other implications, we discuss below the expected pattern of disclosures over time and the variance and skewness of asset returns. We also provide predictions for variation across firms and industries for several of the predictions. As the main nuance in our setting is the dynamic nature of firm value, the discussion of variation across firms focuses primarily on the innovation in firm value. In particular, we interpret the volatility in the shock to underlying profitability, σ_η, as firm idiosyncratic volatility (i.e., firm-specific risk), a frequently used firm characteristic in the empirical literature. Relatedly, we interpret the proprietary disclosure cost c as the level of competition, or degree of concentration, within an industry.[12] The autocorrelation in profitability κ can be interpreted as the autocorrelation or persistence in cash flows, as profitability and cash flows are correlated. Finally, recall that realized cash flows are announced and distributed as dividends, $s_0 = y_0 + w_0$, at the beginning of the second period. The random shock w_0 is an industry or macroeconomic shock to cash flows, and hence we interpret σ_w as industry- or economy-wide systematic volatility. We note that an alternative, though consistent, interpretation of σ_w is the firm's degree of exposure to systematic risk (i.e., low σ_w implies low firm beta).

5.1 Stock Return Skewness

Skewness of returns upon initial disclosure

The model provides several predictions regarding the skewness of asset returns. We begin with the case in which the manager discloses in the first period. Upon disclosure in $t = 0$, the results imply that skewness will always be *positive*. To see this, note that the dividend announcement s_0 does not provide additional information to the market following first-period disclosure. Moreover, the second-period price is bounded below due to the disclosure threshold strategy, which implies that future returns are positively skewed following early disclosure. We additionally examine variation in the level of positive skewness among

[12]A number of papers in the empirical literature link competition to high proprietary costs of disclosure, such as Guo et al. (2004), Jin (2005), Ellis et al. (2012), and Aobdia (2018).

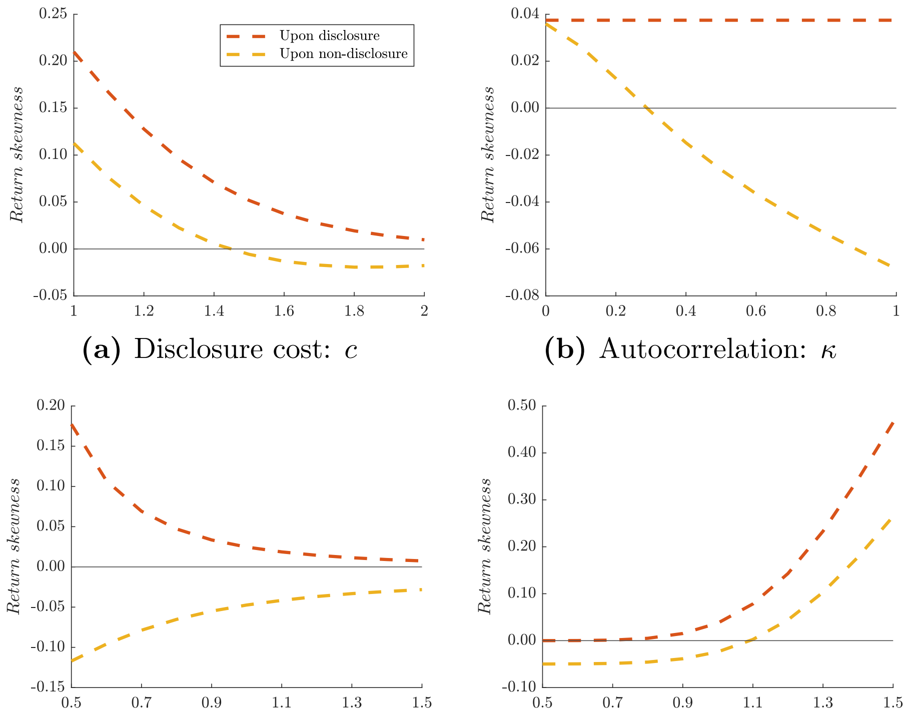
disclosing firms. Figure 5 illustrates the return skewness on the dividend announcement date conditional on the manager's $t = 0$ disclosure decision for various levels of idiosyncratic volatility, disclosure cost, autocorrelation, and systematic risk. We see that return skewness is always positive upon initial disclosure, but declines as idiosyncratic volatility decreases. Under high firm-specific risk, the manager is more likely to benefit from higher realizations of the innovation in cash flows. However, the manager can continue to hide the downside of heightened volatility—extreme bad news—through non-disclosure. As a consequence, return skewness is increasing in σ_η.

Similarly, return skewness upon disclosure is decreasing in the disclosure cost. This is somewhat counter-intuitive as the manager should have greater ability to conceal negative information when c is high. However, the manager is also less inclined to disclose in the second period following disclosure in the first period due to the higher cost. As a consequence, the shock to profitability must be exceptionally high to induce disclosure, and the manager more often accepts the non-disclosure price in the second period, p^{dn}. Hence, skewness following disclosure is decreasing in industry competition. Next, we see that return skewness is unaffected by changes in the autocorrelation. Following disclosure, the market uses the level of autocorrelation to adjust only the conditional mean of second-period profitability y_1. Consequently, the effect on the manager's second-period disclosure propensity is uniform for all levels of persistence, conditional on first-period disclosure (Lemma 1).

Finally, while the informational role of the dividend announcement becomes irrelevant following disclosure, heightened systematic risk affects realized cash flows and thus the second-period return. An increase in the variance of realized cash flows counteracts the truncation effect from the disclosure strategy, leading to lower return skewness. Hence, our predictions regarding stock return skewness conditional on initial disclosure are the following: *Early disclosure is followed by positive future return skewness. This skewness is greater for firms with greater idiosyncratic volatility, firms in less competitive industries, and firms in industries with lower systematic risk.*

Return skewness upon initial non-disclosure

We next consider return skewness conditional on non-disclosure in $t = 0$. Unlike the previous case, upon initial non-disclosure the dividend announcement $s_0 = y_0 + w_0$ provides relevant information about y_0 and affects market beliefs. A poor realization of s_0, perhaps due to a negative systematic shock w_0, leads to a downward revision in market beliefs, thus lowering the threshold level of disclosure. Consequently, the manager discloses her private information



(a) Disclosure cost: c

(b) Autocorrelation: κ

(c) Systematic volatility: σ_w

(d) Idiosyncratic volatility: σ_η

Fig. 5. The conditional return skewness at time 1 with changes in the disclosure cost c, autocorrelation κ, systematic volatility σ_w, and idiosyncratic volatility σ_η. The baseline parameters are: $\sigma_\eta = 1$, $\sigma_y = 0.5$, $c = 1.6$, $\rho = 0.5$, and $\kappa = 0.5$.

to counteract the damage from the negative shock, but this disclosure may be information that she otherwise would have preferred to withhold. This implies that return skewness can be *negative* following non-disclosure. This prediction is similarly noted in Acharya et al. (2011).

Conversely, industry conditions may be favorable and push the dividend announcement to be unexpectedly strong. While we would expect the manager to be more inclined to withhold information following a strong public signal, this may not be the case. Recall that the firm's underlying profitability evolves between the two periods. Due to this property, positive skewness can arise if the innovation in firm profitability overtakes a high systematic shock w_0 to the public signal. For example, suppose that the dividend announcement vastly overstates y_0. In this case, the news is positive and market beliefs improve. However, underlying profitability may also be improving, and to such a magnitude that it surpasses the second-period disclosure threshold, resulting in disclosure by the manager. This implies that, even though the public signal is releasing good news, there is also disclosure of good news by the firm, which thus leads to *positive* skewness in the stock return following non-disclosure in $t = 0$. We note that this feature arises due to the evolving nature of firm value and is one of the novel insights of this setting. Moreover, this helps to explain the findings of Miller (2002), who documents that firms often issue voluntary disclosures immediately following positive shocks to earnings. Specifically, Miller (2002) finds that an increase in earnings is met with a contemporaneous increase in voluntary disclosure of forward-looking information at the time of the earnings announcement. In the context of our model, realized earnings can be interpreted as s_0. A shock to earnings through w_0 when s_0 is announced is followed by disclosure regarding future cash flows y_1 if the innovation to firm profitability η is sufficiently strong. Hence, our model helps to explain this pattern where positive earnings shocks are followed by voluntary disclosure by the firm, and also implies that conditional return skewness can be positive after such events.

Figure 5 helps to distinguish when the two scenarios discussed above can arise. Under low levels of idiosyncratic volatility, return skewness is negative following non-disclosure. When σ_η is low, the value of y_1 is expected to be closer to y_0, and hence it is unlikely that the shock to fundamentals will overtake a positive systematic shock. As a result, the manager is more likely to face disclosure following a bad news release from the dividend announcement, leading returns to be negatively skewed. As idiosyncratic risk rises, so does the likelihood of observing extreme realizations of the innovation, η. This brings us closer to the second case, where the innovation in fundamental value is more likely to outweigh a positive systematic shock. As above, the manager can disclose extreme positive realizations while hiding extreme negative realizations through non-disclosure. Consequently, return skewness increases in σ_η and becomes positive.

In contrast, return skewness in periods of non-disclosure is decreasing in the disclosure cost. Return skewness is positive for low levels of c, where the manager can more readily release positive improvements in underlying profitability. However, as the disclosure cost increases, innovations to the fundamental must be sufficiently strong to overcome positive news from the announcement. At the same time, due to the higher threshold from a high cost, negative news announcements are relatively more informative for the market about fundamental value because of the high level of pooling. This leads the manager to be more receptive to disclosure following bad news, resulting in negative skewness. Next, an increase in the level of autocorrelation leads to more negative return skewness following non-disclosure. As κ increases, an unfavorable dividend announcement carries greater implications for second-period profitability, leading the manager to be more inclined to disclose following bad news announcements. Finally, an increase in systematic risk leads the market to place less emphasis on unfavorable realizations of the dividend announcement, resulting in less bad news disclosure by the manager and an increase in the conditional return skewness.

We thus have the following predictions: *During periods of non-disclosure, return skewness is decreasing in industry competition and autocorrelation in cash flows, and is increasing in idiosyncratic risk and systematic risk. In particular, the conditional return skewness tends to be positive (negative) for firms with high (low) idiosyncratic volatility, low (high) autocorrelation in cash flows, and in less (more) competitive industries.*

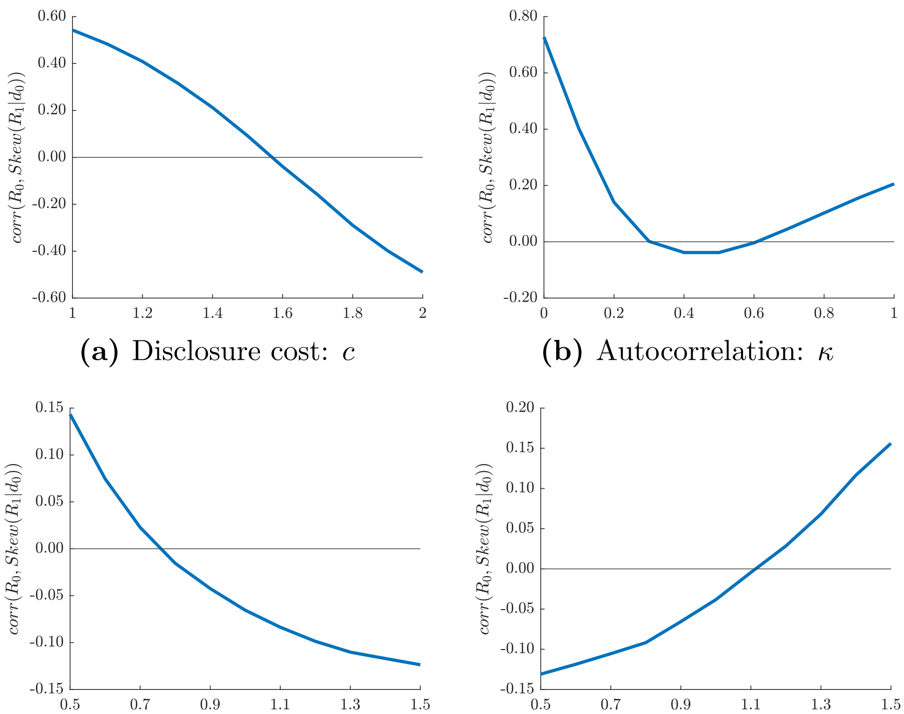
Unconditional skewness

We next consider the unconditional return skewness, which incorporates both of the conditional patterns discussed above. The patterns are similar to those shown in Figure 5 and we therefore omit the figure for brevity. We find that unconditional skewness is negative for low levels of idiosyncratic volatility and increases and becomes positive as σ_η increases. This helps to explain the findings of Engle and Mistry (2014) and Amaya et al. (2015), who document cross-sectional dispersion in realized return skewness. In particular, they find that stock returns are more positively skewed for smaller firms, value firms, highly levered firms, and firms with poor credit ratings. These types of firms are likely to have higher idiosyncratic volatility, consistent with our prediction that such firms should have positive unconditional return skewness. Likewise, Harvey and Siddique (2000), Chen et al. (2001), and Dennis and Mayhew (2002) find that larger firms tend to have more negatively skewed returns. Our results help to explain this somewhat puzzling empirical regularity, as larger firms tend to have lower idiosyncratic risk (e.g., Fu (2009)), and thus negative return skewness.

Additionally, we find that unconditional skewness is decreasing in the disclosure cost. As the cost of disclosure increases, the manager is less likely to disclose in the first period, which exposes her to the possibility of negative systematic shocks. At a broader level, these implications also help to explain how unconditional skewness in returns can be negative. Our predictions are thus: *unconditional return skewness is decreasing in the level of industry competition and in the persistence of cash flows. Return skewness is increasing in firm-specific idiosyncratic risk and in the sensitivity to systematic risk.*

Skewness and returns

These findings also have implications for the relation between skewness and lagged returns. In Figure 6, we examine the correlation between the first-period return R_0 and skewness of the second-period return, $skew(R_1|d_0)$, conditional on the first-period disclosure decision. Before proceeding, we note that skewness conditional on non-disclosure in the first period is



(a) Disclosure cost: c **(b)** Autocorrelation: κ

(c) Systematic volatility: σ_w **(d)** Idiosyncratic volatility: σ_η

Fig. 6. The correlation between the time 0 return and time 1 conditional return skewness: $corr(R_0, Skew(R_1|d_0))$, with changes in the disclosure cost c, autocorrelation κ, systematic volatility σ_w, and idiosyncratic volatility σ_η. The baseline parameters are: $\sigma_\eta = 1,\ \sigma_y = 0.5,\ c = 1.6,\ \rho = 0.5$, and $\kappa = 0.5$.

always lower than skewness conditional on disclosure. This occurs due to the possibility that the manager discloses unfavorable information in the second period following a low public signal s_0 in the former case, whereas this situation does not arise upon disclosure in the first period.

We first observe that skewness is negatively correlated with lagged returns under low levels of idiosyncratic volatility. To see this, recall that disclosure can result in a lower average return than from non-disclosure ($\overline{R}^d < R^n$) when the critical type y_0^* is sufficiently high relative to the threshold x_0, as discussed in Section 4. This is more likely to be the case under low idiosyncratic volatility (Figure 4). Moreover, upon disclosure, firms have higher conditional skewness than under non-disclosure. Hence, disclosing firms with low idiosyncratic risk have a low return in time 0 followed by high skewness in time 1. Correspondingly, non-disclosing firms enjoy a high return R^n relative to disclosing firms, but this is met with low, and often negative, return skewness. Hence, we expect a *negative* correlation between skewness and lagged returns for firms with low idiosyncratic volatility. This helps to explain the findings of Harvey and Siddique (2000) and Chen et al. (2001), who document that firms which experience positive returns tend to exhibit negative return skewness in the following period. Our findings provide additional texture to this regularity and suggest that the relation varies by firm and industry characteristics.

Furthermore, as firm-specific risk rises, the average return for disclosing firms begins to exceed that of non-disclosing firms, $\overline{R}^d > R^n$. As a result, disclosing firms have high returns and high positive skewness, while non-disclosing firms have low returns and low skewness (relative to disclosing firms). In turn, the average correlation between skewness and lagged returns becomes positive for firms with high idiosyncratic risk. Similarly, under low disclosure costs, we have that $\overline{R}^d > R^n$ and high return skewness for disclosing firms, which implies that high returns are followed by high skewness. As competition increases, R^n begins to exceed $\overline{R}^d$ and the correlation between lagged returns and return skewness turns negative. Similar explanations hold for the other parameters as well.

Hence, *we expect a negative correlation between lagged returns and return skewness for firms with low idiosyncratic risk, in more competitive industries, and in industries with greater systematic uncertainty. Likewise, we predict a positive correlation between lagged returns and return skewness for firms with high idiosyncratic risk, in less competitive industries, and in industries with lower systematic uncertainty.*

Aggregate skewness

While the introduction of evolving private information helps to explain the cross section of individual stock return skewness, our model provides an additional implication on the timing of disclosures across firms and aggregate skewness (i.e., the skewness of portfolio returns). Recall that the systematic component in our model is captured by w_0, which affects all firms as a shock to their cash flows in the beginning of the second period. For firms that withheld disclosure in the first period, a negative systematic shock to cash flows leads the market to revise its beliefs concerning underlying profitability. If the downward revision is strong enough, this will compel firms to disclose negative information in the second period. As the negative event impacts most firms in the industry or portfolio who had initially kept quiet, such negative disclosure events are likely to be *homogeneous*, or common, across firms.

Conversely, a systematic positive shock raises the disclosure threshold for firms that initially kept quiet. At the same time, firms learn the idiosyncratic innovation in their underlying profitability. For firms with a strong enough innovation, this may lead to disclosure in the second period, even following a positive systematic shock, as discussed above. However, firms that did not see sufficient improvement in profitability will continue to withhold disclosure in the second period. As shocks to each firm's profitability are idiosyncratic, this generates cross-sectional *heterogeneity* in such positive disclosure events following systematic positive shocks.

The above discussion implies that the portfolio return is expected to exhibit *negative* return skewness. This is because negative or bad news disclosures induced by negative systematic shocks are homogeneous across firms. Meanwhile, the positive disclosure events induced by idiosyncratic positive shocks, which lead these firms to have positive conditional skewness individually, are not found in the market portfolio since such events are heterogeneous across firms. As a result, the portfolio return should be more negatively skewed than the average skewness of individual stock returns. This helps to explain the corresponding well-documented empirical regularity (see, e.g., Badrinath and Chatterjee (1991), Alles and Kling (1994), Chen et al. (2001), Bakshi et al. (2003), Albuquerque (2012)).[13]

5.2 Stock Return Variance

Our dynamic setting allows us to examine the relationship between conditional return variance and asset returns as affected by disclosure decisions. We first discuss a preliminary property. As shown in Section 3, the second-period threshold is higher following disclosure in $t = 0$ than from non-disclosure (Proposition 4). Following disclosure in $t = 0$, we are more likely to observe the non-disclosure price in $t = 1$, and hence there is a steeper truncation of the set of disclosed values in that period. Consequently, the conditional return variance is always *lower* following disclosure in $t = 0$, i.e., $Var(R_1|d_0 = 1) < Var(R_1|d_0 = 0)$, as illustrated in Figure 7.

While the conditional return variance always decreases upon disclosure, the asset return related to this decrease can vary. As discussed in Section 4, when the critical type y_0^* is sufficiently large relative to the threshold x_0, disclosing firms on average have a lower return than non-disclosing firms (i.e., $\overline{R}^d < R^n$), which is accompanied by a decrease in the future return variance due to disclosure in $t = 0$. Meanwhile, non-disclosing types receive a high return, but also see high volatility in returns. Hence, the expected relation is *positive*. Conversely, when y_0^* and x_0 are sufficiently close, disclosing firms receive a higher return on average than non-disclosing firms (i.e., $\overline{R}^d > R^n$). In this case, the expected relation between returns and conditional return variance is *negative*—high returns are associated with low conditional return variance.

We investigate when the relation is expected to be positive or negative. In Figure 8, we examine the correlation between the first-period return R_0 and the conditional variance

[13]We note that a similar prediction is offered in Acharya et al. (2011). However, the presence of both idiosyncratic and systematic shocks in our setting allows this feature to perhaps be more apparent.

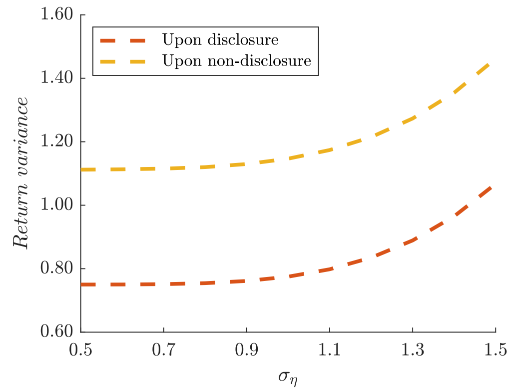
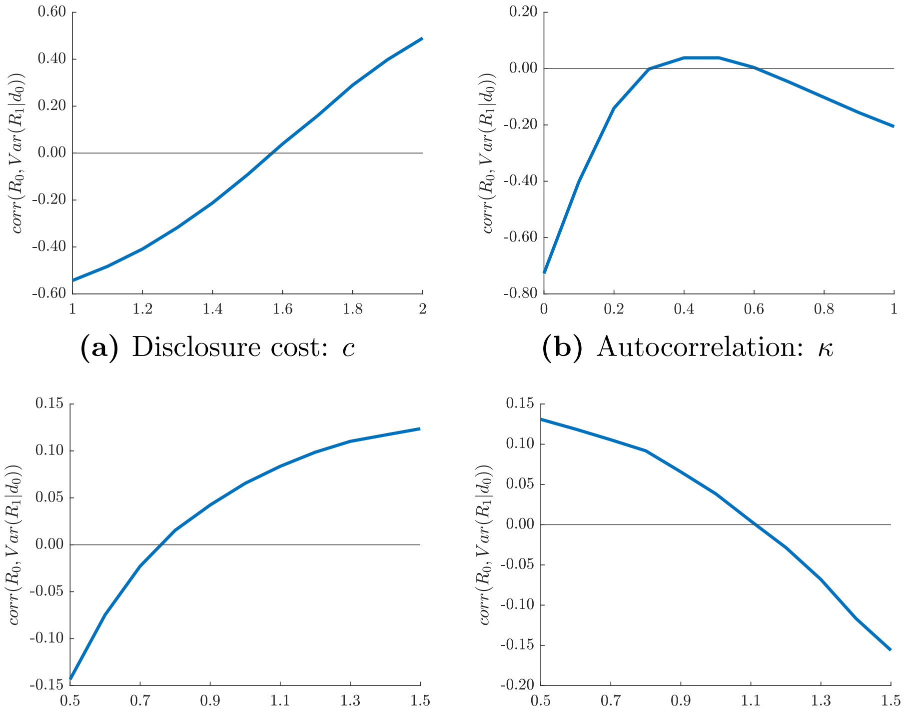


Fig. 7. The time 1 conditional return variance with changes in the idiosyncratic volatility σ_η. The baseline parameters are: $\sigma_\eta = 1,\ \sigma_y = 0.5,\ c = 1.6,\ \rho = 0.5$, and $\kappa = 0.5$.

of the second-period return, $Var(R_1|d_0)$. The relation is negative for high idiosyncratic volatility, low systematic uncertainty, and low levels of the disclosure cost, and is inverse-U shaped in the autocorrelation. Firms with low idiosyncratic volatility are more inclined to disclose, as disclosure has a greater impact on market beliefs. This implies that there is a greater difference between the critical type y_0^* and the threshold x_0, and thus disclosing firms are more likely to have a return that is below the non-disclosure return, $\overline{R}^d < R^n$. As discussed above, disclosing firms have a subsequent return variance that is lower than the return variance of non-disclosing firms. Accordingly, the average relation between the conditional return variance and returns is positive for firms with low idiosyncratic volatility, σ_η. However, as σ_η rises, the distance between y_0^* and x_0 shrinks, and disclosing firms generate a higher return than non-disclosing firms. This leads high returns to be associated with low conditional return variance for firms with high idiosyncratic volatility. The expected variation in the correlation with changes in the disclosure cost, systematic volatility, and autocorrelation have similar interpretations.

These results help to reconcile conflicting findings in the empirical literature with respect to the relation between stock returns and return variance. In particular, some studies have documented a negative relation (e.g., Black (1976), Campbell (1987), Nelson (1991), Glosten et al. (1993), Whitelaw (1994), Brandt and Kang (2004), Lettau and Ludvigson (2010)), while other studies have found a positive relation (e.g., French et al. (1987), Campbell and Hentschel (1992), Ghysels et al. (2005), Guo and Whitelaw (2006), Lundblad (2007), Pástor et al. (2008), Brandt and Wang (2010)). The results provide implications for variation in the relation. Specifically, *the correlation between returns and conditional return variance is (i) negative for firms with high idiosyncratic risk, in less competitive industries, and in industries with low systematic volatility (or for firms with low sensitivity to systematic risk);*



(a) Disclosure cost: c

(b) Autocorrelation: κ

(c) Systematic volatility: σ_w

(d) Idiosyncratic volatility: σ_η

Fig. 8. The correlation between the time 0 return and the time 1 conditional return variance: $corr(R_0, Var(R_1|d_0))$, with changes in the disclosure cost c, autocorrelation κ, systematic volatility σ_w, and idiosyncratic volatility σ_η. The baseline parameters are: $\sigma_\eta = 1$, $\sigma_y = 0.5$, $c = 1.6$, $\rho = 0.5$, and $\kappa = 0.5$.

and (ii) positive for firms with low idiosyncratic risk, in more competitive industries, and in industries with high systematic volatility.

5.3 Correlation of disclosure over time

The model provides implications for firm voluntary disclosure patterns over time. A key result of the model is that disclosure in the present period always results in a higher second-period non-disclosure price (Proposition 4). This implies that voluntary disclosure today lowers the likelihood of disclosure in the future. Interestingly, we find that *all* disclosing manager types ($y_0 \geq x_0$) have a strictly lower likelihood of disclosing in the second period following disclosure in the first period than if they had kept quiet. This follows from the property that the second-period disclosure probability is independent of the disclosed first-period value (Lemma 1). Intuitively, disclosure of y_0 in time 0 shifts the conditional mean of the market's beliefs concerning y_1. As a consequence, the market factors the disclosed value into their beliefs when determining the time 1 threshold x_d, and hence each disclosing man-

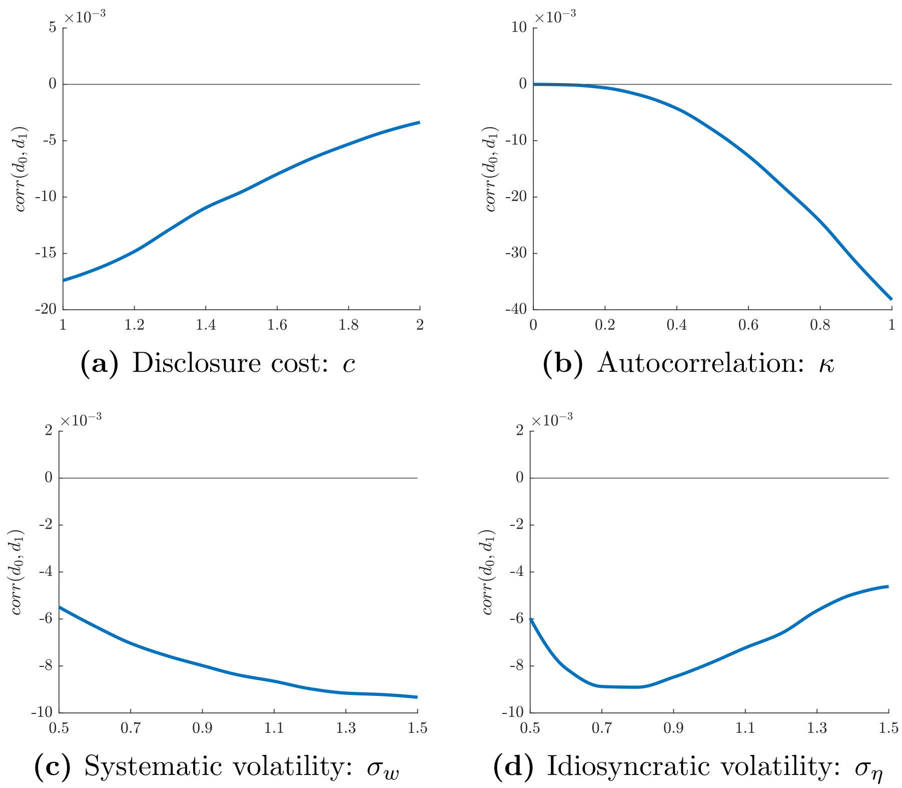


Fig. 9. The autocorrelation in disclosure decisions: $corr(d_0, d_1)$, with changes in the disclosure cost c, autocorrelation κ, systematic volatility σ_w, and idiosyncratic volatility σ_η. The baseline parameters are: $\sigma_\eta = 1$, $\sigma_y = 0.5$, $c = 1.6$, $\rho = 0.5$, and $\kappa = 0.5$.

ager type has the same likelihood of non-disclosure in $t = 1$. This implies that an extremely favorable time 0 disclosure continues to be followed by a lower likelihood of disclosure in time 1.

However, we note that, while all managers above the threshold have a lower probability of disclosing in the future after disclosure in the present (i.e., $\theta_d < \theta_n$ for all $y_0 \geq x_0$), this property does not necessarily hold for all manager types that *withhold* disclosure in equilibrium. Specifically, for very small y_0, the likelihood of non-disclosure continues to be low in the second period as well, and indeed can be lower than the disclosure probability following first-period disclosure (i.e., for some $y_0 < x_0$, we can have $\theta_n < \theta_d$). This is because these extremely low types are able to more effectively pool through non-disclosure in both periods; if they were to disclose in the first period, this would shift market beliefs of the conditional mean sufficiently downward such that they have less of a "cushion" before initiating disclosure in that period.

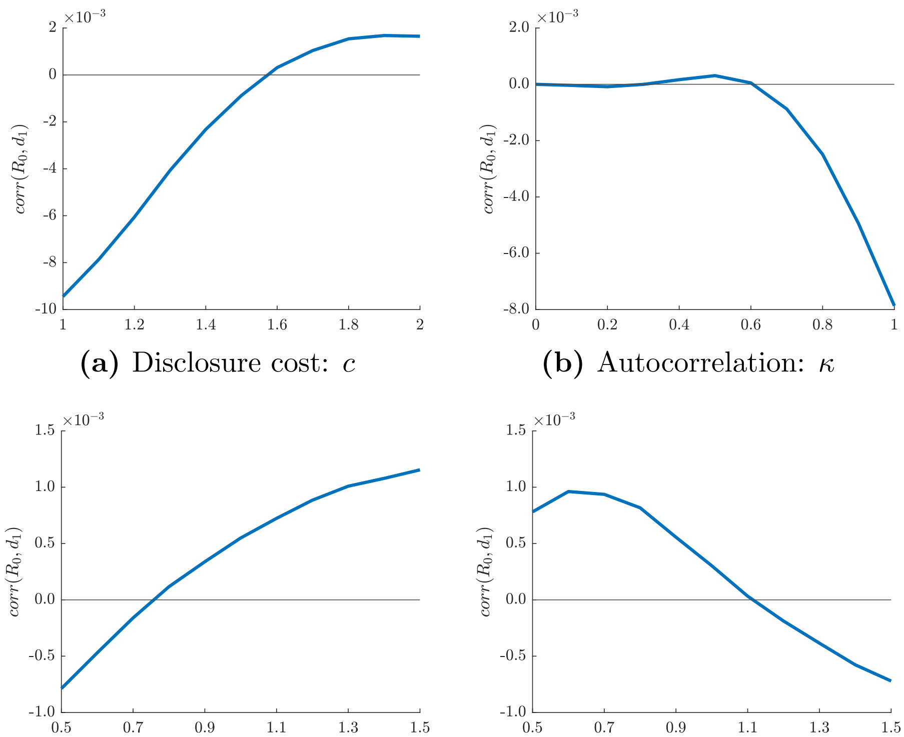
Our numerical exercises, however, indicate that the likelihood of such non-disclosing firms is relatively small. Indeed, we consistently observe unconditional *negative* autocorrelation in disclosure. We further investigate conditions under which this negative autocorrelation in disclosure is expected to be strong. The likelihood of disclosure in the second-period depends on that period's disclosure threshold level. The greater the manager's influence on this threshold, the less likely she is to disclose in the second period. This implies that the belief impact of the current disclosure—how strongly the disclosure influences market beliefs of future profitability—determines the likelihood of future disclosure.

As previously discussed, the belief impact of current-period disclosure is increasing in the level of autocorrelation in profitability, κ. This is natural as greater persistence in the profitability of underlying fundamentals leads to heavier weight on the first-period disclosure when forming beliefs. Moreover, as κ tends to zero, the autocorrelation in disclosure also tends to zero, as firm profitability becomes independent in time. Accordingly, we expect a stronger negative correlation of voluntary disclosure in time for firms or industries where cash flows are more highly autocorrelated.

Interestingly, we find that the negative correlation in disclosure is U-shaped in idiosyncratic risk, whereby the strongest negative correlation occurs under intermediate risk levels. Under low idiosyncratic risk, the manager does not often receive substantial shocks to firm profitability. Consequently, managers with low realizations of y_0 are inclined to withhold in both the first and second period, implying a weaker negative correlation. Under high risk, the manager's disclosure in the first period has less influence on the second-period disclosure threshold, leading to less correlation in disclosure. However, under intermediate volatility levels, disclosure in $t = 0$ strongly influences market beliefs, which decreases second-period disclosure. Meanwhile, first-period non-disclosing firms are more likely to disclose in the second period due to the higher possibility of incurring a sizable shock to profitability.

Next, as systematic risk increases, the dividend announcement becomes less informative, leading to greater disclosure in the first period and thus stronger negative correlation in disclosure. Finally, as the cost increases, it is more likely that the manager withholds information in both periods, which weakens the negative correlation in disclosure. To summarize, we have the following predictions: *Voluntary disclosures are negatively correlated in time. This negative correlation is stronger for firms that have intermediate idiosyncratic volatility, more persistent cash flows, in industries which are less competitive, and are less sensitive to systematic risk (or in industries with lower systematic risk).*

These findings also predict a relation between the firm's voluntary disclosure behavior and lagged returns. As discussed above, the manager is less likely to disclose in $t = 1$ after disclosure in $t = 0$. Moreover, as we saw in Section 4, disclosure results in a lower first-



(a) Disclosure cost: c **(b)** Autocorrelation: κ

(c) Systematic volatility: σ_w **(d)** Idiosyncratic volatility: σ_η

Fig. 10. The correlation between the time 0 return and the time 1 disclosure decision: $corr(R_0, d_1)$, with changes in the disclosure cost c, autocorrelation κ, systematic volatility σ_w, and idiosyncratic volatility σ_η. The baseline parameters are: $\sigma_\eta = 1,\ \sigma_y = 0.5,\ c = 1.6,\ \rho = 0.5$, and $\kappa = 0.5$.

period return, i.e., $\overline{R}^d < R^n$, for low levels of idiosyncratic volatility (Figure 4). This implies a *positive* relation between current period returns and future disclosure. This is exemplified in Figure 10; under low idiosyncratic risk, low returns in the current period predict non-disclosure (i.e., low disclosure) in the future, while high returns should be followed by a higher likelihood of future disclosure. However, as firm-specific risk increases, the expected return from disclosure exceeds that of non-disclosure and we have the opposite pattern—low (resp. high) returns predict future disclosure (resp. non-disclosure).

Similarly, with regard to the disclosure cost c, under low cost levels, the return from disclosure exceeds that from non-disclosure, implying that high current period returns predict future non-disclosure for firms in less competitive industries. The autocorrelation κ follows a more subtle pattern. We see in Figure 4 that the return from disclosure $\overline{R}^d$ is U-shaped in κ. However, as the autocorrelation in profitability decreases towards zero, the correlation in disclosure tends to zero (Figure 9). Accordingly, we should not expect correlation between lagged returns and disclosure for low levels of κ, even if $\overline{R}^d > R^n$. As κ rises, the correlation in disclosure becomes more strongly negative while the return from disclosure exceeds the non-disclosure return. Therefore, we expect a negative association between past returns and disclosure for high levels of persistence in cash flows, and no significant relation when autocorrelation in cash flows is low.

The above predictions are summarized as follows: *For firms with low idiosyncratic volatility, low sensitivity to systematic risk, or in more competitive industries, voluntary disclosure is positively correlated with lagged returns (i.e., low returns predict low subsequent disclosure). For firms with high idiosyncratic volatility, high persistence in cash flows, high sensitivity to systematic risk, or in less competitive industries, voluntary disclosure is negatively correlated to lagged returns.* The empirical literature has largely not considered the relation between disclosure and past returns, however the notion that current performance may help predict future disclosure decisions seems natural.[14] Hence, our results may help to guide empirical investigation.

5.4 Autocorrelation in prices

The results provide implications for the correlation of asset prices over time. As shown in Proposition 4, firms that disclose in the first period have a higher threshold for non-disclosure in the second period, and thus a higher non-disclosure price in that period. Moreover, for disclosing firms, the information conveyed in the dividend announcement s_0 does not impact prices. As a result, disclosure in the second period following first-period disclosure can only be brought by sufficiently strong improvements in the underlying fundamental. Meanwhile, the manager can hide bad news and accept the second-period non-disclosure price when profitability declines. This suggests that prices tend to drift *upward* following disclosure in the first period.

In the case where the average disclosure return exceeds the return from non-disclosure, $\overline{R}^d > R^n$, the *average* price response is positive following disclosure in the first period. Moreover, prices tend to drift upward in the second-period due to the higher non-disclosure price and the manager's ability to capitalize on positive innovations while hiding declines. This is consistent with a number of studies which document serial correlation in prices following corporate information releases, where prices continue to drift upward following the disclosure of good news (e.g., Ball and Brown (1968), Bernard and Thomas (1989), Sadka (2006)). Figure 11 depicts this positive serial correlation in prices, which occurs for low values of c where $\overline{R}^d > R^n$. This is also illustrated in Figure 5, where prices tend to exhibit

[14] Related to our predictions, Miller (2002) finds that voluntary disclosure increases following positive earnings shocks, but does not consider the relation between stock returns and subsequent disclosure.

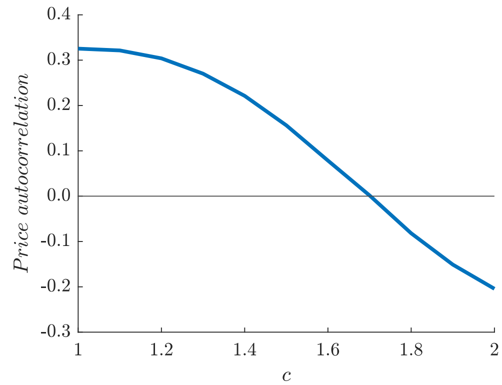
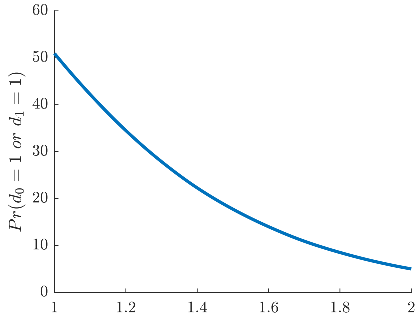


Fig. 11. The autocorrelation in prices: $corr(p_0, p_1)$, with changes in the disclosure cost c. The baseline parameters are: $\sigma_\eta = 1,\ \sigma_y = 0.5,\ \rho = 0.5$, and $\kappa = 0.5$.

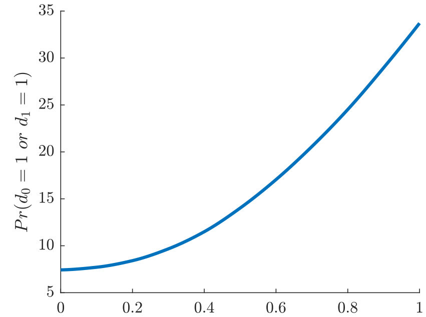
positive skewness following disclosure in the first period.

Conversely, in the case where the non-disclosure return exceeds the disclosure return, $\overline{R}^d < R^n$, disclosing firms on average are met with a price and return decline. However, the higher disclosure threshold in the second period and protection from potential bad news through the public announcement s_0 implies that disclosing managers enjoy higher prices in the future. Likewise, non-disclosing firms receive a higher first-period price than disclosing firms, but prices tend to drift downward in the second period. De Bondt and Thaler (1985, 1987) and Lee and Swaminathan (2000) document that firms which experience recent price declines tend to have higher prices in the future than firms which experience recent price increases, while Jegadeesh (1990) documents negative serial correlation in prices. Our results help to explain this kind of "price reversal" and negative serial correlation as disclosing firms incur a price decline in the present period, but are then met with a higher price in the future.

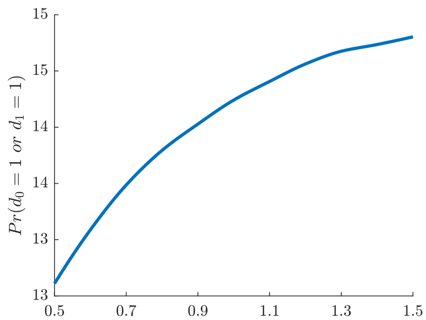
We find that this negative autocorrelation occurs when $\overline{R}^d < R^n$ and the disclosure cost is sufficiently high. A high disclosure cost implies that the first-period threshold is also very high, which results in a greater level of pooling by non-disclosing firms. As a consequence, the downward price drift for these firms is more prominent. Likewise, disclosing firms observe a more severe price drop in the first period due to the high cost of disclosure, which amplifies the upward price drift in time 1. Accordingly, in Figure 11 we observe that prices exhibit negative serial correlation under high levels of disclosure cost and $\overline{R}^d < R^n$. This is also exemplified in panel (a) of Figure 5, where returns are always positively skewed upon disclosure, but are negatively skewed upon non-disclosure in the region where $\overline{R}^d < R^n$ and the disclosure cost is high. Hence, *we expect negative autocorrelation in prices for firms in more competitive industries, and positive autocorrelation in prices for firms in less competitive industries.*



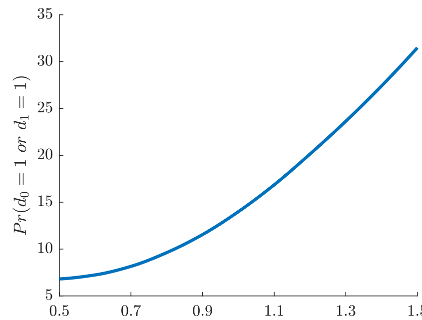
(a) Disclosure cost: c



(b) Autocorrelation: κ



(c) Systematic volatility: σ_w



(d) Idiosyncratic volatility: σ_η

Fig. 12. Price informativeness, defined as the probability that the manager discloses at least once in each of the two periods (%): $Pr(d_0 = 1\ or\ d_1 = 1)$, with changes in the disclosure cost c, autocorrelation κ, systematic volatility σ_w, and idiosyncratic volatility σ_η. The baseline parameters are: $\sigma_\eta = 1$, $\sigma_y = 0.5$, $c = 1.6$, $\rho = 0.5$, and $\kappa = 0.5$.

5.5 Price Informativeness

The model considers private information disclosure and thus has natural implications for the amount of managerial or firm-specific information conveyed in stock prices. Several papers in the empirical literature have considered price informativeness (from the perspective of investors), or how representative the share price is of the firm's fundamental value (e.g., Morck et al. (2000), Durnev et al. (2003), Chan and Chan (2014), Kelly (2014)). Price informativeness in our setting translates to greater voluntary information release by the manager. In particular, we consider the manager's probability of disclosing at least once in the two periods as parameters vary. This analysis provides implications for variation in the level of price informativeness across firms and industries.

We see in Figure 12 that price informativeness is increasing in idiosyncratic volatility. This is somewhat paradoxical since the first-period threshold is increasing in σ_η, implying less first-period disclosure. However, the manager is then more inclined to disclose in the second period, as disclosures are negatively correlated in time. We find that the increase in the second-period disclosure propensity outweighs the corresponding decrease in first-period disclosure likelihood, and hence the probability of making at least one disclosure increases.

This helps to explain the findings of Durnev et al. (2003), who document that prices are more informative for firms that have greater idiosyncratic risk.

Relatedly, the manager is more inclined to disclose in the first-period as the autocorrelation κ increases. While the second-period disclosure probability declines due to increased disclosure in the first period, the first-period effect dominates and leads to a net increase in the disclosure likelihood. As a result, prices become more informative as the persistence in cash flows increases. We also find that price informativeness increases in systematic risk (or equivalently, in the firm's exposure to systematic risk), as first-period disclosure becomes more valuable. Finally, price informativeness is decreasing in the disclosure cost c. This is natural as the manager's incentive to disclose in either period is decreasing in c, leading to lower overall disclosure.

In sum, *prices contain greater (lower) levels of firm-specific information, and are thus more (less) informative to investors, for firms with higher (lower) idiosyncratic volatility, greater (lower) persistence in cash flows, in less (more) competitive industries, or with higher (lower) exposure to systematic risk.*

6 Extensions

In this section, we consider two extensions to the baseline setting. We first allow discounting of the second-period cash flows by the manager and the market. We then consider a three-period extension of our baseline model to examine disclosure policies over a longer horizon. For ease of exposition, we assume that the autocorrelation is equal to one, $\kappa = 1$, in the following extensions.

6.1 Discounting

We introduce discounting in both the first-period market price and the manager's utility function. Suppose that the market discounts time 1 cash flows with a discount factor $\beta \in [0, 1]$, and the manager discounts the time 1 price with a discount factor $\lambda \in [0, 1]$. The time 1 disclosure decision is not affected by discounting. However, the time 0 prices are now given by

$$\begin{aligned} p^d &= (1+\beta)x_0 - c(1+\beta\alpha_d), \\ p^n &= -(1+\beta)\sigma_y \delta\left(\frac{x_0}{\sigma_y}\right) - c\beta E[\alpha_n|\Omega^n]. \end{aligned}$$

Similarly, the manager is now maximizing $p_0 + \lambda E[p_1|y_0]$. Then, the time 0 disclosure threshold is determined by

$$p^d = p^n + \lambda(u_n - u_d).$$

We characterize the equilibrium disclosure behavior of the manager in the limit cases in the following proposition.

Proposition 5 *As $\beta \to 0$ and $\lambda \to 0$, we have that $x_0 \to x^{**}$, where x^{**} is the static disclosure threshold and solves the following equation:*

$$c = \sigma_y v\left(\frac{x^{**}}{\sigma_y}\right). \tag{15}$$

As $\beta \to 1$ and $\lambda \to 0$, we have that $x_0 \to x^*$. As $\beta \to 0$ and $\lambda \to 1$, we have that $x_0 < x^{**}$. As $\beta \to 1$ and $\lambda \to 1$, the model becomes the baseline one. The first-period threshold x_0 is decreasing in β if $E[\alpha_n|\Omega^n] > \alpha_d$, and is decreasing in λ.

As $\beta \to 0$ and $\lambda \to 0$, both the market and the manager become myopic. The market cares only about the first-period cash flow when pricing the firm and the manager also maximizes the first-period price. Thus, the first-period threshold approaches the static threshold x^{**}. As $\beta \to 1$ and $\lambda \to 0$, only the manager becomes myopic, in which case we attain the myopic benchmark presented in Section 3.1. As $\beta \to 0$ and $\lambda \to 1$, the market becomes myopic, and the manager continues to have two real options. Since the option value upon disclosure is higher, the equilibrium continues to exhibit excessive disclosure relative to the static case. Finally, as $\beta \to 1$ and $\lambda \to 1$, the model becomes the baseline one.

As long as $E[\alpha_n|\Omega^n] > \alpha_d$, as the market becomes less myopic, the more excessively the manager discloses in the first period. As β increases, the difference between the disclosure and non-disclosure price increases given that the likelihood of disclosure in the second period is higher upon non-disclosure: $E[\alpha_n|\Omega^n] > \alpha_d$. Thus, the manager is compelled to disclose excessively to increase the option value generated from disclosure, u_d. Conversely, as the manager becomes less myopic, the more she discloses in the first period. This occurs because the difference between the two option values is weighed more heavily in the manager's first-period utility, and hence she is willing to begin disclosure at lower realizations of y_0.

We briefly discuss implications of our model that relate to managerial myopia. We see from the above analysis and from Theorem 1 that the myopic manager discloses less often in the first period than a forward-looking manager. Moreover, as discussed in Sections 5.1 and 5.2, less disclosure in time 0 implies a greater return variance and lower return skewness

This suggests that *higher levels of managerial myopia are correlated with higher stock return variance and lower return skewness.* Recently, the empirical literature has used executive pay duration (Gopalan et al. (2014)) or the level of equity vesting in a certain period (Edmans et al. (2017)) as measures for the extent of short-term incentives and thus myopia. Our results suggest that periods of equity vesting should precede negative skewness in returns and high return variance.

6.2 Three-period extension

While the baseline setting captures the main economic insights of disclosure under evolving information in a simple two-period model, a natural question is how the manager's disclosure policy behaves in a longer horizon. We explore this further and examine a three-period extension of our baseline setting. Firm profitability y_t follows the process

$$y_{t+1} = y_t + \eta_t,$$

and is privately observed by the manager at the start of each period $t = 0, 1, 2$, with probability one. A complicating feature of the model is that market beliefs of underlying profitability depend on the past history of disclosure decisions. This lack of stationarity complicates the analysis as the manager has eight different possible paths by the end of the third period. For tractability and to simplify the exposition, we assume that there is one dividend at the end of the third period, which is equal to profitability y_2, rather than intermediate dividends at the end of each period as in our baseline setting. Following the analysis, we offer discussion on the role of additional public information through an intermediate dividend.

The remainder of the model proceeds as in our baseline setting. The manager makes a disclosure decision, d_t, at the beginning of each period after observing the evolved firm profitability. The market prices the firm based on available information and the manager's disclosure strategy at the end of each period:

$$p_t = E\left[y_2 - c\sum_{j=t}^{2} d_j|\Omega_t\right],$$

where disclosure costs are incurred in the period of disclosure. The manager continues to maximize the sum of expected prices of each period:

$$\max_{d_t} \sum_{t=0}^{2} E[p_t|y_0].$$

At the start of the third period ($t = 2$), we have four different states that depend on the disclosure history: $\omega_2 \in \{dd, dn, nd, nn\}$, where the first letter represents the time 0 disclosure decision and the second one represents the time 1 disclosure decision (d denotes disclosure, n denotes non-disclosure).

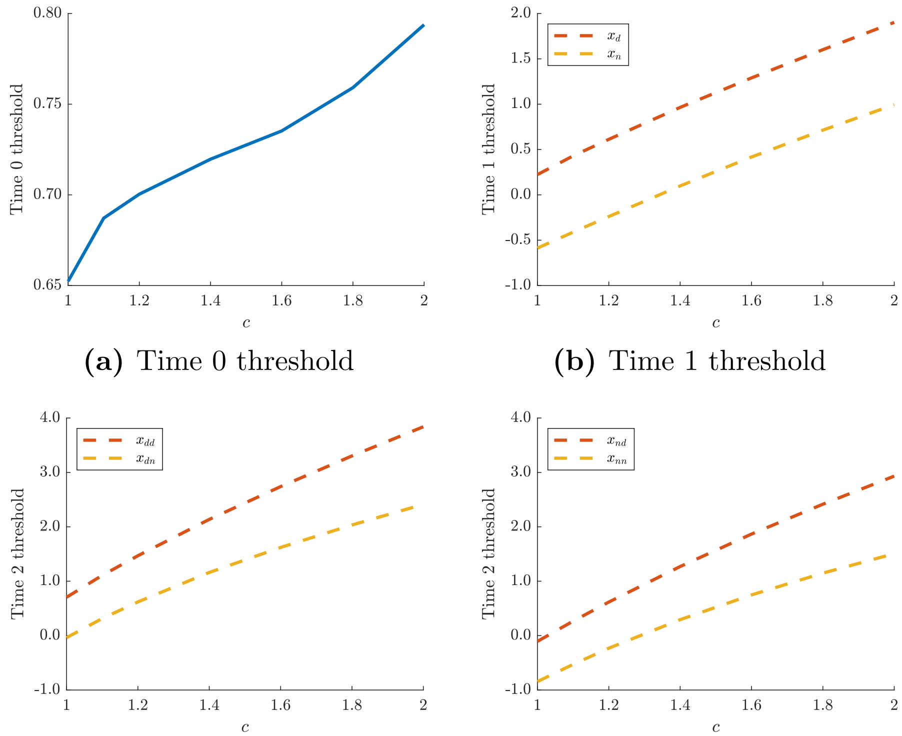
In Appendix B, we analyze the manager's disclosure decision backwards given a time 0 disclosure decision and then derive the time 0 equilibrium condition. This involves determining the disclosure threshold under each possible history in each period. In contrast to the baseline setting, we now define the manager's option value in terms of the value function from continuation under a particular history. We ultimately find that the equilibrium indifference condition for the threshold-type is given by

$$p^d + E[\max(v^{dd}, v^{dn})|y_0 = x_0] = p^n + E[\max(v^{nd}, v^{nn})|y_0 = x_0],$$

where v denotes the manager's value function at time 1, with a superscript that denotes the disclosure decisions in the first and second period.

As in the baseline setting, the manager can influence market beliefs of future value by disclosing in the present period, and thus early disclosure generates a real option. Likewise, by withholding disclosure, the manager saves on disclosure costs and can continue to capitalize on future improvements to firm value, which provides an option value to withholding disclosure. Due to the complexity of the model, we use numerical solutions to investigate the manager's equilibrium disclosure strategy. We first examine if the main qualitative properties of our baseline setting continue to hold in this extended setting.

In Figure 13, we plot the disclosure thresholds in each period as based on the disclosure history. The corresponding prices are shown in Figure 14. We see that the disclosure threshold in time 1 is consistently higher upon disclosure in time 0 than upon non-disclosure, i.e., $x_d > x_n$. Similarly, the third-period threshold is higher upon disclosure in the second period, $x_{dd} > x_{dn}$ and $x_{nd} > x_{nn}$. Likewise, the non-disclosure price in the current period is always higher following disclosure in the previous period. This suggests that one of the main features of the baseline setting, whereby present-period disclosure improves the option value of withholding disclosure in the future, continues to be preserved in this extended setting.



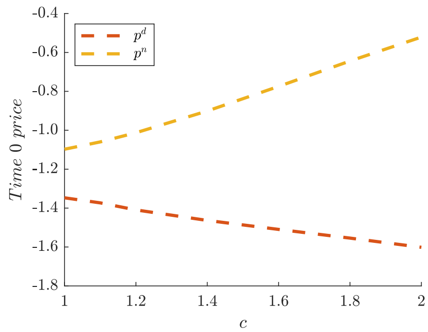
(a) Time 0 threshold **(b)** Time 1 threshold

(c) Time 2 threshold upon $d_0 = 1$ **(d)** Time 2 threshold upon $d_0 = 0$

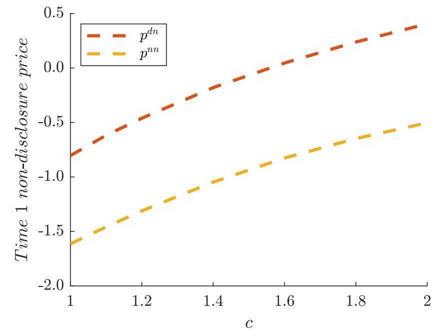
Fig. 13. The disclosure thresholds in each period and after each history in the three-period extended setting. We illustrate the change in the time 0 disclosure threshold (panel (a)), time 1 threshold (panel (b)), time 2 threshold upon disclosure in time 0 (panel (c)), and the time 2 threshold upon non-disclosure in time 0 (panel (d)) with changes in the disclosure cost c. The baseline parameters are: $\sigma_y = 0.5$ and $\sigma_\eta = 1$.

Moreover, even though the manager generates an option value in $t = 1$ from disclosure, she is more inclined to withhold information in that period after disclosure in $t = 0$, as evidenced by the fact that $x_d > x_n$. However, the real option generated from disclosure, and thus excessive disclosure, continues to persist as the non-disclosure price in time 0 exceeds the disclosure price (panel (a) of Figure 14).

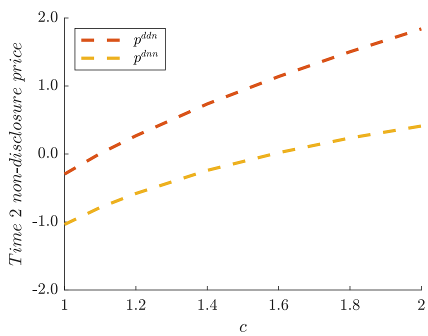
This extended setting also provides additional implications concerning time series patterns of disclosure. Notably, we see that the disclosure threshold in $t = 2$ is higher after disclosure in $t = 0$, even if this is followed by non-disclosure in $t = 1$, i.e., $x_{dn} > x_{nn}$. That is, in the case where the manager withheld disclosure in $t = 1$, the non-disclosure price in $t = 2$ is higher if the manager had disclosed in $t = 0$ than if she had kept quiet. Hence, even though the manager made the same decision in time 1 (non-disclosure), the manager who disclosed at time 0 will receive a higher non-disclosure price at time 2 (also shown in Figure 14, where $p^{dnn} > p^{nnn}$). For this reason, the manager is more inclined to conceal at time 1 upon disclosure in time 0, as mentioned above. This suggests that early disclosure has a "ripple effect" and long-term consequences for future periods, whereby the non-disclosure price in later periods continues to be higher if the manager has not disclosed recently but in two periods prior. Relatedly, we see that the lack of early disclosure also has long-term



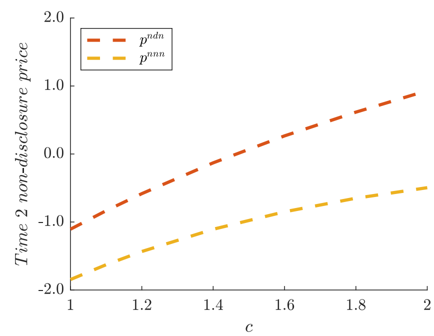
(a) Time 0 price



(b) Time 1 non-disclosure price



(c) Time 2 non-disclosure price upon $d_0 = 1$



(d) Time 2 non-disclosure upon $d_0 = 0$

Fig. 14. The non-disclosure price at time 0, 1, and 2. We illustrate the time 0 price, p^n and p^d for the threshold-type manager, $y_0 = x_0$ (panel (a)), the time 1 non-disclosure price, p^{dn} and p^{nn} (panel (b)), the time 2 non-disclosure price, p^{ddn} and p^{dnn}, upon disclosure in time 0, for the threshold-type manager, $y_1 = x_d$ (panel (c)), and the time 2 non-disclosure price, p^{ndn} and p^{nnn} upon non-disclosure in time 0, for the threshold-type manager, $y_1 = x_n$ (panel (d)) with changes in disclosure cost c. The baseline parameters are $\sigma_y = 0.5$, and $\sigma_\eta = 1$.

consequences on the manager's future disclosure behavior. This is captured by the property $x_{dd} > x_{nd}$. Even though the manager has disclosed in the second period in both histories, the third-period threshold following disclosure in the first period is greater. This suggests that disclosure in the second period is not sufficient to "un-do" the effects of non-disclosure in the first period.

Another notable feature we find is that the minimum innovation to profitability necessary to induce disclosure *increases* the longer that the manager has disclosed, as captured by the property that $x_{dd} - x_d > x_d - x_0$. To see this, consider the case in which the manager has disclosed in the first period. After disclosure in $t = 0$, the manager faces two countervailing incentives in $t = 1$. On the one hand, the manager is incentivized to withhold information in the second period due to the higher threshold level following disclosure in the first period (i.e., $x_d > x_n$). However, the manager also has an incentive to disclose in $t = 1$, as this raises the disclosure threshold in $t = 2$. In contrast, only the first effect is present in the terminal period, $t = 2$. This implies that the minimum innovation in profitability required for disclosure in $t = 1$ must be *lower* than that in $t = 2$, even conditional on the history that the manager has disclosed in every prior period. Hence, after consecutive disclosure, larger innovations to the fundamental are necessary to induce further disclosure. This implies that firms which have disclosed frequently have the greatest cushion for non-disclosure, and will only disclose following the arrival of a major breakthrough. Conversely, in the continual absence of disclosure, market beliefs deteriorate. Indeed, the profitability level necessary for disclosure continues to decrease as the manager consecutively hides information, as evidenced by the feature that $x_{nn} < x_n < x_0$. As the manager continually withholds, the market grows increasingly pessimistic, and this reaches a point where even minor improvements in profitability induce disclosure.

These findings consistently suggest that early disclosure continues to be valuable for the manager and generates higher prices in the absence of disclosure in future periods. While we have not included an intermediate dividend in this setting for tractability, we expect that a public signal would behave similarly as in the baseline setting. In particular, this provides additional incentive for the manager to withhold disclosure in periods prior to the release of public news through dividends. This would be reflected in the manager's disclosure threshold in each period and history. For example, if an intermediate dividend was to be released at the beginning of time 2, this would raise both of the time 1 thresholds x_d and x_n, as withholding disclosure becomes more appealing.

7 Concluding remarks

Voluntary information release is an ubiquitous activity by firms and is central to the process of price discovery. Firms frequently disclose information voluntarily that is met with a negative market reaction. We capture this feature in a parsimonious setting where the firm's underlying profitability evolves over time. We find that the manager may voluntarily disclose information even if this results in a lower price than if she had concealed the information. The manager endures this price drop for the purpose of generating a real option which allows her to conceal information more often in the future. The implication is that disclosure in the present period positively influences the market's beliefs concerning the evolution of the firm's value, and thus increases the price upon non-disclosure in a future period. We find that this result holds even in the face of a public signal which may overstate the firm's value, thus providing the manager with an option value from delaying disclosure.

The results of this study provide a rich set of avenues for future research. In particular, the results provide asset pricing implications regarding conditional and unconditional stock return skewness, return variance, and price autocorrelation, as well as predictions regarding the pattern of disclosures over time and price informativeness. We tie the moments and properties we consider to lagged returns to provide additional insights. These include a negative correlation between lagged returns and return skewness for firms with low idiosyncratic risk, which helps to explain findings in the empirical literature. The predictions also consider future disclosure behavior based on past returns, which helps to link the firm's past performance to future expected disclosure behavior.

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Appendix

A Proofs

We first establish the following lemmas that will be used to prove the formal statements.

Lemma A1 *Define $v(x) = x + \delta(x)$. Then, $v(x)$ is non negative, and increasing in x. Furthermore, $\delta(x)$ is weakly decreasing in x. Finally, $\lim_{x\to-\infty} v(x) = 0$, $\lim_{x\to\infty} \delta(x) = 0$, $\lim_{x\to-\infty} \delta(x)v(x) = 1$, and $\lim_{x\to\infty} \delta(x)v(x) = 0$.*

Proof of Lemma A1. First, we want to show that $\delta(x) \geq -x$ so that $v(x) \geq 0$. When $x \geq 0$, clearly it holds. For $x < 0$, define $R(x) = \delta(x)^{-1}$. Then, we want to show that $R(x) \leq -1/x$ for $x < 0$. The first derivative is

$$R'(x) = 1 + xR(x), \tag{A.1}$$

and we also have

$$\lim_{x\to-\infty} xR(x) = -1. \tag{A.2}$$

Suppose that at any point $x_1 < 0$, $R(x_1) > -1/x_1$, i.e. $x_1R(x_1) < -1$ by contradiction. Then, by (A.1) $R'(x) < 0$ and $R(x)$ would continue to increase with decreasing x. This also implies that $xR(x)$ would continue to decrease, hence we should have $xR(x) < -1$ for any $x \leq x_1$, which contradicts (A.2). Therefore we show that $R(x) \leq -1/x$, i.e. $\delta(x) \geq -x$ for $x < 0$ also.

Next, we want to show that $v'(x) > 0$. The first derivative of $v(x)$ is given by

$$v'(x) = 1 - \delta(x)v(x).$$

Notice that this is the variance of a standard normal variable ξ conditional on $\xi < x$. Since this must be positive, we have $v'(x) > 0$. This also implies that $\delta(x)v(x) < 1$ and $-1 < \delta'(x) = -\delta(x)v(x) \leq 0$ since $\delta(x) > 0$ and $v(x) \geq 0$.

Finally, since $\delta(x)$ is the negative mean of a standard normal variable with one-sided truncation of the upper tail at x, we have that $v(x) \to 0$ as $x \to -\infty$ and $\delta(x) \to 0$ as $x \to \infty$. This also implies that $\delta'(x) = -\delta(x)v(x) \to -1$ as $x \to -\infty$ and $\delta'(x) = -\delta(x)v(x) \to 0$ as $x \to \infty$. ■

Lemma A2 *Define the following function for any $\xi > 0$:*

$$F(x, y; \xi) = \int_{-\infty}^{y} \Phi\left(-x + \xi z\right) \frac{\phi(z)}{\Phi(y)} dz.$$

Then, we have the following properties: $F(x, y; \xi) < \Phi(-x+\xi y)$, $F_x(x, y; \xi) < 0$, $F_y(x, y; \xi) > 0$, $\xi F_x(x, y; \xi) + F_y(x, y; \xi) < 0$, $\lim_{y\to-\infty} F(x, y; \xi) = \lim_{y\to-\infty} \Phi(-x + \xi y)$.

Proof of Lemma A2. The first property holds since

$$F(x, y; \xi) < \int_{-\infty}^{y} \Phi\left(-x + \xi y\right) \frac{\phi(z)}{\Phi(y)} dz = \Phi\left(-x + \xi y\right).$$

Take the partial derivative with respect to x:

$$F_x = -\int_{-\infty}^{y} \phi\left(-x+\xi z\right)\frac{\phi(z)}{\Phi(y)}dz < 0.$$

Take the partial derivative with respect to y:

$$\begin{aligned} F_y &= \delta(y)\left[\Phi(-x+\xi y) - \int_{-\infty}^{y} \Phi\left(-x+\xi z\right)\frac{\phi(z)}{\Phi(y)}dz\right] \\ &= \xi\delta(y)\int_{-\infty}^{y} \phi\left(-x+\xi z\right)\frac{\Phi(z)}{\Phi(y)}dz > 0. \end{aligned}$$

Thus, we have

$$\xi F_x(x,y;\xi) + F_y(x,y;\xi) = \xi\int_{-\infty}^{y} \phi\left(-x+\xi z\right)\left(\delta(y)-\delta(z)\right)\frac{\Phi(z)}{\Phi(y)}dz < 0.$$

Finally, $F(x,y;\xi)$ can be expressed as

$$F(x,y;\xi) = \Phi(-x+\xi y) + \xi\int_{-\infty}^{y} \phi\left(-x+\xi z\right)\frac{\Phi(z)}{\Phi(y)}dz,$$

which implies that $\lim_{y\to-\infty} F(x,y;\xi) = \lim_{y\to-\infty}\Phi(-x+\xi y)$. ■

Lemma A3 *The market's belief of the probability of disclosure at time 1 given non-disclosure and cash flows, α_n, is a function of $g = x_0 - \rho^2 s_0$ and has the following properties: (i) $\alpha_n(g) \to \alpha_d$ and $d\alpha_n(g)/dg < 0$ as $g \to -\infty$, and (ii) $\alpha_n(g) \to \bar{\alpha} < 1$ and $d\alpha_n(g)/dg > 0$ as $g \to \infty$, where $\bar{\alpha}$ is defined in (A.3).*

Proof of Lemma A3. The market's belief of the likelihood of disclosure at time 1 given non-disclosure and cash flows at time 0 can be expressed using the function $F(x,y;\xi)$:

$$\begin{aligned} \alpha_n(g) &= E[\theta_n(y_0)|s_0, y_0 < x_0] \\ &= E\left[\Phi\left(-\frac{\epsilon^*(g) - \kappa(y_0-\rho^2 s_0)}{\sigma_\eta}\right)|s_0, y_0 < x_0\right] \\ &= \int_{-\infty}^{g}\Phi\left(-\frac{\epsilon^*(g)-\kappa z}{\sigma_\eta}\right)\frac{\phi\left(\frac{z}{\sigma_z}\right)}{\sigma_z\Phi\left(\frac{g}{\sigma_z}\right)}dz \\ &= F\left(\frac{\epsilon^*(g)}{\sigma_\eta}, \frac{g}{\sigma_z}; \frac{\kappa\sigma_z}{\sigma_\eta}\right). \end{aligned}$$

The second equality follows from Lemma 3 and the third equality follows from the observation that given cash flows, y_0 can be expressed as $y_0 = \rho^2 s_0 + z$, where z is a normal variable with mean zero and variance $\sigma_z^2 = (1-\rho^2)\sigma_y^2$, and the additional non-disclosure information implies that $y_0 = \rho^2 s_0 + z < x_0 \leftrightarrow z < x_0 - \rho^2 s_0 = g$. Taking g to $-\infty$, then we have

$$\lim_{g\to-\infty}\alpha_n(g) = \lim_{g\to-\infty} F\left(\frac{\epsilon^*(g)}{\sigma_\eta}, \frac{g}{\sigma_z}, \frac{\kappa\sigma_z}{\sigma_\eta}\right) = \lim_{g\to-\infty}\Phi\left(-\frac{\epsilon^*(g)-\kappa g}{\sigma_\eta}\right) = \Phi\left(-\frac{\eta^*}{\sigma_\eta}\right) = \alpha_d.$$

The second equality is due to Lemma A2 and the third one is due to Proposition 4. Taking g to ∞, then we have $\alpha_n(g) \to \bar{\alpha}$, where $\bar{\alpha}$ is given by

$$\bar{\alpha} = \lim_{g\to\infty} F\left(\frac{\epsilon^*(g)}{\sigma_\eta}, \frac{g}{\sigma_z}, \frac{\kappa\sigma_z}{\sigma_\eta}\right) = \int_{-\infty}^{\infty} \Phi\left(-\frac{\bar{\epsilon}-\kappa z}{\sigma_\eta}\right) \frac{\phi\left(\frac{z}{\sigma_z}\right)}{\sigma_z} dz < 1, \tag{A.3}$$

where $\bar{\epsilon}$ is defined in Proposition 4. Lastly, we can take the first derivative of $\alpha_n(g)$:

$$\frac{d\alpha_n(g)}{dg} = \frac{1}{\sigma_z}\left[\frac{\sigma_z}{\sigma_\eta}\frac{d\epsilon^*(g)}{dg}F_x + F_y\right].$$

By Proposition 4 and A2, we have as $g \to -\infty$

$$\frac{d\alpha_n(g)}{dg} \to \frac{1}{\sigma_z}\left[\frac{\kappa\sigma_z}{\sigma_\eta}F_x + F_y\right] < 0,$$

and as $g \to \infty$, we have $d\alpha_n(g)/dg \to F_y/\sigma_z > 0$. ■

Lemma A4 *Define a function $k(x, y; \xi)$ for any $\xi > 0$*

$$k(x, y; \xi) = \int_{-\infty}^{y} v\left(x - \xi z\right) \frac{\phi(z)}{\Phi(y)} dz.$$

We have the following properties of $k(x, y; \xi)$:

$$k_x(x, y; \xi) > 0, \quad \textit{and} \quad k_x(x, y; \xi) + \frac{1}{\xi}k_y(x, y; \xi) > 0.$$

Proof of Lemma A4. The first derivative with respect to x is given by

$$k_x(x, y; \xi) = \int_{-\infty}^{y} (1 - \delta(x - \xi z)v(x - \xi z))\frac{\phi(z)}{\Phi(y)} dz > 0.$$

The second inequality is due to Lemma A1. Take the first derivative with respect to y:

$$\begin{aligned}
k_y(x, y; \xi) &= \delta(y)\left[v(x - \xi y) - \int_{-\infty}^{y} v(x - \xi z)\frac{\phi(z)}{\Phi(y)}dz\right] \\
&= \delta(y)\left[v(x - \xi y) - v(x - \xi z)\frac{\Phi(z)}{\Phi(y)}|_{-\infty}^{y} - \xi\int_{-\infty}^{y} (1 - \delta(x - \xi z)v(x - \xi z))\frac{\Phi(z)}{\Phi(y)}dz\right] \\
&= -\xi\delta(y)\int_{-\infty}^{y} (1 - \delta(x - \xi z)v(x - \xi z))\frac{\Phi(z)}{\Phi(y)}dz.
\end{aligned}$$

The third equality holds since we have

$$\Phi(z) = \left(1 - \frac{1}{z^2} + \mathcal{O}\left(\frac{1}{z^2}\right)\right)\frac{\phi(z)}{|z|},$$

and thus

$$\lim_{z\to-\infty} v(x - \xi z)\Phi(z) = \lim_{z\to-\infty} \frac{x - \xi z}{|z|}\left(1 - \frac{1}{z^2} + \mathcal{O}\left(\frac{1}{z^2}\right)\right)\phi(z) = 0.$$

Finally, we have

$$k_x(x,y;\xi)+\frac{1}{\xi}k_y(x,y;\xi)=\int_{-\infty}^{y}(1-\delta(x-\xi z)v(x-\xi z))(\delta(z)-\delta(y))\frac{\Phi(z)}{\Phi(y)}dz>0.$$

The second inequality is due to $\delta(z)>\delta(y)$ for $z<y$ and $\delta(\cdot)v(\cdot)<1$. ■

A.1 Proof of Proposition 1

The result follows from Lemmas A1 and A3.

A.2 Proof of Proposition 2 and Lemma 1

Proposition 2 follows from Lemma A1. To prove Lemma 1, notice that $p^{dn}=x_d-c=\kappa y_0+\eta^*-c$ and thus it is increasing in y_0 at a rate equal to κ and is independent of s_0 and x_0. The manager discloses at time 1 upon initial disclosure only when $y_1=\kappa y_0+\eta\geq x_d=\kappa y_0+\eta^*$, i.e. $\eta\geq\eta^*$. Given the market's information set, it also happens when $\eta\geq\eta^*$. Thus, the market's belief coincides with the true probability of disclosure in the second period which is given by

$$\alpha_d=\theta_d=\Pr(\eta>\eta^*)=\Phi\left(-\frac{\eta^*}{\sigma_\eta}\right).$$

We can see that θ_d is independent of y_0, s_0, and x_0.

A.3 Proof of Proposition 3 and Lemma 2

Notice that x_0 and s_0 are public information and can be summarized as one variable $g=x_0-\rho^2 s_0$. Thus, ϵ^* should be a function of g. We can express (10) using $k(x,y;\xi)$:

$$\begin{aligned}
c &= \epsilon^*(g)-E\left[E[\kappa z+\eta|z,\eta<\epsilon^*(g)-\kappa z]|z<g\right]\\
&= \epsilon^*(g)-E\left[\kappa z-\sigma_\eta\frac{\phi(\frac{\epsilon^*(g)-\kappa z}{\sigma_\eta})}{\Phi(\frac{\epsilon^*(g)-\kappa z}{\sigma_\eta})}|z<g\right]\\
&= \epsilon^*(g)+\sigma_\eta\int_{-\infty}^{g}\left[-\frac{\kappa z}{\sigma_\eta}+\delta\left(\frac{\epsilon^*(g)-\kappa z}{\sigma_\eta}\right)\right]\frac{\phi(\frac{z}{\sigma_z})}{\sigma_z\Phi(\frac{g}{\sigma_z})}dz\\
&= \sigma_\eta k\left(\frac{\epsilon^*(g)}{\sigma_\eta},\frac{g}{\sigma_z};\frac{\kappa\sigma_z}{\sigma_\eta}\right).
\end{aligned}\tag{A.4}$$

By Lemma A4, given g, the right hand side of (A.4) is increasing in the first argument so that there exists a unique fixed point, which proves Proposition 3.

Next, totally differentiate (A.4), then we have

$$0<\frac{d\epsilon^*(g)}{dg}=-\frac{\frac{\sigma_\eta}{\sigma_z}k_y\left(\frac{\epsilon^*}{\sigma_\eta},\frac{g}{\sigma_z};\frac{\kappa\sigma_z}{\sigma_\eta}\right)}{k_x\left(\frac{\epsilon^*}{\sigma_\eta},\frac{g}{\sigma_z};\frac{\kappa\sigma_z}{\sigma_\eta}\right)}<\kappa,$$

by Lemma A4. Since $p^{nn} = \kappa\rho^2 s_0 + \epsilon^*(g) - c$, this implies

$$0 < \frac{\partial p^{nn}}{\partial x_0} < \kappa \quad \text{and} \quad 0 < \frac{\partial p^{nn}}{\partial s_0} < \kappa\rho^2.$$

A.4 Proof of Lemma 3

The probability of disclosure in the second period conditional on initial non-disclosure and cash flows is given by

$$\theta_n = \Pr(d_1 = 1|s_0, y_0, d_0 = 0) = \Pr(\eta > \epsilon^*(g) - \kappa(y_0 - \rho^2 s_0)|s_0, y_0) = \Phi\left(-\frac{\epsilon^*(g) - \kappa(y_0 - \rho^2 s_0)}{\sigma_\eta}\right).$$

We can see that θ_n is increasing in y_0, and decreasing in s_0 and x_0. Finally, we have

$$\alpha_n(g) = F\left(\frac{\epsilon^*(g)}{\sigma_\eta}, \frac{g}{\sigma_z}; \frac{\kappa\sigma_z}{\sigma_\eta}\right) < \Phi\left(-\frac{\epsilon^*(g) - \kappa g}{\sigma_\eta}\right), \tag{A.5}$$

by Lemma A2, and the right-hand side of (A.5) is the threshold type's true probability of disclosure conditional on initial non-disclosure and cash flows s_0.

A.5 Proof of Proposition 4

We begin by computing the option values. The option value upon initial disclosure is given by

$$\begin{aligned} u_d = E[\max(\eta^* - \eta, 0)] &= \int_{-\infty}^{\eta^*} (\eta^* - \eta) \frac{1}{\sqrt{2\pi\sigma_\eta^2}} e^{-\eta^2/2\sigma_\eta^2} d\eta \\ &= \eta^* \Phi\left(\frac{\eta^*}{\sigma_\eta}\right) + \sigma_\eta \delta\left(\frac{\eta^*}{\sigma_\eta}\right) \Phi\left(\frac{\eta^*}{\sigma_\eta}\right) \\ &= \sigma_\eta \Phi\left(\frac{\eta^*}{\sigma_\eta}\right) v\left(\frac{\eta^*}{\sigma_\eta}\right) \\ &= c\Phi\left(\frac{\eta^*}{\sigma_\eta}\right). \end{aligned}$$

The last equality holds by the definition of η^*. We similarly compute the manager's option value upon non-disclosure:

$$\begin{aligned} u_n &= E\left[\max\left(\epsilon^*(g) - \kappa g - \eta, 0\right)|y_0 = x_0\right] \\ &= E\left[E\left[(\epsilon^*(g) - \kappa g - \eta)^+ |g\right]|y_0 = x_0\right] \\ &= E\left[\sigma_\eta \Phi\left(\frac{\epsilon^*(g) - \kappa g}{\sigma_\eta}\right) v\left(\frac{\epsilon^*(g) - \kappa g}{\sigma_\eta}\right)|y_0 = x_0\right], \end{aligned}$$

where the last conditional expectation is done with respect to w_0 and g is given by $(1 - \rho^2)x_0 - \rho^2 w_0$ conditional on $y_0 = x_0$. Notice that u_n is a function of the time 0 threshold. Since $\Phi(x)v(x)$ is an increasing function in x, it is enough to show that $\eta^* > \epsilon^*(g) - \kappa g$ for any g in order to prove $p^{dn} > p^{nn}$, $\theta_d < \theta_n$, and

$u_n < u_d$ for the threshold-type manager. Notice that as $g \to -\infty$, $\epsilon^*(g)$ solves

$$\begin{aligned}
c &= \lim_{g\to-\infty} \sigma_\eta \int_{-\infty}^{g} v\left(\frac{\epsilon^*(g)-\kappa z}{\sigma_\eta}\right) \frac{\phi(\frac{z}{\sigma_z})}{\sigma_z \Phi(\frac{g}{\sigma_z})} dz \\
&= \lim_{g\to-\infty} \left[\sigma_\eta v\left(\frac{\epsilon^*(g)-\kappa z}{\sigma_\eta}\right) \frac{\Phi(\frac{z}{\sigma_z})}{\Phi(\frac{g}{\sigma_z})}|_{-\infty}^{g} - \kappa \int_{-\infty}^{g} \left\{1-\delta\left(\frac{\epsilon^*(g)-\kappa z}{\sigma_\eta}\right) v\left(\frac{\epsilon^*(g)-\kappa z}{\sigma_\eta}\right)\right\} \frac{\Phi(\frac{z}{\sigma_z})}{\Phi(\frac{g}{\sigma_z})} dz\right] \\
&= \lim_{g\to-\infty} \sigma_\eta v\left(\frac{\epsilon^*(g)-\kappa g}{\sigma_\eta}\right).
\end{aligned}$$

Thus, we have $\epsilon^*(g) - \kappa g \to \eta^*$ as $g \to -\infty$. Now, suppose that $g \to \infty$. Then, we have $\epsilon^*(g) \to \bar{\epsilon}$, where $\bar{\epsilon}$ solves

$$c = \lim_{g\to\infty} \sigma_\eta \int_{-\infty}^{g} v\left(\frac{\bar{\epsilon}-\kappa z}{\sigma_\eta}\right) \frac{\phi(\frac{z}{\sigma_z})}{\sigma_z \Phi(\frac{g}{\sigma_z})} dz = \sigma_\eta \int_{-\infty}^{\infty} v\left(\frac{\bar{\epsilon}-\kappa z}{\sigma_\eta}\right) \frac{\phi(\frac{z}{\sigma_z})}{\sigma_z} dz.$$

We can now show that $\eta^* > \epsilon^*(g) - \kappa g$ for any g since we have that $\lim_{g\to-\infty} \epsilon^*(g) - \kappa g = \eta^*$ and that $d(\epsilon^*(g) - \kappa g)/dg < 0$.

A.6 Proof of Theorem 1

The equilibrium condition (14) for the first-period disclosure threshold can be rewritten as

$$(1+\kappa)\sigma_y v\left(\frac{x_0}{\sigma_y}\right) - u_n + cE[\alpha_n(g)|\Omega^n] = c(1+\alpha_d) - u_d = 2c\Phi\left(-\frac{\eta^*}{\sigma_\eta}\right). \tag{A.6}$$

We define a function

$$f(x) = (1+\kappa)\sigma_y v\left(\frac{x}{\sigma_y}\right) - u_n + cE[\alpha_n(g)|\Omega^n],$$

where the last conditional expectation is done with respect to s_0 conditional on Ω^n. Differentiating, we have

$$f'(x) = (1+\kappa)\left[1-\delta\left(\frac{x}{\sigma_y}\right) v\left(\frac{x}{\sigma_y}\right)\right] - (1-\rho^2)E\left[\Phi\left(\frac{\epsilon^*(g)-\kappa g}{\sigma_\eta}\right)\left(\frac{d\epsilon^*(g)}{dg} - \kappa\right)|y_0 = x_0\right] + cE[\alpha_n(g)'|\Omega^n].$$

Note that we use $(\Phi(x)v(x))' = \Phi(x)$. Take x to $-\infty$, then by Lemmas A1 and A3,

$$\begin{aligned}
\lim_{x\to-\infty} f(x) &= -u_d + c\alpha_d = c\left[2\Phi\left(-\frac{\eta^*}{\sigma_\eta}\right) - 1\right], \\
\lim_{x\to-\infty} f'(x) &= \lim_{x\to-\infty} cE[\alpha_n(g)'|\Omega^n] < 0.
\end{aligned}$$

Taking x to ∞, we have

$$\begin{aligned}
\lim_{x\to\infty} f(x) &= \lim_{x\to\infty} (1+\kappa)\sigma_y v\left(\frac{x}{\sigma_y}\right) + c\bar{\theta} = \infty, \\
\lim_{x\to\infty} f'(x) &= 1 + \kappa + \lim_{x\to\infty} cE[\alpha_n(g)'|\Omega^n] > 0.
\end{aligned}$$

Thus, there exists a unique x solving $f(x) = 2c\Phi\left(-\eta^*/\sigma_\eta\right)$. Suppose that x_0 is such x. Finally, compare equilibrium conditions (4) and (14) and notice that $u_n < u_d$, which implies that the myopic threshold x^* should be higher than x_0 since $f(x)$ is increasing at $x = x_0$.

A.7 Proof of Corollaries 1 and 2

When $\rho \to 1$ or $\sigma_w \to 0$, upon observing s_0, investors can recover y_0 perfectly. Thus, the two option values are identical, which implies $x_0 = x^*$. When $\kappa \to 0$, y_t is independent across time and the first-period disclosure decision is irrelevant for determining the second-period price. This implies $x_0 = x^*$.

For the threshold-type manager, we have

$$p^n = p^d(x_0) + u_d - u_n = (1+\kappa)x_0 - c(1+\alpha_d) + u_d - u_n.$$

By definition of y_0^*, we also have

$$p^n = p^d(y_0^*) = (1+\kappa)y_0^* - c(1+\alpha_d).$$

By combining above two equations, we have

$$(1+\kappa)(y_0^* - x_0) = u_d - u_d = p^n - p^d(x_0),$$

which completes the proof.

A.8 Proof of Proposition 5

The equilibrium condition for the first-period disclosure threshold can be rewritten as

$$c = (1+\beta)\sigma_y v\left(\frac{x_0}{\sigma_y}\right) + \lambda(u_d - u_n) + c\beta(E[\alpha_n|\Omega^n] - \alpha_d). \tag{A.7}$$

Clearly, the equilibrium condition becomes (15) when $\beta = \lambda = 0$. When $\beta = 1$ and $\lambda = 0$, the model becomes the myopic benchmark and we have $x_0 = x^*$. When $\beta = 0$ and $\lambda = 1$, the equilibrium condition becomes

$$c = \sigma_y v\left(\frac{x_0}{\sigma_y}\right) + u_d - u_n.$$

Since $u_d > u_n$, the first-period threshold x_0 should be lower than the static one x^{**}. Finally, when $\beta = \lambda = 1$, the model becomes the baseline one.

Set the function $f(x_0)$ equal to the right hand side of (A.7). Totally differentiate $f(x_0)$, then we have

$$\frac{\partial x_0}{\partial \beta} = -\frac{\sigma_y v\left(\frac{x_0}{\sigma_y}\right) + c(E[\alpha_n|\Omega^n] - \alpha_d)}{f'(x_0)} < 0,$$

when $E[\alpha_n|\Omega^n] > \alpha_d$ since $f'(x_0) > 0$ if x_0 satisfies (A.7). Again, totally differentiate $f(x_0)$, then we have

$$\frac{\partial x_0}{\partial \lambda} = -\frac{u_d - u_n}{f'(x_0)} < 0,$$

since $u_d > u_n$ and $f'(x_0) > 0$ if x_0 satisfies (A.7).

B Three-period extension: equilibrium conditions

In this Appendix, we provide the equilibrium conditions for the three-period extension discussed in Section 6.2.

Upon disclosure in $t = 0$

We first consider the case in which the manager disclosed in the first period. We have two states in the third period that depend on first-period disclosure, $\omega_2 \in \{dd, dn\}$. In the state $\omega_2 = dd$, the manager has disclosed in both of the previous periods. As in the baseline setting, the disclosure threshold is given by

$$x_{dd} = c + E[y_2 | y_1, y_2 < x_{dd}] = y_1 + \eta^*,$$

where x_{dd} is the threshold that the manager uses in this state, y_1 is the disclosed value at time 1, and η^* is defined similar to equation (5). As firm profitability follows a Markov process, the only relevant information to the market after disclosure in time 1 is y_1.

Next, in state $\omega_2 = dn$, the manager has disclosed in $t = 0$ but not in $t = 1$. The threshold in this state, x_{dn}, is given by

$$x_{dn} = y_0 + \epsilon^*,$$

where y_0 is the disclosed value at time 0 and ϵ^* solves

$$c = \epsilon^* - E[\eta_1 + \eta_2 | \eta_1 < x_d - y_0, \eta_1 + \eta_2 < \epsilon^*].$$

We can interpret ϵ^* as the mean-adjusted threshold in state $\omega_2 = dn$ which is analogous to that of equation (10) in the baseline setting. These thresholds characterize the manager's third-period disclosure policy following disclosure in $t = 0$. We next analyze the second-period disclosure strategy based on the manager's continuation value in that period. At time 1, the threshold-type manager ($y_1 = x_d$) must be indifferent between disclosure and non-disclosure:

$$v^{dd} = v^{dn},$$

where v^{dd} denotes the manager's value function at time 1 if her disclosure decision is $d_1 = d_0 = 1$, and can be expressed as

$$v^{dd} = p^{dd} + E[\max(p^{ddd}, p^{ddn}) | y_1 = x_d].$$

The manager's value function includes the contemporaneous price upon her disclosure decision and the value of the real option following disclosure. The latter is determined by the expected third-period prices, which is either p^{ddd} from disclosure, or p^{ddn} from non-disclosure in that period. The manager's value function if she withholds disclosure in the second period (after disclosure in the first) is similarly defined.

Upon non-disclosure in $t = 0$

We next consider the case where the manager kept quiet in the first period. The state at the beginning of the third period in this case is given by $\omega_2 \in \{nd, nn\}$. Following disclosure in the second period ($\omega_2 = nd$), the disclosure threshold in $t = 2$ is

$$x_{nd} = c + E[y_2|y_1, y_2 < x_{nd}] = y_1 + \eta^*.$$

We see that the threshold has the same form as x_{dd}. However, the disclosed value y_1 has a different distribution depending on the $t = 0$ disclosure decision. Next, in state $\omega_2 = nn$, the threshold x_{nn} solves

$$c = x_{nn} - E[y_2|y_0 < x_0, y_1 < x_n, y_2 < x_{nn}],$$

where the expectation on the right-hand side is the non-disclosure price given that the manager is not disclosing in time 0, 1, and 2, i.e., $y_0 < x_0$, $y_1 < x_n$, and $y_2 < x_{nn}$. Going one step backward, in the state $\omega_1 = n$, the threshold-type manager ($y_1 = x_n$) must be indifferent between disclosure and non-disclosure:

$$v^{nd} = v^{nn},$$

where v^{nd} and v^{nn} are the manager's value function upon disclosure and non-disclosure following non-disclosure in $t = 0$, respectively. These continuation values are similarly defined as the in the preceding case where the manager disclosed in $t = 0$.

Time 0 decision

We have thus far defined the indifference conditions in terms of the manager's continuation value under a particular history rather than directly through the following period's price. Moreover, the second-period decision now embeds a real option. Consequently, the first-period decision function embeds this option as well as the option that arises from the first-period decision. The option value in the first period is now captured through differences in the continuation value in the second period. Specifically, at time 0, the threshold-type manager's ($y_0 = x_0$) equilibrium indifference condition is given as

$$p^d + E[\max(v^{dd}, v^{dn})|y_0 = x_0] = p^n + E[\max(v^{nd}, v^{nn})|y_0 = x_0].$$

The manager's expected utility is again the sum of the current market price upon disclosure decision and the corresponding real option value, which depends on the manager's value functions at time 1. We see that the main difference between the three-period extension and the baseline model is that the real option at time 0 is now defined by the manager's value function at time 1 rather than the market price at time 1, which implicitly embeds the second-period real option.