Ambiguity and the Tradeoff Theory of Capital Structure

Yehuda Izhakian[*], David Yermack[†], and Jaime F. Zender[‡§]

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Abstract

We examine the impact ambiguity, or Knightian uncertainty, has on the capital structure decision. A static tradeoff theory model is developed in which agents are both ambiguity and risk averse. The model supports the well known prediction that increased risk—the uncertainty over known possible outcomes—leads firms to decrease leverage. Conversely, the model predicts that greater ambiguity—the uncertainty over the probabilities associated with the outcomes—leads firms to increase leverage. Using a theoretically motivated measure of ambiguity, our empirical analysis presents evidence consistent with the prediction that ambiguity is positively related to leverage and shows that ambiguity has an important and distinct impact on capital structure choice.

Keywords and Phrases: Capital Structure, Ambiguity aversion, Ambiguity measure.

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[*]Zicklin School of Business, Baruch College, yud@stern.nyu.edu

[†]Stern School Of Business, New York University and NBER, dyermack@stern.nyu.edu

[‡]Leeds School of Business, University of Colorado Boulder, jaime.zender@colorado.edu

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1 Introduction

Almost every financial decision entails ambiguity (Knightian uncertainty).[1] In an influential paper, Ellsberg (1961) demonstrates that individuals act as if they are averse to this aspect of uncertainty.[2] Nevertheless, the canonical results in finance have been developed using expected utility theory, a theory that can be interpreted as either assuming away ambiguity or assuming that individual preferences are neutral to ambiguity. If indeed ambiguity and aversion to it play a significant role in financial decision-making, ignoring this aspect of uncertainty implies that our characterizations of optimizing behavior are at best incomplete and at worst misleading.

We introduce ambiguity and aversion to ambiguity into the static tradeoff theory of corporate capital structure in hopes of providing new insights into capital structure decisions. Examining leverage decisions through the lens of a financial market in which investors are averse to both ambiguity and risk may serve to further enhance our understanding of corporate decision-making. The market's valuation of the firm's chosen mix of securities is the focus of the capital structure question, making the consequences of ambiguity and aversion to ambiguity a central concern. Furthermore, most recent empirical studies of leverage (e.g., Rajan and Zingales, 1995; Faulkender and Petersen, 2006; Lemmon et al., 2008; Frank and Goyal, 2009) or even seminal papers in this literature (e.g., Titman and Wessels, 1988) take no account of this aspect of uncertainty. To test the predictions of the theoretical model, we adapt recent empirical models of the static tradeoff theory of corporate capital structure. Our empirical tests provide strong evidence that ambiguity is an important explanatory variable for leverage.

The famous theorem of Modigliani and Miller (1958) asserts that, with perfect capital markets, two firms generating the same distribution of future cash flow will have the same market value regardless of their financing choices. Standard tradeoff theory frictions have been used to establish an interior optimum (e.g., Kraus and Litzenberger, 1973) for the financing decision. We extend these theories to account for ambiguity and aversion to ambiguity. We show that in a perfect capital market, Modigliani and Miller's irrelevance proposition is maintained under ambiguity. This can be seen using the familiar argument that buying the debt and the equity of a levered firm results in the investor obtaining claims to cash flows that are equivalent to the equity cash flow of an equivalent but unlevered firm. Stated

[1] *Risk* refers to as a case in which the event to be realized is *a-priori* unknown, but the odds of all the possible events are perfectly known. *Ambiguity*, or *Knightian uncertainty*, refers to the case where not only is the event to be realized *a-priori* unknown, but also the odds of all possible events are either unknown or not uniquely assigned. Throughout the paper the term "*uncertainty*" is used to refer to the aggregation of risk and ambiguity.

[2] Ellsberg (1961) demonstrates that in the presence of ambiguity, individuals typically violate the independence ("Sure-Thing Principle") axiom of expected utility theory.

another way, while the cost of debt and the cost of equity capital both increase with an increase in ambiguity, the weighted average cost of capital is invariant to changes in leverage for a given level of ambiguity.

The increase in the cost of equity (a lower valuation for levered equity with a debt level) that accompanies an increase in ambiguity can be used to highlight another standard issue in the capital structure literature: the over-investment problem. Agency problems have played a prominent role in the capital structure literature (e.g., Jensen and Meckling, 1976). For a given debt level, increasing the risk of the firm's cash flow increases the value of the levered equity following the usual intuition from option pricing (e.g., Hartman, 1972; Abel, 1983). This suggests that decision-makers who maximize the value of levered equity are willing to trade off the (net present) value of an investment opportunity against its risk; i.e., are willing to invest in negative net present value projects if they provide a sufficient increase in risk. Conversely, an increase in ambiguity reduces the value of levered equity. Viewing equity as a call option, increased ambiguity reduces the value of that option, because the increase in ambiguity causes ambiguity averse investors to reduce the perceived probabilities of outcomes with high payoffs (e.g., Izhakian and Yermack, 2017; Augustin and Izhakian, 2019). Net present value would, therefore, be sacrificed by a manager seeking to maximize the value of the existing equity only if the investment achieved a sufficient *reduction* in ambiguity. Thus, consistent with Garlappi et al. (2017), investments that entail an increase in ambiguity may suffer from an under-investment problem. Investments that increase both ambiguity and risk may or may not be attractive to managers seeking to maximize the value of a firm's levered equity. Ambiguity aversion may therefore mitigate the over-investment incentives of managers acting in the interests of shareholders.

When taxes and bankruptcy costs are introduced into the model, the theory predicts that while an increase in the level of risk results in a reduction in the use of debt financing, an increase in the level of ambiguity is associated with an increase in the use of debt. The usual economic intuition explaining why higher risk is associated with less extensive use of leverage (e.g., Kraus and Litzenberger, 1973) is maintained in the model with ambiguity aversion. At the optimal leverage, the marginal cost of bankruptcy equals the marginal tax benefit associated with the use of debt. An increase in the level of risk associated with the firm's cash flows causes the marginal probability of bankruptcy to increase, decreasing the marginal tax benefit and increasing the marginal cost of debt financing. Obtaining the new optimum, therefore, entails a reduction in the use of debt. The economic intuition for why increased ambiguity results in higher optimal leverage is similar. An increase in ambiguity results in ambiguity averse investors underweighting the perceived probabilities of all states; with high cash flow states being underweighted more than are low cash flow states. This implies that the perceived marginal probability of bankruptcy decreases. Therefore, the marginal tax benefit increases and the marginal bankruptcy cost decreases when the level of ambiguity increases. An increase in leverage is, therefore, required to achieve the new optimum.

One view of the theoretical contribution of this analysis is that, perhaps for want of clear guidance regarding whether and how the different aspects of uncertainty provide distinct implications for corporate decision making, the existing literature has confounded risk and ambiguity. The analysis presented here highlights a sharp distinction in the impacts on capital structure choice for the two aspects of uncertainty: ambiguity and risk. Importantly, the underlying decision theoretic framework provides a measure of ambiguity that can be estimated from the data, allowing empirical examination of the separate roles ambiguity and risk play in the capital structure decision.

To provide a preliminary examination of the predicted positive relation between ambiguity and leverage, we investigate whether a firm's leverage ratio is related to the lagged measure of its ambiguity. We account for the unobservable firm specific component of leverage, identified by Lemmon et al. (2008), transforming the data by taking first differences of all the variables and examining the extent to which innovations in ambiguity are followed by a change in leverage in the subsequent year. Changes in both book and market leverage are considered as the dependent variable. Firms' realized ambiguity (computed from equity market data), as well as a measure of the ambiguity for equivalent unlevered firms (to mitigate concerns that computing ambiguity using equity return is a biased measure of firm ambiguity) are employed as the explanatory variables of interest. In each regression specification using the entire sample the estimated coefficient on the ambiguity measure is positive and significant, consistent with the model's prediction. Furthermore, ambiguity is found to be economically more significant than the explanatory variables for leverage commonly used in the empirical capital structure literature. Tests using subgroups of the sample by firm size and leverage are also consistent with the model's predictions.

We follow the methodology used in recent studies and estimate firm-level ambiguity as the volatility of daily return probabilities estimated from intraday returns data (e.g., Izhakian and Yermack, 2017; Brenner and Izhakian, 2018; Augustin and Izhakian, 2019). Brenner and Izhakian (2018), and Augustin and Izhakian (2019), at the market level and the firm level respectively, conduct extensive tests to allay concerns that this measure of ambiguity captures other well-known dimensions of uncertainty. Similarly, our own robustness tests show that ambiguity represents a distinct aspect of uncertainty in the context of the capital structure decision. The significant effect of our ambiguity measure on leverage is robust to the inclusion of many alternative uncertainty (e.g., volatility of return means, volatility of return volatilities, and disagreement among analysts) and market-microstructure factors (e.g., illiquidity and bid-ask spreads).

Measuring ambiguity independently from aversion to ambiguity, risk, and risk aversion is the main challenge in testing the predictions delivered by the theoretical model. The empirical measure of ambiguity is rooted in the decision theory framework of expected utility with uncertain probabilities (EUUP, Izhakian, 2017). In that framework, aversion to ambiguity takes the form of aversion to mean-preserving spreads in probabilities. Thereby, the degree of ambiguity can be measured by the volatility of the *probabilities* of future outcomes, just as the degree of risk can be measured by the volatility of outcomes. The separation of risk and ambiguity is an important prerequisite for an empirical assessment of the impact of ambiguity on leverage.

In a related study, Lee (2014) considers a similar question to that posed here and finds that increased ambiguity leads to a reduction in leverage. Lee, however, uses the smooth model of ambiguity aversion (Klibanoff et al., 2005) as the underlying preference theory. Within this model, ambiguity and preference toward ambiguity are outcome-dependent and so risk-dependent. It is, therefore, not surprising that his result mirrors the standard result for an increase in risk. Moreover, his model assumes that while the firm's manager is ambiguity averse, market participants are (and so market pricing is) ambiguity neutral. Lee's theoretical result, therefore, reflects an agency cost rather than value maximizing behavior, making it unclear how the model's predictions are expected to associate with observed firm behavior. Finally, Lee's empirical results are based on an event (the 1982 Voluntary Restraint Agreement on steel import quotas) that confounds risk and ambiguity, making it very difficult to interpret his empirical findings.

The remainder of this paper is organized as follows. Section 2 presents a theoretical discussion of ambiguity and develops the model. Section 3 discusses the sample selection and empirical tests. Section 4 presents regression analysis of capital structure, and Section 6 concludes the paper. All proofs are provided in the Appendix.

2 The model

We develop a static model to examine the impact of ambiguity and aversion to ambiguity on corporate capital structure choice. The model has two important, distinguishing features. First, agents' preferences for ambiguity are outcome-independent and therefore independent of risk and attitude toward risk. The outcome-independence of agents' preferences for ambiguity is necessary to completely differentiate the effect of ambiguity from that of risk. Second, agents are averse to both ambiguity and risk.

Capital structure decisions are determined by value (or equivalently perceived expected utility), which is determined by the perceived expected return on risky and ambiguous investments. The perceived expected return on investments is affected by risk and by ambiguity. To illustrate, suppose that a firm considers investing $400,000 in a new enterprise.[3] The payoff of this investment is determined by a flip of an unbalanced coin, for which the investors do not know the odds of heads or tails. The payoff is given as $1,000,000 in the event of heads and $0 in the event of tails. Suppose that new information indicates that the payoff in the event of heads will be $2,000,000 rather than $1,000,000. In this case, both risk and expected payoff increase, such that the investors may find this investment opportunity more attractive. However, note that the ambiguity has not changed. Investors have no reason to change either their assessed probabilities or their assessed degree of ambiguity since there has been no new information regarding likelihoods. Suppose instead that the new information increases the assessed degree of ambiguity about the coin. As investors are ambiguity averse, in response to an increase in ambiguity they lower their perceived probability of the good payoff. As a result, their expected payoff falls, so that investors find this investment opportunity less attractive. In this case, investors have no reason to change their assessments of the payoffs in the different states as their new information concerned only the likelihoods of the outcomes.

This example highlights a critical property: ambiguity is outcome independent, up to a state-space partition. That is, if the outcomes associated with events change, while the induced partition of the state space into events (set of events) remains unchanged, then the degree of ambiguity remains unchanged, since the probabilities remain unchanged. Furthermore, outcome dependence implies risk dependence. To measure ambiguity independently of risk, the underlying preference for ambiguity must apply exclusively to the probabilities of events, independently of the outcomes associated with these events. Outcome independence is a critical property that distinguishes our measure of ambiguity from those previously used in the literature.

To provide a basic intuition for the effect of ambiguity and risk on the value of the firm, consider a simplified structural framework in which the optimal face value of the debt is F. If the firm's risk increases, as is illustrated in Panel (a) of Figure 1, the left-tail probability mass increases, increasing the marginal probability of default, given the initial debt level F. This change increases the marginal

[3] Izhakian (2018) provides a similar example.

bankruptcy cost and decreases the marginal tax benefit from debt financing at the original optimum F. As a result, optimal leverage is lower after the change.

A classic feature of many ambiguity models (e.g., Choquet expected utility, Schmeidler, 1989) is that ambiguity-averse investors act *as if* they underweight the perceived probabilities of states with high outcomes.[4] Under Choquet expected utility (CEU) in particular, they underweight the perceived probabilities of high outcome states to a greater extent than they underweight the perceived probabilities of low outcome states, such that perceived probabilities are subadditive; i.e., perceived probabilities sum to a number smaller than one. In this case, as is illustrated in Panel (b) of Figure 1, in response to an increase in ambiguity, ambiguity averse agents perceive the left-tail probability mass decreases, decreasing the marginal probability of default at F. Therefore, the marginal bankruptcy cost falls and the marginal tax benefit rises at the original optimum implying that optimal leverage increases.[5]

[Figure 1]

2.1 The decision theoretic framework

To develop a model of optimal capital structure with ambiguity and aversion to ambiguity, we employ the preference relation CEU (Schmeidler, 1989) and augment it with the theoretical framework contained in expected utility with uncertain probabilities (EUUP, Izhakian, 2017).[6] In this context, EUUP can be viewed as providing an axiomatic development of the capacities (the nonadditive probabilities) employed in the CEU model. EUUP derives these capacities based upon the ambiguity in the environment (beliefs) and the agents' aversion to (attitude toward) ambiguity. Under EUUP, preferences for ambiguity are outcome-independent, which allows for the separation of ambiguity from risk as well as the separation of attitudes from beliefs. Importantly, a by-product of this approach is a model-derived, risk-independent measure of ambiguity rooted in axiomatic decision theory that can be estimated and employed to test the predictions of the theory.[7]

[4]Experiments show that this weighting holds true for unfavorable and favorable outcomes (Abdellaoui et al., 2010) as well as for unlikely and likely events (Crockett et al., 2019).

[5]Note that the associated intuition within the max-min expected utility model (Gilboa and Schmeidler, 1989) or the smooth model of ambiguity aversion (Klibanoff et al., 2005) would predict the opposite; a negative relation between ambiguity and leverage. This intuition derives from the outcome-dependence and the resulting risk-dependence of preferences over ambiguity within these models. Therefore, in these models, it is difficult to separate the impact of ambiguity from that of risk.

[6]The max-min expected utility model (MEU) of Gilboa and Schmeidler (1989) is a special case of CEU. However, MEU does not allow the separation of beliefs regarding ambiguity from attitudes toward ambiguity.

[7]The measurement of ambiguity independently from risk poses a challenge for other frameworks which do not separate ambiguity from attitude toward ambiguity (e.g., Gilboa and Schmeidler, 1989; Schmeidler, 1989) or in which preferences for ambiguity are outcome-dependent (e.g., Gilboa and Schmeidler, 1989; Klibanoff et al., 2005; Chew and Sagi, 2008).

The main concept behind the CEU framework when augmented with EUUP is that the preferences for ambiguity are applied exclusively to the *uncertain* probabilities of future events. Thus, under the structure of EUUP, aversion to ambiguity is defined as an aversion to mean-preserving spreads in probabilities, which are outcome-independent. As such, the Rothschild and Stiglitz (1970) approach can be employed to measure ambiguity, independently of risk, as the volatility of the uncertain probabilities of future events.

Formally, the investor, who values a risky and ambiguous payoff X, possesses a set $\mathcal{P}$ of prior probability distributions P over events, equipped with a prior probability ξ (a distribution over probability distributions). Each cumulative probability distribution $\mathrm{P} \in \mathcal{P}$ is associated with a marginal probability function $\varphi(\cdot)$. The investor evaluates the expected utility of a risky and ambiguous payoff by the CEU model (Schmeidler, 1989)

$$\mathrm{V}(X) \quad = \quad \int_{\mathcal{S}} \mathrm{U}(\cdot)\, d\mathrm{Q}, \tag{1}$$

where $\mathcal{S}$ stands for the set of states and Q for the capacity. Using the set of priors $\mathcal{P}$ and the second-order prior ξ, by EUUP, the investor assesses the capacities as the certainty equivalent probability of each event. A capacity, referred to in EUUP as the *perceived probability*, is the unique certain probability value that the investor is just willing to accept in exchange for the uncertain probability of a given event. Accordingly, the marginal perceived probability is assessed by[8]

$$d\mathrm{Q} \quad = \quad \mathrm{E}[\varphi(x)]\left(1+\frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(x)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(x)])}\mathrm{Var}[\varphi(x)]\right), \tag{2}$$

where the function Υ, called the *outlook function*, captures the investor's attitude toward ambiguity;[9] the expected marginal and cumulative probability of x are computed using ξ, such that $\mathrm{E}[\varphi(x)] = \int_{\mathcal{P}} \varphi(x)\, d\xi$ and $\mathrm{E}[\mathrm{P}(x)] = \int_{\mathcal{P}} \mathrm{P}(x)\, d\xi$; and $\mathrm{Var}[\varphi(x)] = \int_{\mathcal{P}} \left(\varphi(x) - \mathrm{E}[\varphi(x)]\right)^2 d\xi$ defines the variance of the marginal probability. Using these perceived probabilities, the investor assesses the expected utility of a risky and ambiguous payoff $\mathrm{V}(X)$ by

$$\mathrm{V}(X) \quad = \quad \int \mathrm{U}(x)\underbrace{\mathrm{E}[\varphi(x)]\left(1+\frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(x)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(x)])}\mathrm{Var}[\varphi(x)]\right)}_{\text{Perceived Probability of Outcome } x} dx. \tag{3}$$

Risk aversion is distinctly captured by a strictly-increasing, concave, and twice-differentiable continuous utility function $\mathrm{U}: \mathbb{R} \to \mathbb{R}$, applied to the uncertain outcomes. As the investor is ambiguity-

[8] Equation (2) is an approximation of the perceived probability $\mathrm{Q}(x) = \Upsilon^{-1}\left(\int_{\mathcal{P}} \Upsilon(\mathrm{P}(x))\, d\xi\right)$ (Theorem 2, Izhakian, 2018). The residual of the approximation is $R_2\big(\mathrm{P}(x)\big) = o\left(\mathrm{E}\left[|\mathrm{P}(x) - \mathrm{E}[\mathrm{P}(x)]|^3\right]\right)$ as $|\mathrm{P}(x) - \mathrm{E}[\mathrm{P}(x)]| \to 0$, which is negligible. Therefore, to simplify notation, we use the equal sign instead of the approximation sign.

[9] The outlook function is assumed to satisfy $\left|\frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(x)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(x)])}\right| \leq \frac{1}{\mathrm{Var}[\varphi(x)]}$, which bounds the concavity and convexity of Υ to assure that the approximated marginal perceived probabilities are always positive and satisfy set monotonicity.

averse, she compounds the set of priors $\mathcal{P}$ using the second-order prior ξ over $\mathcal{P}$ in a non-linear way. This aversion is captured by a strictly-increasing, concave, and twice-differentiable continuous function $\Upsilon : [0,1] \to \mathbb{R}$ applied to the probabilities.[10] The investor's (subadditive) perceived probabilities represented in Equation (3) are a function of the extent of ambiguity, measured by $\mathrm{Var}\left[\varphi\left(x\right)\right]$, and the investor's aversion to ambiguity, captured by $-\frac{\Upsilon''(\cdot)}{\Upsilon'(\cdot)} > 0$. Both a higher aversion to ambiguity or a higher extent of ambiguity result in lower (more underweighted) perceived probabilities. When the investor is ambiguity neutral, Υ is linear, the perceived probabilities become the (additive) expected probabilities (the linear reduction of compound lotteries) and Equation (3) collapses to the standard expected utility framework. The same reduction of the model occurs when ambiguity is not present (there is no uncertainty over probabilities). To simplify the analysis and the discussion of the results, it is assumed that relative ambiguity aversion is nonincreasing, i.e., $\left(\frac{\Upsilon''}{\Upsilon'}\right)' \geq 0$. Many classes of outlook functions satisfy this condition, including constant relative ambiguity aversion (CRAA) and constant absolute ambiguity aversion (CAAA).[11]

Based on the functional form in Equation (3), the degree of ambiguity can be measured by the expected probability weighted average (across the relevant events) volatility of probabilities. Formally, the measure of ambiguity is given by

$$\mho^2\left[X\right] = \int \mathrm{E}\left[\varphi\left(x\right)\right] \mathrm{Var}\left[\varphi\left(x\right)\right] dx. \tag{4}$$

A major advantage of this measure is that it can be estimated using high frequency trading data. *Risk-independence* is another major advantage of $\mho^2$ (mho^2); unlike risk measures, it does not depend upon the magnitudes of the outcomes associated with the different events.

2.2 The asset pricing framework

We employ a standard framework, where the only variation in our structure is the specification of probabilities. We assume markets are perfect and an absence of arbitrage opportunities (i.e., the law of one price holds). There are two dates, 0 and 1. The state at date 0 is known, and the states at date 1 are ordered from the associated lowest consumption level to the highest. As is common in the capital structure literature (e.g., Kraus and Litzenberger, 1973; Fischer et al., 1989; Leland, 1994; DeMarzo and Fishman, 2007), we assume a representative investor who is endowed with initial wealth

[10] Ambiguity aversion is reflected in the attitudes the investor has for the expectation of an uncertain probability over the uncertain probabilities. Recall that risk aversion is exhibited when an investor prefers the expectation of the uncertain outcomes over the uncertain outcomes.

[11] Since Υ operates on $1 - \mathrm{E}\left[\mathrm{P}\left(x\right)\right]$, this assumption coincides with the experimental findings of Baillon and Placido (2019) who show that aversion to ambiguity decreases in expected utility. A similar assumption concerning risk is widely used in the literature.

w_0. Her wealth at time 1 is denoted w_1. Since, the asset pricing framework is used only to extract state prices, for simplicity, we assume a single product in the economy, with an uncertain payoff x at time 1. The investor's objective function can then be written in the usual way

$$\max_{c_0,\theta} \quad \mathrm{V}(c_0) + \mathrm{V}(c_1) \tag{5}$$

subject to the budget constraints

$$c_0 = w_0 - \theta \int q(x) x dx \qquad \text{and} \qquad c_1 = w_1 + \theta x,$$

where $q(x)$ is the date 0 price of a pure state contingent claim on state s associated with consumption x; and θ is the investor's position of the single asset. Using the functional form of expected utility in Equation (3), the state prices can be extracted as follows.

Theorem 1 *Suppose a time-separable utility function. The state price of state x is then*

$$q(x) \;\; = \;\; \pi(x) \frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}, \tag{6}$$

where

$$\pi(x) \;\; = \;\; \mathrm{E}[\varphi(x)] \left(1 + \frac{\Upsilon''(1 - \mathrm{E}[\mathrm{P}(x)])}{\Upsilon'(1 - \mathrm{E}[\mathrm{P}(x)])} \mathrm{Var}[\varphi(x)]\right); \tag{7}$$

and $q(x)$ is unique and positive.

The state price $q(x)$ is the price of a claim to one unit of consumption contingent on the occurrence of state s (Arrow security).[12] The representation of state prices in Equation (6) illustrates the distinct impacts that risk, ambiguity, and the attitudes toward these aspects of uncertainty have on market pricing. Ambiguity and aversion to ambiguity impact state prices through the transformation of probabilities to the perceived probabilities, $\pi(x)$. Risk and aversion to risk impact the state price via the curvature of the utility function U and the magnitude of the outcomes x relative to consumption at time 0. When there is no ambiguity, i.e., probabilities are perfectly known, state prices $q(x)$ reduce to the conventional representation $q(x) = \varphi(x) \frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}$.[13]

The following corollary is an immediate consequence of Theorem 1.

Corollary 1 *The risk neutral probability of state x is*

$$\pi^*(x) \;\; = \;\; \frac{q(x)}{\int q(x) dx} \;\; = \;\; \frac{\pi(x) \frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}}{\int \pi(x) \frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}} dx}, \tag{8}$$

[12] Chapman and Polkovnichenko (2009), for example, extract the state prices in a rank-dependent expected utility framework.

[13] In the presence of ambiguity with ambiguity neutral investors state prices are $q(x) = \mathrm{E}[\varphi(x)] \frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}$, which is also consistent with the conventional representation.

and the risk-free rate of return is

$$r_f \;=\; \frac{1}{\displaystyle\int q(x)dx} - 1 \;=\; \frac{1}{\displaystyle\int \pi\left(x\right)\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}dx} - 1. \tag{9}$$

Note that, while they are dependent upon the subadditive perceived probabilities, the risk neutral probabilities are additive. Similarly to the state prices in risk-only framework, ambiguity and aversion to ambiguity transform the standard representation of risk neutral probabilities via the $\pi\left(x\right)$ term.

2.3 The capital structure decision in a perfect capital market

The capital structure model developed here is based on the canonical tradeoff theory model of Kraus and Litzenberger (1973). Consider a one-period project that requires a capital investment I at time 0. The payoff of this project, realized at time $t = 1$, is a risky and ambiguous variable from the perspective of time $t = 0$. As discussed above, it is assumed that the owner of the project is a representative investor, whose set of priors (the set of possible probability distributions $\mathcal{P}$ on X) is identical to the set of priors of each investor in the economy, and her beliefs (second-order probability distribution) over $\mathcal{P}$ is identical to that of each investor.

At time $t = 0$, a decision to invest in a project is made if and only if

$$\frac{1}{1+r_f}\mathbb{E}^*\left[X\right] > I,$$

where $\mathbb{E}^*$ is the expectation taken with respect to the risk neutral probabilities, π^*, defined in Corollary 1.[14] Given the decision to invest in the project, its owner must decide at time $t = 0$ what mix of debt and equity financing the firm will use to establish the project. The objective of this decision is to maximize the value of the firm:

$$\begin{aligned} \max_{F} \qquad & S_0\left(F\right) + D_0\left(F\right) \\ \text{s.t.} \quad S_0 \; &= \frac{1}{1+r_f}\mathbb{E}^*\left[\max\left(X - F\ ,\ 0\right)\right] \\ D_0 \; &= \frac{1}{1+r_f}\mathbb{E}^*\left[\min\left(F\ ,\ X\right)\right], \end{aligned}$$

where S_0 is the market value of the common shares; D_0 is the market value of debt; and F is the debt's face value. Without loss of generality, the debt is assumed to be a one-period zero-coupon bond. The

[14] The double-struck capital font is used to designate expectation or variance of outcomes, taken with respect to the expected probabilities or with respect to risk-neutral probabilities, while the regular straight font is used to designate expectation or variance of probabilities, taken with respect to the second-order probabilities, ξ.

optimization problem can be written more explicitly as

$$\begin{aligned}
\max_{F} \quad & S_0(F) + D_0(F) \\
\text{s.t.} \quad S_0 &= \frac{1}{1+r_f}\int_F^{\infty} \pi^*(x)\,(x-F)\,dx \\
D_0 &= \frac{1}{1+r_f}\left(\int_0^F \pi^*(x) x dx + \int_F^{\infty} \pi^*(x) F dx\right).
\end{aligned}$$

Any $x < F$ is considered a default state.

Theorem 2 *Suppose that there are no taxes, no bankruptcy costs and no asymmetric information. Modigliani-Miller's capital structure irrelevance proposition is maintained when all agents in the economy are averse to ambiguity.*

Proposition 1 in Modigliani and Miller (1958) shows that, in an economy without taxes, bankruptcy costs or asymmetric information, the capital-structure choice of a firm is irrelevant to its market value. Modigliani and Miller's proof is based on the idea the an investor can generate the return of holding the levered firm's equity by holding the equity of an equivalent unlevered firm and a loan in the same proportion as the leverage ratio of the levered firm. The same argument holds true in an ambiguous economy in which agents are ambiguity averse.

2.4 The over-investment problem

In their seminal paper, Jensen and Meckling (1976) introduce the "over-investment" problem. Central to their idea is the simple notion that, in the equity holders' view, levered equity can be described as a call option on the firm, where the exercise price is the face value of the debt (e.g., Merton, 1974). Maintaining the assumptions of an absence of taxes and bankruptcy costs, in the present model, the value of the equity is the expected value of the future equity payoff (using the risk neutral probabilities) discounted at the risk free rate,

$$\begin{aligned}
S_0 &= \frac{1}{1+r_f}\mathbb{E}^*\left[\max\left(X - F\ ,\ 0\right)\right] \\
&= \frac{1}{1+r_f}\int_F^{\infty} \pi^*(x)\,(x-F)\,dx.
\end{aligned}$$

Thus, this value is affected by ambiguity through $\pi^*(x)$ and by risk through the magnitude of x.

As in Rothschild and Stiglitz (1970), we consider an asset as becoming risker if its payoffs at time t can be written as a mean-preserving spread of its payoff at time $t-1$. The following proposition establishes that the standard relation between the value of levered equity and the level of risk remains in the current setting.

Proposition 1 *For a given face value of debt, the higher is the degree of risk, the higher is the value of the levered equity.*

Levered equity holders benefit from the upside risk and are protected from downside risk. The value of levered equity is, therefore, increasing in the risk of the underlying cash flow and levered equity holders have an interest in increasing the risk of the firm's cash flow. The over-investment problem follows from the recognition that levered equity holders, therefore, benefit from an increase in risk even at the expense of some reduction in the expected value of the cash flow. Namely, levered equity holders benefit if the firm undertakes a negative net present value investment opportunity when the investment provides a sufficient increase in the risk of the firm's cash flow. The next proposition shows that this incentive becomes more complex in a setting with ambiguity and ambiguity averse investors. In Proposition 2, an increase in the degree of ambiguity occurs when the possible probabilities associated with a given outcome at time t can be written as a mean preserving spread of the possible probabilities at time $t-1$.

Proposition 2 *For a given face value of debt, if the degree of ambiguity increases, the value of the levered equity is reduced.*

The intuition for this result is equally straightforward. An increase in the level of ambiguity causes ambiguity-averse investors to further heavily underweight the probabilities of "high" outcomes, reducing the value of the option like payoff represented by levered equity. Similarly, as investors become more averse to ambiguity, the value of levered equity will fall. It is important to note that it is the combination of ambiguity and individuals' aversion to ambiguity that causes the change in perceived probabilities. If individuals are ambiguity neutral, an increase in ambiguity has no impact on the value of levered equity.

Levered equity holders, therefore, see a loss in value from an increase in ambiguity. With ambiguity and aversion to ambiguity, there exists a three way tradeoff between value, risk, and ambiguity. The incentive of levered equity holders regarding an investment that increases uncertainty (the combination of risk and ambiguity) is, therefore, unclear. An often cited example of the over-investment problem is the incentive of a firm's managers to pursue firm value reducing M&A activities due to their potential to increase the risk of the firm's cash flow. However, it is plausible that while a given merger may increase the level of risk of a firm's cash flow, the merger will also increase the level of ambiguity associated with the firm's cash flow. In such a case, the incentive of a manager acting in the interests of the levered equity holders is unclear. A variety of outcomes are possible. At one extreme, the manager may avoid a merger that would increase firm risk and firm value if it will increase ambiguity significantly, implying an "under-investment" problem. A deeper examination of the effect of ambiguity on M&A decisions is the subject of ongoing research.

A related result is provided in Garlappi et al. (2017) who model heterogeneous beliefs among a corporate board or management team as a decision-making group (DMG) that possesses a set of priors. In that case, the DMG can be viewed as facing ambiguity, à la Gilboa and Schmeidler (1989). Therefore, the greater is the disagreement (or the more heterogenous are the beliefs) amongst the DMG, the higher is the ambiguity or the aversion to ambiguity of the "corporation." When the DMG must decide whether to invest in a new project, Garlappi et al. (2017) show that, even though individuals within the group may independently believe that the investment provides value, collectively, the group may decide not to invest.

2.5 Taxes and bankruptcy costs

Consider now an economy with corporate taxes and bankruptcy costs. Recall that any state $x < F$ is considered a default state. Bankruptcy costs are assumed to be proportional to the firm's output in the event of default

$$B(x;F) = \begin{cases} \alpha x, & x < F \\ 0, & x \geq F \end{cases},$$

where $0 < \alpha < 1$. The tax benefit associated with the use of debt is, for simplicity, based on the entire debt service in non-default states,[15]

$$T(x;F) = \begin{cases} 0, & x < F \\ \tau F, & x \geq F \end{cases},$$

where τ is the corporate income tax rate. In such an economy, firm value is a function of the chosen capital structure. In particular, for a given face value of debt F, firm value is written

$$\begin{aligned} V(F) &= S_0(F) + D_0(F) \qquad (10) \\ &= \frac{1}{1+r_f}\left(\int_F^{\infty} \pi^*(x)(1-\tau)(x-F)\,dx + \int_0^F \pi^*(x)(1-\alpha)\,x dx + \int_F^{\infty} \pi^*(x) F dx\right). \end{aligned}$$

[15] See, for example, Kraus and Litzenberger (1973).

To maximize the firm's value, the capital structure choice problem can be written

$$\begin{aligned} \max_{F} \quad & S_0(F) + D_0(F) \\ \text{s.t.} \quad S_0(F) &= \frac{1}{1+r_f}\mathbb{E}^*\left[\max\left((1-\tau)x - F + T(x;F)\ ,\ 0\right)\right] \\ D_0(F) &= \frac{1}{1+r_f}\mathbb{E}^*\left[\min\left(F\ ,\ x - B(x;F)\right)\right]. \end{aligned}$$

Notice that S_0 can also be written

$$S_0(F) = \frac{1}{1+r_f}\mathbb{E}^*\left[\max\left((1-\tau)(x-F)\ ,\ 0\right)\right].$$

With the capital structure choice problem defined above, the next theorem identifies the optimal level of leverage for the firm.

Theorem 3 *The optimal leverage satisfies*

$$F = \frac{\tau}{\pi^*(F)\alpha}\int_F^{\infty} \pi^*(x)dx. \tag{11}$$

Alternatively, the optimal leverage can be written in terms of a risk neutral hazard rate for default. It can be immediately observed from Equation (11) that a higher tax rate, τ, or a lower bankruptcy cost, α, positively affect the use of leverage. Our focus is, however, on the implications of changes in ambiguity or risk for capital structure choice. Proposition 3 describes the consequence of a comparative static change in risk.

Proposition 3 *The higher is the degree of risk, the lower is the optimal leverage.*

Proposition 4 describes the consequence of a comparative static change in ambiguity.

Proposition 4 *The higher is the degree of ambiguity, the higher is the optimal leverage.*

Propositions 3 and 4 demonstrate that increased risk has a *negative* effect on the chosen level of leverage, while increased ambiguity has a *positive* effect. The intuition for Proposition 3 is standard: greater volatility in the project payoff has a symmetric effect, pushing probability mass from the center of the distribution to its tails. The increase in volatility has the effect of increasing the perceived marginal bankruptcy cost and decreasing the perceived marginal tax benefit. This effect is illustrated in Panel (a) of Figure 1. A reduction in the use of debt, to F', is required to once again equate the cost and benefit of the use of debt financing at the margin.

It may be tempting to suggest that this intuition can be applied to an increase in ambiguity as well; more ambiguity (uncertainty of probabilities) regarding future cash flow might be thought to have the same effect as an increase in risk. As discussed above, in models of ambiguity in which preferences for ambiguity are outcome-dependant, this will be the case. An increase in ambiguity, however, represents increased uncertainty regarding *probabilities.* Thus, increased ambiguity is independent of the outcomes (unlike an increase in risk), which implies that the standard intuition cannot be applied. A more appropriate intuition derives from the effect of ambiguity when investors are ambiguity averse. Ambiguity averse investors underweight probabilities: they underweight the probabilities of high output states to a greater extent than they do the probabilities of low output states. Therefore, an increase in ambiguity has an asymmetric effect on the perceived probability distribution, as illustrated in Panel (b) of Figure 1. The *total* bankruptcy cost and the *total* value of the tax shields are both reduced as a result of this change. An increase in ambiguity decreases the marginal perceived probability of default at the original optimum (i.e., the marginal perceived probability of $x = F$). Conversely to the standard intuition for an increase in risk, this implies that at the original optimum, the marginal bankruptcy cost is reduced and that the marginal tax benefit associated with debt financing is increased. An increase in leverage is necessary to attain the new optimum.

3 Empirical design

We turn now to test the predictions of Propositions 3 and 4 empirically. For consistency with the majority of the empirical capital structure literature, we examine how annual observations of firms' leverage ratios are related to annual observations of ambiguity, risk, and cash flow volatility. Similar results are obtained using quarterly data.

3.1 Sample selection

We estimate the relation between ambiguity and leverage using a sample of all nonfinancial firm-year observations in the annual Compustat database between 1993-2018. The time window 1993-2018 is dictated by the intraday stock data available on the TAQ database. This data is used in estimating the degree of ambiguity associated with each firm. After a process of data cleaning and filtering described below, we analyze $54,691$ annual firm-year observations for 5,092 unique firms over the 26 years between 1993 and 2018.

To construct our sample, we begin with all records from the Compustat fundamentals annual database. We drop all financial firms as well as all government entities. We also drop all observations missing any of the following descriptors: fiscal year end (FYR), total assets (AT), long-term debt

(DLTT) or stock price (PRCC). Finally, we drop all observations with negative values for sales, total assets or debt, and all observations with a negative leverage ratio or leverage ratio greater than 100% (for both book or market leverage). In addition, we drop all duplicate records or records for which we are unable to match identifiers to the CRSP or TAQ stock price databases. All observations for which the annual degree of ambiguity, risk or cash flow volatility cannot be estimated are also dropped. To avoid potential biases that might be caused by outlier observations, for each variable that can take on only positive values we trim the highest 1% of observation and for variables taking any value the highest and lowest 1% of observations are trimmed. Applying these filters leaves us with $54,691$ observations.

We estimate the annual degree of ambiguity, risk and cash flow volatility for each firm and each year as detailed below. For ambiguity and risk, we estimate the monthly values and then use their average over the fiscal year as the annual ambiguity and risk associated with each firm-year observation. As described below, firm levered and unlevered measures of ambiguity and risk are computed to confirm that the use of equity based measures are not responsible for the reported findings.

3.2 Estimating ambiguity

A main motivation for our use of the CEU model augmented with the EUUP framework is that Equation (3) naturally implies a risk-independent measure of ambiguity, denoted by $\mho^2$. Within the EUUP framework, the degree of ambiguity can be measured by the volatility of uncertain *probabilities*, just as the degree of risk can be measured by the volatility of uncertain *outcomes*. Formally, the measure of ambiguity is defined by Equation (4), which represents an expected probability weighted average of the variances of probabilities. We follow Izhakian and Yermack (2017) and Augustin and Izhakian (2019), and estimate the monthly degree of ambiguity for each firm using intraday stock return data from the TAQ database.[16]

The challenge in estimating ambiguity as identified in Equation (4) (or the implementation version in Equation (12) below) is to measure the expectation of and the variation in probabilities across the set of possible prior probability distributions. Each prior in the set of possible distributions is assumed to be represented by the observed daily intraday returns on the firm's (levered or unlevered)

[16]The measure of ambiguity, defined in Equation (4), is distinct from aversion to ambiguity. The former, which is a matter of beliefs (or information), is estimated from the data, while the latter is a matter of subjective attitudes. Baillon et al. (2018) suggest an elicitation of "matching probabilities," which are similar to the perceived probabilities in EUUP. Based upon matching probabilities, they suggest an ambiguity insensitivity index. By construction, this index supports only three possible events, and is attitude dependent, making it inappropriate for this application. The measure employed here supports unlimited events and is attitude independent.

equity, and the number of priors in the set is assumed to depend on the number of trading days in the month. The set of priors thus consists of 18-22 realized distributions over a month. For practical implementation, we discretize return distributions into n bins $B_j = (r_j, r_{j-1}]$ of equal size, such that each distribution is represented as a histogram, as illustrated in Figure 2. The height of the bar of a particular bin is computed as the fraction of daily intraday returns observed in that bin, and thus represents the probability of the outcomes in that bin. Equipped with these 18-22 daily return histograms, we compute the expected probability of being in a particular bin across the daily return distributions for each month, $\mathrm{E}[\mathrm{P}(B_j)]$, as well as the variance of these probabilities, $\mathrm{Var}[\mathrm{P}(B_j)]$, by assigning an equal likelihood to each histogram.[17] Using these values, the monthly degree of ambiguity of firm i is then computed as follows:

$$\mho^2[r_i] \equiv \frac{1}{\sqrt{w(1-w)}} \sum_{j=1}^{n} \mathrm{E}[\mathrm{P}_i[B_j]] \mathrm{Var}[\mathrm{P}_i[B_j]]. \tag{12}$$

To minimize the impact of the selected bin size on the value of ambiguity, we apply a variation of Sheppard's correction and scale the weighted-average volatilities of probabilities to the size of the bins by $\frac{1}{\sqrt{w(1-w)}}$, where $w = r_{i,j} - r_{i,j-1}$.

[Figure 2]

In our implementation, we sample five-minute stock returns from 9:30 to 16:00, as this frequency has been shown to eliminate microstructure effects (Andersen et al., 2001; Ait-Sahalia et al., 2005; M.Bandi and R.Russell, 2006; Y.Liu et al., 2015). Thus, we obtain daily histograms of up to 78 intraday returns. If we observe no trade in a specific time interval for a given stock, we compute returns based on the volume-weighted average of the nearest trading prices. We ignore returns between closing and next-day opening prices to eliminate the impact of overnight price changes and dividend distributions. We drop all days with less than 15 different five-minute returns; we also drop months with less than 15 intraday return distributions. In addition, we drop extreme returns (plus or minus 5% log returns over five minutes), as many such returns are due to improper orders that are subsequently canceled by the stock exchange. We normalize the intraday five-minute rates of return to daily returns.[18]

[17] This is consistent with the *principle of insufficient reason*, which states that given n possibilities that are indistinguishable except for their names, each possibility should be assigned a probability equal to $\frac{1}{n}$ (Bernoulli, 1713; de Laplace, 1814). It is also consistent with the idea of the simplest non-informative prior in Bayesian probability (Bayes et al., 1763), which assigns equal probabilities to all possibilities; and the principle of maximum entropy (Jaynes, 1957), which states that the probability distribution which best describes the current state of knowledge is the one with the largest entropy.

[18] Our results are robust to the inclusion of extreme price changes, as well as for cutoffs at lower levels of 1% in terms of log returns over five minutes. Note that the extreme 5% return over five-minutes, normalized to daily returns, implies a 390% daily return.

For the bin formation, we divide the range of daily returns into 162 intervals. We form a grid of 160 bins, from -40% to 40%, each of width 0.5%, in addition to the left and right tails, defined as $(\infty, -40\%]$ and $(+40\%, +\infty)$, respectively. We compute the mean and the variance of probabilities for each interval by assigning equal likelihood to each histogram.[19] Some bins may not be populated with return realizations, which makes it difficult to compute their probability. Therefore, we assume a normal return distribution and use its moments to extrapolate the missing return probabilities. That is, $\mathrm{P}_i[B_j] = \left[\Phi(r_j; \mu_i, \sigma_i) - \Phi(r_{j-1}; \mu_i, \sigma_i)\right]$, where $\Phi(\cdot)$ denotes the cumulative normal probability distribution, characterized by its mean μ_i and the variance σ_i^2 of the returns. As in French et al. (1987), the variance of the returns is computed by applying the adjustment for non-synchronous trading, as proposed by Scholes and Williams (1977).[20]

An important characteristic of the measure of ambiguity implied by EUUP is that it is outcome-independent (up to a state-space partition), which allows for a risk-independent examination of the impacts of ambiguity on financial decisions. Other proxies for ambiguity that have been used in the literature for empirical applications include the volatility of the mean return (Franzoni, 2017), the volatility of return volatility (Faria and Correia-da Silva, 2014), or the dispersion in analyst forecasts (Drechsler, 2013). As these measures are sensitive to changes in the set of outcomes (i.e., are outcome-dependent), they are risk-dependent and therefore less useful for this study. Furthermore, these proxies are conceptually different from and only weakly related to our measure of ambiguity. Similarly, other moments of the return distribution such as skewness, kurtosis, etc. are also outcome-dependent and so conceptually different from $\mho^2$. Jumps, time varying mean and time varying volatility are also outcome-dependent measures.

Brenner and Izhakian (2018) and Augustin and Izhakian (2019) study the implications of ambiguity for the equity premium and for the spreads on credit default swaps. Their results indicate that in these contexts $\mho^2$ does not simply reflect other well-known "uncertainty" factors including skewness, kurtosis, variance of variance, variance of mean, downside risk, mixed data sampling measure of forecasted volatility (MIDAS), jumps, or investors' sentiment, among many others. Their tests also help to mitigate the concern that observed returns are generated by a single (additive) probability distribution. In our robustness tests, we examine many of these uncertainty factors at the firm level.

[19]The assignment of equal likelihoods is equivalent to assuming that the daily ratios $\frac{\mu_i}{\sigma_i}$ are Student's-t distributed. When $\frac{\mu}{\sigma}$ is Student's t-distributed, cumulative probabilities are uniformly distributed (e.g., Proposition 1.27, page 21 Kendall and Stuart, 2010).

[20]Scholes and Williams (1977) suggest adjusting the volatility of returns for non-synchronous trading as $\sigma_t^2 = \frac{1}{N_t}\sum_{\ell=1}^{N_t}(r_{t,\ell} - \mathrm{E}[r_{t,\ell}])^2 + 2\frac{1}{N_t - 1}\sum_{\ell=2}^{N_t}(r_{t,\ell} - \mathrm{E}[r_{t,\ell}])(r_{t,\ell-1} - \mathrm{E}[r_{t,\ell-1}])$. We perform all estimations without the Scholes-Williams correction for non-synchronous trading. The results are essentially the same.

In particular, in Table 5 we examine the explanatory power of our measure of ambiguity relative to others that have been proposed, and show that ambiguity represents a distinct aspect of uncertainty in the present context.

It has long been recognized that leverage magnifies measures of volatility, creating cross-sectional variation in volatility that is related to leverage. It is also possible that leverage may affect the measure of ambiguity, however, due to the outcome independence of ambiguity the nature of any such impact is unclear. Furthermore, taking first differences of the data removes some of any cross-sectional explanatory power of ambiguity (or risk) created by leverage and so we do not predict that this will have a material impact on the reported findings. Nevertheless, for completeness, we estimate all regression tests using both a measure of ambiguity based the firm's levered returns and based on a proxy for the firm's unlevered returns. This is done by combining the book value of total debt and the market value of equity to represent firm value for every five-minute interval.[21] The resulting periodic firm values are used to estimate unlevered or firm level returns for each interval. Unlevered ambiguity is computed from these unlevered returns as described above.

3.3 Estimating risk

Following many earlier studies of leverage (e.g., Frank and Goyal, 2009), we use equity return volatility (or unlevered return volatility) as a proxy for default risk. Return volatility has also been used in the literature on pricing default risk for debt (e.g., Bharath and Shumway, 2008) and as an input for the "distance to default" measure derived from the Merton (1974) model. Other empirical examinations of leverage have used a measure of cash flow volatility (e.g., the variance of operating income) rather than return volatility as the relevant proxy. Cash flow volatility is a more direct measure of default risk but does not take account of other firm characteristics that impact default risk (i.e., the amount and timing of principal repayments and other debt service). Because we take first differences of the variables in our empirical tests the cash flow volatility measure will, by construction, exhibit very little time-series variation. We therefore use return volatility as our primary measure of risk. In the Online Appendix, we present results using cash flow volatility as the measure of default risk. In these tests, cash flow volatility is measured by the volatility of the past three annual cash flow realizations. Similar findings are obtained using the volatility of the past twelve quarterly cash flow realizations or measuring the volatility of annual cash flow realizations measured over longer horizons.

[21] We consider three different frequencies of estimating the market value of the firm: updating the value every five-minutes, every day, and every month. Our results are robust across these alternatives.

For consistency, the variance of equity returns is computed using the same five-minute returns that we use to measure ambiguity. For each individual stock i on each day, we compute the variance of intraday returns, applying the Scholes and Williams (1977) correction for non-synchronous trading and a correction for heteroscedasticity.[22] In a given year, we then compute the annual variance of stock returns using the average of daily variances, scaled to a monthly frequency. For robustness, we also estimate risk as the average monthly variance of daily returns with no change in (not tabulated) results.

3.4 Leverage and firm specific characteristics

The annual leverage ratio of each firm is the main dependent variable in our empirical tests; and we consider both book value and market value versions of this ratio. Book leverage is computed as "debt in current liabilities" plus "long-term debt" divided by the total book value of assets. Market leverage is computed as "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt."

In addition to the leverage ratio and return volatility measures, we obtain a set of annual firm characteristics using variables that are standard in the empirical capital structure literature. Firm size is measured by the log of one plus the level of the firm's sales.[23] Profitability is measured by operating income before depreciation divided by book assets. Asset tangibility is measured by property, plant and equipment divided by book assets. The market-to-book ratio is measured by the market value of equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is measured by R&D, expenses relative to sales and normalized by $10,000$. In addition, to explore the firms' leverage ratios relative to their industry, we also include industry median annual leverage ratio, where firms are classified by their four-digit SIC codes. Expected marginal tax rates are obtained from John Graham's website. When the expected marginal tax rate is missing, we use the industry median annual marginal tax rate if available or the market-wide median annual marginal tax rate if necessary.

[22] See, for example, French et al. (1987).

[23] Alternatively, firm size can be measured by the log of book assets. We test all predictions using this alternate measure of firm size and the findings are virtually identical.

3.5 Summary statistics

Table 1 presents summary statistics for the key variables used in the paper. Based on the statistics presented in the table, our sample is broadly consistent with those used in recent studies of leverage.

[Table 1]

3.6 Regression tests

The main focus of our empirical tests is Proposition 4. The main empirical model considers a version of the time series, cross-sectional regression that is standard in the empirical capital structure literature, augmented to include the measure of ambiguity as an explanatory variable

$$L_{i,t} \quad = \quad \alpha + \beta \cdot \mho^2_{i,t-1} + \gamma \cdot Z_{i,t-1} + \varepsilon_{i,t}. \tag{13}$$

However, in order to address this model's omitted variables problem, identified by Lemmon et al. (2008), we utilize first differences of the dependent and independent variables. To this end, we estimate

$$\Delta L_{i,t} \quad = \quad \alpha + \beta \cdot \Delta\mho^2_{i,t-1} + \gamma \cdot \Delta Z_{i,t-1} + \varepsilon_{i,t}, \tag{14}$$

where $\Delta L_{i,t}$ is the change in variable L between time $t-1$ and t, and $\Delta Z_{i,t-1}$ is the change in the vector of variables Z between time $t-2$ and time $t-1$.

Lemmon et al. (2008) employ firm fixed effects to control for the omitted variables bias. The use of first differences in Equation (14) is conceptually analogous to adding firm fixed effects to Equation (13). If the unobserved variation is truly time invariant, then the use of firm fixed effects offers a more robust solution to the problem then does a first differences approach. However, recent evidence (e.g., DeAngelo and Roll, 2015) indicates that the unobserved variation in leverage highlighted by Lemmon et al. (2008) is *not* strictly time invariant. This result suggests that the use of first differences offers a superior solution (e.g., Grieser and Hadlock, 2019).[24]

The regression specification in Equation (14) is tested using both book and market leverage as the dependent variables. Each model is also tested using both levered and unlevered ambiguity at the firm level as the explanatory variable of interest. All reported standard errors are clustered by firm and are robust to within firm heteroskedasticity.

[24] Furthermore, Wooldridge (2010) notes that in an instrumental variables context (presented in the Online Appendix) the use of first-difference two-stage least squares has a weaker exogeneity restriction than does a fixed effects two-stage least squares estimator. We present the first-difference two-stage analysis in order to provide external validity for the ambiguity measure used in this paper. The instrumental variables approach presented in the Online Appendix shows that variation in the ambiguity measure explained by the instruments is also significantly positively related to the firm's leverage ratio, consistent with the prediction in Proposition 4.

4 Empirical findings

4.1 Main findings

The results of our main empirical specification are reported in Table 2. Results for the examination of the book leverage ratio are contained in Panel A of the table and the examination of the market leverage ratio are in Panel B. Consistent with our theoretical predictions, Table 2 reports that in every variation of the model, the ambiguity (or unlevered ambiguity) variable has a positive and statistically significant coefficient estimate. Furthermore, in each of the regressions in panel A (book leverage) and panel B (market leverage), the coefficient estimates for the measure of risk are all negative and significant.

The coefficient estimates for the control variables in Table 2 provide conclusions that are generally in line with prior studies. Leverage tends to be higher in larger firms and for firms with more tangible assets, while more profitable firms and firms with higher market-to-book ratios tend to use less leverage. However, asset tangibility and firm size are the only reliably significant control variables in these regression specifications. Using the first differences of the explanatory variables to explain the change in book leverage, we find that only the coefficient estimates for asset tangibility, the market-to-book ratio and firm size are statistically significant. In the model explaining market leverage, the estimated coefficient for R&D expenditures also becomes statistically significant while the coefficient estimate for the market-to-book ratio becomes insignificant. It is somewhat surprising that the estimated relation between R&D expenditures and the leverage ratio (both market and book) is essentially zero. Prior studies (e.g., Mackie-Mason, 1990; Berger et al., 1997) have found a negative relation, in line with the Myers (1977) prediction that growth opportunities will be financed primarily by equity. This is likely due to the fact that the first differences transformation sweeps out much of the cross-sectional variation which has been shown to represent the majority of the variation in leverage (Lemmon et al., 2008).

[Table 2]

The reported results are also robust across a variety of restrictions on the sample included in the tests or alternative measures of the explanatory variables. If we winsorize rather than trim outliers in the data, require more consecutive data points to emphasize the time-series component to the estimates, or use quarterly rather than annual data, the estimated coefficient on the ambiguity measure remains positive and significant. Further, using the variance of cash flow (see Table OA.1 in the Online Appendix) or a measure of the "distance to default" derived from the Merton (1974) model to measure risk (e.g., Bharath and Shumway, 2008), or if we measure firm size using the natural log of book assets rather than sales, the estimated coefficient on the ambiguity measure remains positive and significantly related to leverage.[25] However, the relation between the measure of risk and leverage varies depending upon the specification. The somewhat erratic significance of risk as an explanatory variable for leverage is consistent with Frank and Goyal (2009), who find that risk, measured as the variance of equity returns, is not one of the "core factors" determining leverage. Note that their analysis is primarily based on the incremental contribution to R-squared of the explanatory variables rather than on the economic or statistical significance of their coefficient estimates.

To further examine the relation between ambiguity and leverage, Table 3 reports the results of an estimation of the model in Equation (14) using sub-samples of the data based on the median level of firm size or leverage ratio. The first four columns of the table consider subgroups based on leverage ratio. In particular, firms with leverage below the median value of leverage in the sample are labeled as low leverage and those with a leverage ratio above the median value are labeled as high leverage. The second four columns of the table consider subgroups based on firm size, where firms below the median firm size in the full sample are labeled as small and those whose size is above the median are labeled as large. The table examines the relation between leverage and ambiguity (unlevered ambiguity). Panel A of Table 3 reports the findings for book leverage and Panel B reports the findings for market leverage.

The results using book leverage in the sub-sample analysis partially support the theoretical predictions and the results using the full sample. Consistent with Propositions 3 and 4, in columns (1) and (2) ambiguity is significantly positively related to leverage and risk is significantly negatively related to leverage, for subgroups of firms with low and high leverage ratios. In columns (3) and (4), unlevered ambiguity is significantly positively related to leverage only for the low leverage firms while for high leverage firms unlevered ambiguity is insignificantly related to leverage. Conversely, columns (3) and (4) show that, unlevered risk is insignificantly related to leverage for the low leverage subgroup and significantly negatively related to leverage for the high leverage subgroup.

Columns (5) and (6) of Table 3 show that ambiguity is insignificantly related to leverage for the subgroup of small firms while ambiguity is significantly positively related to leverage amongst the large firms. Columns (5) and (6) also show that risk is significantly negatively related to leverage for small firms but insignificantly so for the large firm subgroup. Similarly, Columns (7) and (8) show that

[25] These restrictions on the sample or specifications of the explanatory variables have all been used in recent empirical examinations of leverage.

unlevered ambiguity is insignificantly related to leverage in the small firm subgroup and significantly positively related to leverage in the large firm subgroup. Unlevered risk is significantly negatively related to leverage within both the small and large firm subgroups.

Panel A of Table 3 also shows that the standard control variables are not consistently significantly related to leverage when we divide the sample by size and leverage. Asset tangibility is significantly positively related to leverage across all subgroups, however, this is the only control variable with a consistently significant relation to leverage. In particular, firm size (the other consistently significant control variable in Table 2) is a significant explanatory variable for leverage in only half of the reported regressions. The market-to-book ratio is significantly negatively related to leverage in most of the regressions while R&D expenditures are insignificant in all of the subgroup regresssions. For the examination of book leverage, dividing the sample into subgroups seems to provide erratic significance for many of the standard explanatory variables.

Panel B of Table 3 indicates that when we examine market leverage, the results are much more consistent across the subgroups. Consistent with Propositions 3 and 4, across all of the subgroups ambiguity (or unlevered ambiguity) is significantly positively related to leverage and risk (or unlevered risk) is significantly negatively related to leverage.

The results for the standard control variables are also more consistent with the results reported in Table 2. Specifically, asset tangibility and firm size are significantly positively related to leverage across all the subgroups. Overall, the subgroup analysis is generally consistent with the results from the full sample analysis. However, there are some interesting patterns that indicate the value of further exploration of this issue.

[Table 3]

4.2 Economic significance

We now turn to an investigation of the economic significance of our measure of ambiguity in explaining capital structure. Table 4 presents measures of the economic significance of the independent variables we employ in our specification for the leverage ratio. The table is constructed by multiplying the standard deviation of each explanatory variable, shown in the second column, by the corresponding coefficient estimate from Table 2 (Panels A and B), with the product of these two quantities displayed in the third column under the heading "Significance."[26] In the fourth and fifth columns we report the

[26] Recall that we are using the first difference of the variables and so the standard deviation reported in the table, "Std", is the standard deviation of the innovations or changes in the explanatory variables rather than the standard deviations of the variables themselves.

significance of the ambiguity measure and the unlevered ambiguity measure relative to the significance of the different explanatory variables. These relative measures are reported to aid in interpretation of the significance levels.

The table reveals three interesting results. First, the two measures of ambiguity or unlevered ambiguity have greater levels of economic significance than do risk or unlevered risk in explaining leverage choice. This suggests that ambiguity may be a more important aspect of uncertainty than is risk if we are to understand the capital structure choice. Second, the measure of ambiguity displays a greater level of economic significance in explaining leverage than does the measure of unlevered ambiguity. This result (as well as the consistency in the sign and significance of ambiguity and unlevered ambiguity reported in Table 2) suggests that the ambiguity measure based on equity returns is not a bias measure of firm level ambiguity. Finally, of the standard control variables, only asset tangibility has a roughly equivalent level of economic significance as does ambiguity or unlevered ambiguity. Both ambiguity and unlevered ambiguity display greater levels of economic significance than do all of the other common control variables. Overall, the ambiguity faced by the firm appears to have an important impact on its leverage decision.

[Table 4]

5 Robustness

We next turn to examinations of the robustness of our analysis to measures of ambiguity.

5.1 Alternative proxies for ambiguity

Table 5 reports robustness tests using alternative proxies for ambiguity that have been employed in the literature. We test each of the following four factors as a substitute for our ambiguity measure as well as a factor alongside our measure. The variance of the mean (Var Mean) is the variance of daily mean returns (computed from 5-minute returns) over a month and averaged over the year. The variance of the variance (Var Var) is the variance of daily variance of returns (computed from 5-minute returns) over a month and averaged over the year. Bid-ask spread is the annual average of the effective bid-ask spread for the firm's equity. Disagreement or dispersion of analysts' forecasts is the variance among analyst forecasts regarding the future stock price.

We run our regression tests using the first differences of book and market leverage, as explained by year lagged differences in the ambiguity proxies. All regressions include an intercept and changes in the standard control variables that are included in the full model reported in Table 2. Panel A presents the findings using book leverage and panel B presents findings using market leverage. When the alternative proxies for ambiguity substitute for our measure of ambiguity in the regressions explaining book leverage the bid-ask spread and the dispersion of analyst forecasts are significantly related to leverage. However, the estimated coefficients on these variables show a negative rather than the predicted positive relation to leverage. When the alternative proxies for ambiguity are substituted for our measure and used to explain market leverage (columns 2 - 5 of panel B), only the bid-ask spread has a significant coefficient estimate. The estimated coefficient on the bid-ask spread variable is again significantly negative.

The final four columns in Panels A and B in Table 5 include both our measure of ambiguity and each of the previously used proxies in regressions as well as the measure of risk and the other control variables. Importantly, our measure of ambiguity remains positive and statistically significant in all the regressions for both book and market leverage. The other proxies for ambiguity are all have either significantly negative coefficient estimates or insignificant coefficient estimates. The variance of mean is insignificantly related to both book and market leverage when included alongside our measure of ambiguity. Since the variance of the mean has been considered a proxy for time varying probability distributions, this finding rules out the possibility that the significance of our measure is driven solely by a time varying mean. Similarly, the variance of variance continues to be insignificantly related to leverage when it is included with our measure of ambiguity in the regression. The bid-ask spread has a significantly negative relation to both book and market leverage when included along side our measure of ambiguity. Finally, the disagreement among analysts is significantly negatively related to book leverage and insignificantly to market leverage when included with our measure.

These findings indicate that our measure of ambiguity and the various proxies for ambiguity capture distinctly different aspects of uncertainty. Overall, the results presented in Table 5 confirm that our measure of ambiguity is not simply a proxy for previously used measures. We also conclude that the previously used proxies for ambiguity, while plausibly reflecting some aspects of ambiguity, do not seem to capture the salient features of ambiguity in the context of the capital structure decision. The main reason might be that all the aforementioned alternative measures are outcome-dependent and therefore risk-dependent, while our measure is outcome-independent.

[Table 5]

5.2 Alternative ambiguity estimates

To construct our ambiguity measure, we assume that intraday returns are normally distributed, and therefore fully characterized by the first and the second moments of the return distribution. For robustness, we explore alternative parametric assumptions of the intraday return distribution, allowing for skewness, kurtosis, or both. First, we compute ambiguity assuming that intraday returns follow a Laplace distribution. To eliminate jump effects, we also compute both measures of ambiguity (normal and leptokurtic) by truncating five-minute intraday returns larger than 1% instead of 5%. Second, we compute ambiguity non-parametrically by constructing the statistical histograms of the intraday return distributions without imposing a particular distributional form. Third, we compute ambiguity using thirty-second and ten-minute return frequencies, as alternatives to the five-minute return frequency used in the results reported here. Fourth, we compute ambiguity using 82, 322 and 1002 bin histograms, instead of 162 bin histograms.[27] We run the main regression specifications using these variations, and the unreported results are essentially the same. The economic significance weakens for the statistical (non-parametric) distribution, as many empty histogram bins are not extrapolated as compared to the use of the parametric distributions. These alternative estimates of ambiguity indicate that the significance of our results is not tied to a specific parametric assumption regarding the intraday return distribution.

6 Conclusion

Uncertainty plays a role in the capital structure decision as in many other financial decisions. Until recently, most studies examined uncertainty only by considering risk. However, ambiguity, or Knightian uncertainty, represents a separate and distinct aspect of uncertainty. We contribute to a growing literature by showing that both ambiguity and risk play important roles in capital structure choice.

We present a model that predicts a positive relation between ambiguity and leverage, and provide preliminary evidence in support of this hypothesis using pooled time series cross-sectional data covering 54,000 firm-year observations from more than 5,000 individual firms. Consistent with the model's prediction, we find a positive association between ambiguity and leverage. The coefficient estimates suggest that the ambiguity facing the firm has an economically meaningful impact on the capital structure decision. Furthermore, the economic significance of ambiguity is at leat as large as all other

[27]For each bin grid, we examine intraday returns sampled at thirty-second, five-minute, and ten-minute intervals. For consistency, alongside each alternative ambiguity measure, we also compute a corresponding measure of risk using the same frequency of returns.

known capital structure explanatory variables. Robustness tests demonstrate that other proxies for ambiguity that have been proposed are not meaningful explanatory variables in the context of the leverage decision.

Our results are consistent with other recent papers analyzing variables affected by financial uncertainly, including the pricing of credit default swaps and the timing of the exercise of executive stock options. Together, these studies suggest that the role of ambiguity in financial decisions is richer and more nuanced than previously believed, and that further investigation of ambiguity has the promise of yielding additional insights that may improve our understanding of basic financial decision-making.

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Appendix

Proofs

Proof of Theorem 1.

Substituting the budget constraints into the objective function in Equation (5) and solving the maximization problem, differentiation with respect to θ conditional on a given x (state of nature), provides

$$q(x)\partial_0 \mathrm{U} \;=\; \mathrm{E}\left[\varphi\left(x\right)\right]\left(1+\frac{\Upsilon''\left(1-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}{\Upsilon'\left(1-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}\mathrm{Var}\left[\varphi\left(x\right)\right]\right)\partial_x \mathrm{U}.$$

Organizing terms completes the proof. Since markets are complete and the law of one price holds, the payoff pricing functional assigns a unique price to each state contingent claim. ■

Proof of Theorem 2.

Obtained from the same arguments in Proposition 1 of Modigliani and Miller (1958). ■

Proof of Theorem 3.

The first-order condition, obtained by differentiating the firm value in Equation (10) with respect to its leverage, is

$$\frac{\partial V\left(\cdot\right)}{\partial F} \;=\; \frac{1}{1+r_f}\left(\begin{array}{l} -\int_F^{\infty}\pi^*(x)\left(1-\tau\right)dx \\ +\pi^*(F)F\left(1-\alpha\right)+\int_F^{\infty}\pi^*(x)dx-\pi^*(F)F \end{array}\right) \;=\; 0.$$

Rearranging terms provides

$$F \;=\; \frac{\tau}{\pi^*(F)\alpha}\int_F^{\infty}\pi^*(x)dx.$$

■

Proof of Proposition 1.

Consider a payoff $F \leq x$. Suppose that risk increases such that instead of x the payoff is $x+\Delta$ or $x-\Delta$, with equal probabilities, i.e., $x\pm\Delta$ is mean-preserving spread of x. If $\Delta \leq x-F$ this will not affect to value of the equity. If $\Delta > x-F$, then $\frac{1}{2}\left(x-F+\Delta\right) > x-F$. This holds true for any $F \leq x$, Thus, by Equation (10), the value of the equity increases in risk. ■

Proof of Proposition 2.

By Equation (7), for every outcome $\pi^*\left(x\right)$ decreases in ambiguity. Thus, by Equation (10), S_0 decreases in ambiguity. ■

Proof of Proposition 3.

Substitute the explicit expression of $\pi^*\left(\cdot\right)$ into the optimal leverage in Equation (11) to obtain

$$F \;=\; \frac{\tau}{\alpha q(F)}\int_F^{\infty}q(x)dx. \qquad (15)$$

The first-order condition (FOC) can be defined to be the implicit function

$$G\left(F, \mho^2, \mathcal{R}\right) = \tau \int_F^\infty q(x)dx - \alpha q(F)F, \tag{16}$$

where $\mathcal{R}$ stands for the level of risk. By the second-order condition, at a maximum

$$\frac{\partial G}{\partial F} < 0.$$

Suppose that for any event with an outcome x the outcome is instead $x \times t$, where $0 < t$. The greater it t the greater is the risk and so is the expected outcome. Differentiating the implicit function in Equation (16) with respect to t, provides

$$\frac{\partial G}{\partial t} = \tau \int_F^\infty \mathrm{E}\left[\varphi\left(x\right)\right]\left(1 + \frac{\Upsilon''\left(1 - \mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}{\Upsilon'\left(1 - \mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}\mathrm{Var}\left[\varphi\left(x\right)\right]\right)\frac{\partial_{xx}\mathrm{U}}{\partial_0 \mathrm{U}} x dx.$$

By risk aversion, $\partial_{xx}\mathrm{U} < 0$; Thus, $\frac{\partial G}{\partial t} \leq 0$. By the second-order condition, $\frac{\partial G}{\partial F} < 0$. By the implicit function theorem,

$$\frac{\partial G}{\partial F}dF + \frac{\partial G}{\partial t}dt = 0.$$

Therefore,

$$\frac{dF}{dt} = -\frac{\frac{\partial G}{\partial t}}{\frac{\partial G}{\partial F}} \leq 0.$$

■

Proof of Proposition 4.

Suppose that the ambiguity increases such that the variance $\mathrm{Var}\left[\varphi\left(x\right)\right] \times t$ of the probability of each x increases, where $t > 0$. Differentiating the implicit function in Equation (16) with respect to t, provides

$$\begin{aligned}\frac{\partial G}{\partial t} = {} & \tau \int_F^\infty \mathrm{E}\left[\varphi\left(x\right)\right]\frac{\Upsilon''\left(1 - \mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}{\Upsilon'\left(1 - \mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}\mathrm{Var}\left[\varphi\left(x\right)\right]\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}dx - \\ & \alpha \mathrm{E}\left[\varphi\left(F\right)\right]\frac{\Upsilon''\left(1 - \mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}{\Upsilon'\left(1 - \mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}\mathrm{Var}\left[\varphi\left(F\right)\right]\frac{\partial_F \mathrm{U}}{\partial_0 \mathrm{U}}F.\end{aligned}$$

Since the FOC, defined by Equation (16), is equal to zero,

$$\frac{\partial G}{\partial t} = -\tau \int_F^\infty \mathrm{E}\left[\varphi\left(x\right)\right]\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}dx + \alpha \mathrm{E}\left[\varphi\left(F\right)\right]\frac{\partial_F \mathrm{U}}{\partial_0 \mathrm{U}}F.$$

Thus, $\frac{\partial G}{\partial t} \geq 0$ when

$$F \geq \frac{\tau}{\alpha}\frac{1}{\mathrm{E}\left[\varphi\left(F\right)\right]\frac{\partial_F \mathrm{U}}{\partial_0 \mathrm{U}}}\int_F^\infty \mathrm{E}\left[\varphi\left(x\right)\right]\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}dx. \tag{17}$$

The right hand side of this inequality is the optimal F assuming there is no ambiguity (alternatively, no aversion to ambiguity). Since $\left(\frac{\Upsilon''}{\Upsilon'}\right)' \geq 0$, $0 \geq \frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(F)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(F)])} \geq \frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(x)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(x)])}$ for any $x \geq F$. Therefore, this inequality holds when ambiguity is present. By the implicit function theorem,

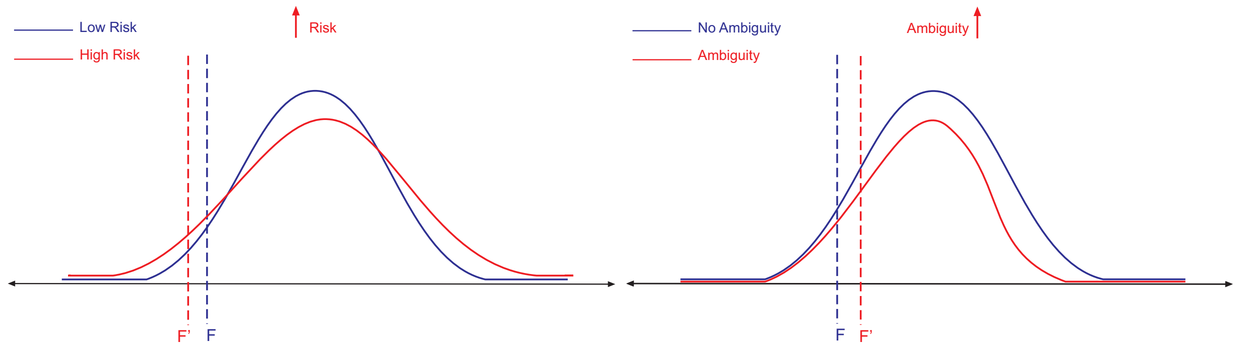
$$\frac{\partial G}{\partial F} dF + \frac{\partial G}{\partial t} dt \quad = \quad 0.$$

Therefore, since $\frac{\partial G}{\partial F} < 0$,

$$\frac{dF}{dt} \quad = \quad -\frac{\frac{\partial G}{\partial t}}{\frac{\partial G}{\partial F}} \geq 0.$$

■

6.1 Tables and figures



Panel A: Increasing Risk

Panel B: Increasing Ambiguity

Figure 1: **Optimal leverage**

This figure depicts the effect of an increase in risk (left panel) and an increase in ambiguity (right panel) on optimal leverage. Each panel represents a stylized perceived probability distribution in which the x-axis illustrates the range of the firm's possible future values. The point F is the optimal choice for the firm's leverage, at which the expected marginal benefit of interest tax shields just equals the expected marginal cost of bankruptcy. In the left panel, the blue series shows a lower-risk perceived probability distribution, which is assumed to shift to the red series, representing a higher risk. The optimal choice of leverage then moves to the left on the x-axis. In the right panel, an increase in ambiguity is assumed to occur, resulting in lower perceived probabilities of expected outcomes, as shown by the change from the blue to the red probability distribution. In this case the optimal choice of leverage moves to the right.

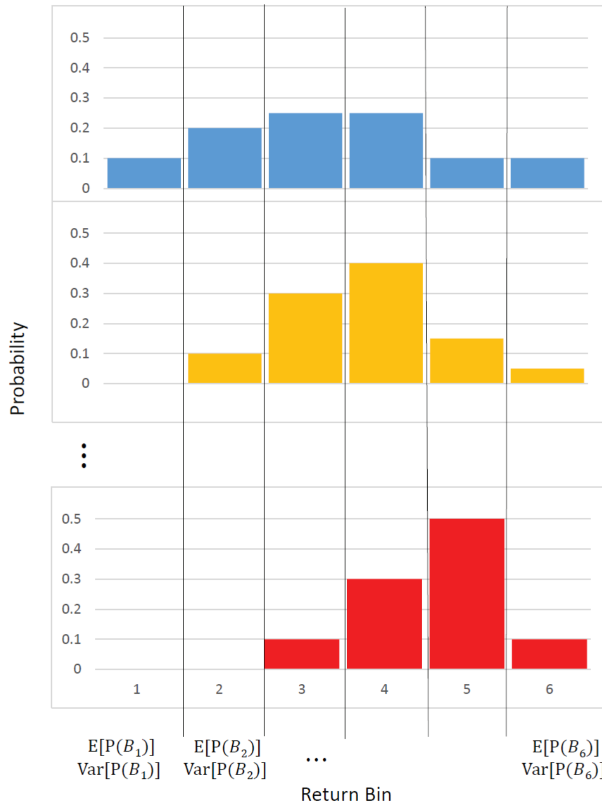


Figure 2: **Ambiguity measurement**

This figure illustrates the computation of the ambiguity measure, which is derived for each firm-month based on intraday stock-returns sampled at a five-minute frequency from 9:30 to 16:00. Thus, we obtain up to 22 daily histograms of up to 78 intraday returns in each month. We discretize the daily return distributions into n bins of equal size $B_j = (r_j, r_{j-1}]$ across histograms. The height of the histogram for a particular bin is computed as the fraction of daily intraday returns observed in that bucket, and thus represents the probability of that particular bin outcome. We compute the expected probability of being in a particular bin across the daily return distributions, $\mathrm{E}[\mathrm{P}_i(B_j)]$, as well as the variance of these probabilities, $\mathrm{Var}[\mathrm{P}_i(B_j)]$. Ambiguity is then computed as $\mho^2[r_i] \equiv 1/\sqrt{w(1-w)} \sum_{j=1}^{n} \mathrm{E}[\mathrm{P}_i(B_j)] \mathrm{Var}[\mathrm{P}_i(B_j)]$, where we scale the weighted-average volatilities of probabilities to the bins' size $w = r_{i,j} - r_{i,j-1}$.

Table 1: **Descriptive statistics**

The full sample includes 54,691 annual observations associated with 5,092 individual firms between 1993 and 2018, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt". Sale is the log of one plus the firm's reported sales. Profitability is operating income before depreciation divided by book assets. Tangibility is the plant property divided by book assets. The market-to-book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is R&D expenses relative to sales. Median leverage is the median of the annual leverage in the industry.

Statistic	N	Mean	St. Dev.	Median	Min	Max
Book Leverage	54,691	0.201	0.202	0.157	0.000	0.968
Market Leverage	54,691	0.176	0.206	0.102	0.000	0.959
Ambiguity	54,691	0.020	0.021	0.013	0.001	0.180
Ambiguity Unlev.	54,691	0.029	0.035	0.016	0.001	0.288
Risk	54,691	0.108	0.148	0.039	0.001	1.042
Risk Unlev.	54,691	0.091	0.133	0.029	0.001	0.894
Cash Flow Vol.	47,297	0.042	0.694	0.0002	0.000	44.967
Median Book Lev.	54,691	0.171	0.152	0.139	0.000	0.950
Median Market Lev.	54,691	0.152	0.162	0.100	0.000	0.953
Profitability	54,691	0.048	0.270	0.111	−3.242	0.787
Tangibility	54,691	0.251	0.229	0.174	0.000	0.958
Market to Book	54,691	1.998	2.038	1.368	0.002	28.783
RD	54,691	0.016	0.049	0.001	0	1
Sales	54,691	1.025	0.203	1.080	0.000	1.234
Tax Rate	54,691	0.147	0.152	0.044	0.000	0.395
Firm Age	54,691	16.823	12.527	13	2	61

Table 2: **First Difference regression estimates of capital structure**

Linear model regression estimates of the changes in annual leverage ratio explained by one-year lagged changes in the variables. The full sample includes 54,691 annual observations associated with 5,092 individual firms between 1993 and 2018, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt". Sale is the log of one plus the firm's reported sales. Profitability is operating income before depreciation divided by book assets. Tangibility is the plant property divided by book assets. The market-to-book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is R&D expenses relative to sales. Median leverage is the median of the annual leverage in the industry. Standard errors, clustered by firm and robust to heteroscedasticity, appear in parentheses below each coefficient estimate. Standard errors are in parentheses, $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Panel A: Book Leverage

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Ambiguity	0.109***		0.129***	0.144***				
	(0.033)		(0.032)	(0.032)				
Ambiguity Unlev.					0.056***		0.058***	0.068***
					(0.021)		(0.021)	(0.021)
Risk		−0.028***	−0.031***	−0.038***				
		(0.008)	(0.008)	(0.008)				
Risk Unlev.						−0.019**	−0.020**	−0.027***
						(0.009)	(0.009)	(0.009)
Median Book Lev.				−0.005				−0.006
				(0.007)				(0.007)
Profitability				−0.003				−0.003
				(0.005)				(0.005)
Tangibility				0.080***				0.078***
				(0.012)				(0.012)
Market to Book				−0.001***				−0.001***
				(0.000)				(0.000)
RD				0.000				0.000
				(0.000)				(0.000)
Sales				0.031***				0.032***
				(0.011)				(0.011)
Tax Rate				−0.000				−0.000
				(0.004)				(0.004)
Constant	0.006***	0.006***	0.006***	0.005***	0.006***	0.006***	0.006***	0.006***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Observations	38,813	38,813	38,813	38,813	38,813	38,813	38,813	38,813
R^2	0.000	0.001	0.001	0.004	0.000	0.000	0.000	0.004

Panel B: Market Leverage

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Ambiguity	0.388***		0.484***	0.488***				
	(0.040)		(0.040)	(0.041)				
Ambiguity Unlev.					0.370***		0.385***	0.385***
					(0.028)		(0.029)	(0.029)
Risk		−0.138***	−0.149***	−0.154***				
		(0.009)	(0.009)	(0.009)				
Risk Unlev.						−0.121***	−0.125***	−0.128***
						(0.009)	(0.009)	(0.009)
Median Market Lev.				−0.009				−0.012
				(0.007)				(0.007)
Profitability				−0.006				−0.004
				(0.004)				(0.004)
Tangibility				0.087***				0.084***
				(0.013)				(0.013)
Market to Book				−0.000				−0.000
				(0.000)				(0.000)
RD				0.000**				0.000**
				(0.000)				(0.000)
Sales				0.037***				0.039***
				(0.008)				(0.008)
Tax Rate				0.003				0.004
				(0.005)				(0.005)
Constant	0.007***	0.007***	0.006***	0.006***	0.007***	0.007***	0.006***	0.006***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Observations	38,813	38,813	38,813	38,813	38,813	38,813	38,813	38,813
R^2	0.003	0.009	0.013	0.016	0.005	0.006	0.010	0.013

Table 3: **First Difference regression estimates of capital structure on subgroups by median**

Linear model regression estimates of the changes in annual leverage ratio explained by one-year lagged changes in the variables using subgroups of above and below median size and above and below median leverage. The full sample includes 54,691 annual observations associated with 5,092 individual firms between 1993 and 2018, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt". Sale is the log of one plus the firm's reported sales. Profitability is operating income before depreciation divided by book assets. Tangibility is the plant property divided by book assets. The market-to-book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is R&D expenses relative to sales. Median leverage is the median of the annual leverage in the industry. Standard errors, clustered by firm and robust to heteroscedasticity, appear in parentheses below each coefficient estimate. Standard errors are in parentheses, *p<0.1; **p<0.05; ***p<0.01

Panel A: Book Leverage

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Leverage Subgroups				Size Subgroups			
	Low	High	Low	High	Small	Large	Small	Large
Ambiguity	0.105**	0.130***			0.116	0.168***		
	(0.043)	(0.045)			(0.083)	(0.034)		
Ambiguity Unlev.			0.064*	−0.017			0.097	0.065***
			(0.036)	(0.026)			(0.069)	(0.022)
Risk	−0.024***	−0.101***			−0.036***	−0.026		
	(0.007)	(0.018)			(0.009)	(0.023)		
Risk Unlev.			−0.008	−0.121***			−0.023**	−0.053*
			(0.007)	(0.026)			(0.009)	(0.032)
Median Book Lev.	−0.014	−0.027***	−0.015	−0.030***	−0.016	0.005	−0.018	0.004
	(0.010)	(0.009)	(0.010)	(0.009)	(0.012)	(0.008)	(0.012)	(0.008)
Profitability	0.001	−0.014	0.002	−0.014	−0.003	−0.010	−0.002	−0.010
	(0.004)	(0.015)	(0.004)	(0.015)	(0.006)	(0.013)	(0.006)	(0.013)
Tangibility	0.061***	0.097***	0.059***	0.095***	0.070***	0.099***	0.068***	0.099***
	(0.012)	(0.019)	(0.012)	(0.019)	(0.017)	(0.015)	(0.017)	(0.015)
Market to Book	−0.001***	−0.003**	−0.001***	−0.003*	−0.001***	−0.001	−0.001***	−0.001
	(0.000)	(0.001)	(0.000)	(0.001)	(0.000)	(0.001)	(0.000)	(0.001)
RD	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Sales	0.011	0.103**	0.012	0.103**	0.030***	0.079	0.031***	0.074
	(0.008)	(0.040)	(0.008)	(0.041)	(0.011)	(0.075)	(0.011)	(0.075)
Tax Rate	−0.001	0.002	−0.000	0.002	0.004	−0.005	0.004	−0.005
	(0.004)	(0.006)	(0.004)	(0.006)	(0.006)	(0.005)	(0.006)	(0.005)
Constant	−0.012***	0.023***	−0.012***	0.023***	0.007***	0.004***	0.007***	0.004***
	(0.000)	(0.001)	(0.000)	(0.001)	(0.001)	(0.000)	(0.001)	(0.000)
Observations	19,404	19,405	19,404	19,405	19,404	19,405	19,404	19,405
R^2	0.005	0.008	0.004	0.008	0.004	0.005	0.004	0.004

Panel B: Market Leverage

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Leverage Subgroups				Size Subgroups			
	Low	High	Low	High	Small	Large	Small	Large
Ambiguity	0.173***	0.670***			0.574***	0.358***		
	(0.034)	(0.064)			(0.091)	(0.045)		
Ambiguity Unlev.			0.099***	0.405***			0.752***	0.251***
			(0.026)	(0.038)			(0.079)	(0.030)
Risk	−0.070***	−0.335***			−0.133***	−0.375***		
	(0.007)	(0.021)			(0.010)	(0.038)		
Risk Unlev.			−0.054***	−0.353***			−0.115***	−0.428***
			(0.006)	(0.026)			(0.010)	(0.050)
Median Market Lev.	−0.023***	−0.015	−0.025***	−0.023**	−0.011	−0.004	−0.010	−0.009
	(0.008)	(0.010)	(0.008)	(0.010)	(0.013)	(0.009)	(0.013)	(0.009)
Profitability	−0.001	−0.016	−0.000	−0.016	−0.006	−0.021	−0.004	−0.018
	(0.002)	(0.013)	(0.002)	(0.013)	(0.004)	(0.017)	(0.004)	(0.017)
Tangibility	0.022**	0.138***	0.018*	0.136***	0.088***	0.097***	0.083***	0.095***
	(0.010)	(0.021)	(0.010)	(0.021)	(0.015)	(0.023)	(0.015)	(0.023)
Market to Book	0.000***	−0.004***	0.000***	−0.004***	−0.000	0.001**	−0.000	0.001**
	(0.000)	(0.001)	(0.000)	(0.001)	(0.000)	(0.001)	(0.000)	(0.001)
RD	0.000*	0.000	0.000*	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Sales	0.019***	0.164***	0.020***	0.163***	0.028***	0.785***	0.029***	0.787***
	(0.006)	(0.045)	(0.006)	(0.045)	(0.008)	(0.114)	(0.008)	(0.114)
Tax Rate	0.007*	0.002	0.008*	0.004	0.019***	−0.015**	0.020***	−0.014**
	(0.004)	(0.008)	(0.004)	(0.008)	(0.007)	(0.007)	(0.007)	(0.007)
Constant	−0.012***	0.024***	−0.012***	0.024***	0.007***	0.002***	0.007***	0.002***
	(0.000)	(0.001)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Observations	19,404	19,405	19,404	19,405	19,404	19,405	19,404	19,405
R^2	0.016	0.033	0.010	0.027	0.017	0.024	0.016	0.021

Table 4: **Economic significance of the estimates of capital structure**

The levels of economic significance for the measures of ambiguity relative to the economic significance of the control variables. The first column reports the standard deviation of the first differences of each variable, the second column reports the coefficient estimates from Table 2, and the third column reports their product. The final two columns report the levels of economic significance of the measures of ambiguity relative to the economic significance of the control variables. The full sample includes 54,691 annual observations associated with 5,092 individual firms between 1993 and 2018, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt". Sale is the log of one plus the firm's reported sales. Profitability is operating income before depreciation divided by book assets. Tangibility is the plant property divided by book assets. The market-to-book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is R&D expenses relative to sales.

Panel A: Book Leverage

	Std	Coeficient	Significance	Ambiguity Relative To	Unlev. Ambiguity Relative To
Ambiguity	0.013	0.144	11.069	1	0.474
Ambiguity Unlev.	0.018	0.068	3.731	2.110	1
Risk	0.070	-0.038	-0.551	3.752	1.778
Risk Unlev.	0.062	-0.027	-0.438	5.349	2.534
Median Book Lev.	0.067	-0.005	-0.068	31.889	15.110
Profitability	0.147	-0.003	-0.022	44.981	21.314
Tangibility	0.050	0.080	1.602	1.803	0.854
Market to Book	1.699	-0.001	-0.001	122.567	58.078
RD	62.712	0.000	0.000	15,645.150	7,413.384
Sales	0.058	0.031	0.543	4.612	2.185
Tax Rate	0.111	-0.000	-0.004	289.673	137.260

Panel B: Market Leverage

	Std	Coeficient	Significance	Ambiguity Relative To	Unlev. Ambiguity Relative To
Ambiguity	0.013	0.488	37.434	1	0.789
Ambiguity Unlev.	0.018	0.385	21.018	1.267	1
Risk	0.070	-0.154	-2.202	3.173	2.504
Risk Unlev.	0.062	-0.128	-2.082	3.802	3.001
Median Market Lev.	0.082	-0.009	-0.112	52.675	41.581
Profitability	0.147	-0.006	-0.039	85.560	67.540
Tangibility	0.050	0.087	1.747	5.590	4.412
Market to Book	1.699	-0.000	-0.000	1,569.533	1,238.971
RD	62.712	0.000	0.000	55,624.560	43,909.370
Sales	0.058	0.037	0.643	13.186	10.409
Tax Rate	0.111	0.003	0.030	148.028	116.851

Table 5: **Robustness tests**

Linear model regression estimates of the changes in annual leverage ratio explained by one-year lagged changes in the variables. The full sample includes 54,691 annual observations associated with 5,092 individual firms between 1993 and 2018, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt". Each regression includes an intercept and the following control variables (whose coefficient estimates are unreported). Sale is the log of one plus the firm's reported sales. Profitability is operating income before depreciation divided by book assets. Tangibility is the plant property divided by book assets. The market-to-book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is R&D expenses relative to sales. Median leverage is the median of the annual leverage in the industry. The variance of the mean (Var mean) is the variance of daily mean returns (computed from 5-minute return) over the month. The variance of the variance (Var var) is the variance of daily variance returns (computed from 5-minute return) over the month. Bid-ask spread is the effective bid-ask spread. Analysts disagreement is the variance among analyst forecasts of the stock price. Standard errors, clustered by firm and robust to heteroscedasticity, appear in parentheses below each coefficient estimate. Standard errors are in parentheses, *p<0.1; **p<0.05; ***p<0.01

Panel A: Book Leverage

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Ambiguity	0.137***					0.136***	0.137***	0.135***	0.162***
	(0.033)					(0.033)	(0.033)	(0.033)	(0.032)
Risk	−0.039***	−0.039***	−0.036***	−0.013	−0.044***	−0.042***	−0.039***	−0.017	−0.046***
	(0.009)	(0.010)	(0.009)	(0.010)	(0.010)	(0.010)	(0.009)	(0.010)	(0.010)
Var Mean		0.016				0.011			
		(0.022)				(0.022)			
Var Var			0.000				0.000		
			(0.004)				(0.004)		
Bid-ask Spread				−0.335***				−0.332***	
				(0.088)				(0.088)	
Analysts Disagr.					−0.006**				−0.006**
					(0.003)				(0.003)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	39,317	39,317	39,317	39,317	33,331	39,317	39,317	39,317	33,331
R^2	0.004	0.004	0.004	0.004	0.005	0.004	0.004	0.005	0.005

Panel B: Market Leverage

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Ambiguity	0.489***					0.490***	0.489***	0.484***	0.494***
	(0.043)					(0.044)	(0.043)	(0.043)	(0.042)
Risk	−0.157***	−0.145***	−0.144***	−0.087***	−0.180***	−0.154***	−0.157***	−0.101***	−0.187***
	(0.011)	(0.013)	(0.011)	(0.017)	(0.013)	(0.013)	(0.011)	(0.017)	(0.013)
Var Mean		0.006				−0.012			
		(0.024)				(0.026)			
Var Var			0.002				0.001		
			(0.007)				(0.008)		
Bid-ask Spread				−0.831***				−0.821***	
				(0.174)				(0.176)	
Analysts Disagr.					−0.001				−0.001
					(0.002)				(0.002)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	39,317	39,317	39,317	39,317	33,331	39,317	39,317	39,317	33,331
R^2	0.016	0.012	0.012	0.015	0.013	0.016	0.016	0.019	0.017

Online Appendix

In Table OA.1, we report findings for the relation between first differences of leverage and first differences of the explanatory variables using the variance of cash flow as the measure of risk. These results are provided as a robustness check regarding the measure of risk. The coefficient estimates on the measures of ambiguity remain uniformly positive and highly significant in each of the specifications. Cash flow volatility, however, is insignificant in most of the regressions. It is, perhaps, unsurprising that this measure of risk is not a significant explanatory variable for leverage in a first differences estimation because by construction there is limited time-series variation in the change in cash flow volatility. Very similar results are found using different time periods over which to measure cash flow volatility or when we measure the variation in cash flow measured using quarterly data.

Table OA.1: **First Difference regression estimates of capital structure**

Linear model regression estimates of the annual leverage ratio explained by one-year lagged variables using first differences. The full sample includes 54,691 annual observations associated with 5,092 individual firms between 1993 and 2018, using records from the Compustat database. Ambiguity, based on TAQ intraday stock price records, is calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt". Cash flow volatility is the standard deviation of the firm's operating income before depreciation measured over the proceeding three fiscal years. Sale is the log of one plus the firm's reported sales. Profitability is operating income before depreciation divided by book assets. Tangibility is the plant property divided by book assets. The market-to-book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is R&D expenses relative to sales. Median leverage is the median of the annual leverage in the industry. Standard errors, clustered by firm and robust to heteroscedasticity, appear in parentheses below each coefficient estimate. Standard errors are in parentheses, *p<0.1; **p<0.05; ***p<0.01

Panel A: Book Leverage

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Ambiguity	0.117***		0.118***	0.126***				
	(0.035)		(0.035)	(0.035)				
Ambiguity Unlev.					0.055**		0.055**	0.063***
					(0.022)		(0.022)	(0.022)
Cash Flow Vol.		0.001	0.001	0.001*		0.001	0.001	0.001*
		(0.001)	(0.001)	(0.001)		(0.001)	(0.001)	(0.001)
Median Book Lev.				−0.008				−0.009
				(0.007)				(0.007)
Profitability				−0.000				−0.000
				(0.006)				(0.006)
Tangibility				0.076***				0.076***
				(0.013)				(0.013)
Market to Book				−0.001				−0.001
				(0.000)				(0.000)
RD				0.000				0.000
				(0.000)				(0.000)
Sales				0.034***				0.034***
				(0.013)				(0.013)
Tax Rate				−0.000				−0.000
				(0.004)				(0.004)
Constant	0.006***	0.006***	0.006***	0.006***	0.006***	0.006***	0.006***	0.006***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Observations	34,025	34,025	34,025	34,025	34,025	34,025	34,025	34,025
R^2	0.000	0.000	0.000	0.003	0.000	0.000	0.000	0.003

Panel B: Market Leverage

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Ambiguity	0.402***		0.402***	0.389***				
	(0.042)		(0.042)	(0.042)				
Ambiguity Unlev.					0.368***		0.368***	0.361***
					(0.030)		(0.030)	(0.030)
Cash Flow Vol.		−0.000	0.000	0.000		−0.000	0.000	0.000
		(0.001)	(0.001)	(0.001)		(0.001)	(0.001)	(0.001)
Median Market Lev.				−0.019**				−0.016**
				(0.008)				(0.008)
Profitability				0.004				0.004
				(0.004)				(0.004)
Tangibility				0.066***				0.068***
				(0.014)				(0.014)
Market to Book				0.001***				0.001***
				(0.000)				(0.000)
RD				0.000**				0.000**
				(0.000)				(0.000)
Sales				0.047***				0.046***
				(0.009)				(0.009)
Tax Rate				0.004				0.004
				(0.005)				(0.005)
Constant	0.007***	0.008***	0.007***	0.007***	0.007***	0.008***	0.007***	0.007***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Observations	34,025	34,025	34,025	34,025	34,025	34,025	34,025	34,025
R^2	0.003	0.000	0.003	0.005	0.005	0.000	0.005	0.007

OA.2 Instrumental variable regression findings

To provide further evidence regarding the relation between ambiguity and leverage, Table OA.2 presents the findings of a first-difference 2-Stage least squares estimate of the model specified by Equations (OA.1) and (OA.2). The instrumental variables estimation allows us to provide external validation for the ambiguity measures we employ. By considering, only the variation in our ambiguity measures that is explained by the instruments can provide further evidence regarding the relation between the ambiguity facing the firm and the firm's capital structure choice.

$$\Delta \mho_{i,t}^2 = \alpha + \beta_1 \cdot DIVC_t + \beta_2 \cdot DEF_{i,t} + \gamma \Delta Z_{i,t} + \epsilon_{i,t}, \tag{OA.1}$$

and

$$\Delta L_{i,t} = \alpha + \beta \cdot \widehat{\Delta \mho}_{i,t-1}^2 + \gamma \cdot \Delta Z_{i,t-1} + \varepsilon_{i,t}. \tag{OA.2}$$

Specifically, in the first-stage, we instrument for the first difference in the ambiguity measures using two proxies for ambiguity: the number of lawsuits in which the firm is named as a defendant in each year as a proxy for firm level ambiguity (DEF), and an indicator variable set equal to one if the US house of representatives and the US senate are controlled by the same political party as that of the president, and zero otherwise, as a proxy for economy wide ambiguity (Divided Congress, $DEFC$).[28,29] In the second-stage the fitted values of ambiguity and unlevered ambiguity are used as explanatory variables for leverage.

The instruments are chosen because they are exogenous to the firm's leverage decisions. The instrument Divided Congress is the result of popular elections. The instrument Defendant is largely the result of the unintended consequences of past operating decisions. Further, both of these instruments are expected to be closely related to the level of ambiguity firms face. The variable Divided Congress reflects the level of policy uncertainty—uncertainty regarding the "rules of the game" for economic activity for a specific period (Baker et al., 2016). The idea being that a unified government is more likely to be able to enact significant policy changes, increasing the level of policy uncertainty. The Defendant variable is a reflection of the level of uncertainty regarding the firm's cash flow and potentially the nature of restrictions on its operations in the near future.

As with all instrumental variables analysis in a corporate context, the primary difficulty in identifying instruments for ambiguity is that good instruments for ambiguity may also affect, for example, the firm's risk.[30] It is therefore difficult to establish that the "exclusion restriction" holds. A further concern with the use of these instruments in an empirical examination of capital structure is the unobserved heterogeneity inherent in models of leverage choice (Lemmon et al., 2008). Identifying whether our chosen instruments also influence leverage via one of the omitted variables (see Angrist and Pischke (2008) and Atanasov and Black (2016)) is, of course, unfeasible. We therefore think of this as providing external validity to the measure of ambiguity employed in this study.

In each panel of Table OA.2, the first and the third columns present the findings from the first-stage regression estimating the first difference of ambiguity or unlevered ambiguity respectively. The second and fourth columns present the findings from the second-stage regression tests. As before, Panel A examines book leverage and Panel B examines market leverage.

As in our other empirical models, the estimated coefficients are consistent with the predictions of our theoretical model. In the first-stage regressions, the coefficient estimates on the instruments for

[28] The Defendant variable is constructed from the data provided in AuditAnalytics database. The Controlled house of representatives indicator variable is constructed from the data provided at *https://en.wikipedia.org/wiki/Divided_government_in_the_United_States*.

[29] The use of variables in levels as instruments for the first-difference of ambiguity is a benefit of using first-differences model to control for the unobserved heterogeneity. Doing so implies that the exogeneity required for the instrument is only contemporaneous exogeneity rather than strict exogeneity as would be required in a fixed effects 2-Stage least squares estimation (e.g., Wooldridge, 2010).

[30] While both of the chosen instruments have significant correlations with ambiguity or unlevered ambiguity, their correlations with risk are smaller by at least an order of magnitude.

ambiguity and unlevered ambiguity are consistently positive and highly significant for both book and market leverage. The coefficient estimates indicate that our measure of ambiguity is indeed higher when firms are named as defendants in a greater number of lawsuits and when there is a greater level of policy uncertainty. The level of significance of these variables and the reported F-statistics for the first-stage regression suggest the model does not suffer from a weak instruments problem.

In each second-stage regression test, the coefficient estimates for the fitted values of the first difference in ambiguity or unlevered ambiguity indicate a positive and highly significant relation with the first difference in both book and market leverage. The F-statistics are highly significant in each second-stage regression. These estimates again demonstrate that our measures of ambiguity are positively related to firms' subsequent leverage choices. The second-stage regressions also confirm that risk (equity return volatility or unlevered return volatility) continues to have a significantly negative relation to leverage.

Table OA.2: **First Difference 2-Stage Least Squares regression estimates of capital structure**

The full sample includes 54,691 annual observations associated with 5,092 individual firms between 1993 and 2018, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt". Sale is the log of one plus the firm's reported sales. Profitability is operating income before depreciation divided by book assets. Tangibility is the plant property divided by book assets. The market-to-book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is R&D expenses relative to sales. Median leverage is the median of the annual leverage in the industry. The instrumental variable Defendant is the number of lawsuits in which the firm is named as a defendant in each year, taken as a proxy for the firm level ambiguity. The instrumental variable Divided Congress is an indicator variable set equal to one if the US house of representatives and the US senate are controlled by different political parties, and zero otherwise, taken as a proxy for economy wide ambiguity. Standard errors, clustered by firm and robust to heteroscedasticity, appear in parentheses below each coefficient estimate. Standard errors are in parentheses, *p<0.1; **p<0.05; ***p<0.01

Panel A: Book Leverage

	(1)	(2)	(3)	(4)
Ambiguity (Fit)		0.820^{***}		
		(0.199)		
Ambiguity Unlev. (Fit)				0.658^{***}
				(0.151)
Divided Congress	0.005^{***}		0.006^{***}	
	(0.000)		(0.000)	
Defendant	0.000^{***}		0.000^{***}	
	(0.000)		(0.000)	
Risk	0.027^{***}	-0.055^{***}		
	(0.002)	(0.010)		
Risk Unlev.			0.016^{***}	-0.027^{***}
			(0.002)	(0.010)
Median Book Lev.	-0.007^{***}	-0.015^{**}	-0.005^{***}	-0.018^{**}
	(0.001)	(0.008)	(0.002)	(0.008)
Profitability	−0.000	−0.001	−0.001	0.000
	(0.000)	(0.006)	(0.001)	(0.006)
Tangibility	-0.007^{***}	0.081^{***}	-0.011^{***}	0.080^{***}
	(0.001)	(0.014)	(0.002)	(0.014)
Market to Book	0.000^{***}	-0.001^{***}	0.000^{***}	-0.001^{***}
	(0.000)	(0.000)	(0.000)	(0.000)
RD	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)
Sales	0.003^{**}	0.022^{*}	0.004^{***}	0.023^{*}
	(0.001)	(0.013)	(0.001)	(0.013)
Tax Rate	0.001	−0.001	0.001	−0.001
	(0.001)	(0.004)	(0.001)	(0.004)
Observations	38,813	38,813	38,813	38,813
R^2	0.092	0.066	0.082	0.059

Panel B: Market Leverage

	(1)	(2)	(3)	(4)
Ambiguity (Fit)		4.028^{***}		
		(0.270)		
Ambiguity Unlev. (Fit)				3.152^{***}
				(0.209)
Divided Congress	0.005^{***}		0.006^{***}	
	(0.000)		(0.000)	
Defendant	0.000^{***}		0.000^{***}	
	(0.000)		(0.000)	
Risk	0.028^{***}	-0.255^{***}		
	(0.002)	(0.013)		
Risk Unlev.			0.018^{***}	-0.173^{***}
			(0.002)	(0.011)
Median Market Lev.	-0.020^{***}	0.047^{***}	-0.029^{***}	0.054^{***}
	(0.001)	(0.010)	(0.001)	(0.011)
Profitability	−0.000	−0.002	-0.001^{**}	0.001
	(0.000)	(0.004)	(0.001)	(0.004)
Tangibility	-0.006^{***}	0.111^{***}	-0.010^{***}	0.111^{***}
	(0.001)	(0.015)	(0.002)	(0.015)
Market to Book	0.000^{***}	-0.001^{***}	0.000^{***}	-0.001^{***}
	(0.000)	(0.000)	(0.000)	(0.000)
RD	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)
Sales	0.003^{***}	0.017^{*}	0.004^{***}	0.019^{**}
	(0.001)	(0.010)	(0.001)	(0.010)
Tax Rate	0.001	0.001	0.001	0.004
	(0.001)	(0.006)	(0.001)	(0.006)
Observations	38,813	38,813	38,813	38,813
R^2	0.106	−0.111	0.097	−0.158