Real-Time Distribution of Stochastic Discount Factors*

Fousseni Chabi-Yo[a†]

[a]*Isenberg School of Management, University of Massachusetts, Amherst, MA 01003*

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Abstract

I use option prices to infer real-time moments of stochastic discount factors (SDFs). The moments are estimated, from daily SP 500 index option data, in real time, without relying on past observations. These moments are forward-looking and significantly predict the market excess return. The theory suggests that the SDF variance (kurtosis) is positively priced while the SDF skewness is negatively priced in the cross-section of returns. A cross-sectional analysis shows that the price of risks associated with the moments of the SDF are economically and statistically significant after controlling for a comprehensible set of economic variables

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†Tel.:+1-614-753-9072. *E-mail address:* fchabiyo@isenberg.umass.edu

1. Introduction

The stochastic discount factor (SDF) has been the center of asset pricing for decades. Knowledge of the SDF enables investors to price any assets in financial markets. Thus, understanding the distribution of the SDF is the key for understanding risk and return in asset pricing. In their seminal work, Hansen and Jagannathan (1991) focus on the second moment of the distribution of the SDF and derive a lower bound on the variance of the SDF. The bound was designed to discriminate among asset pricing models and has an intuitive interpretation in terms of the Sharpe ratio of risky assets. Schneider and Trojani (2017a) exploit the Hansen and Jagannathan (1991) approach to recover the (almost) model-free conditional minimum variance projection of the pricing kernel on various tradeable realized moments of market returns and show that the recovered conditional moments of the market return predict future market returns. Almeida and Garcia (2017) use discrepancy functions to derive unconditional restrictions on the moments of SDFs. This paper makes both theoretical and empirical contributions to asset pricing.

First, in theory, I argue that the conditional distribution of the SDF is of great help in understanding risk and return in financial markets. The conditional distribution of the SDF can be summarized by the first four conditional moments of the SDF: mean, variance, skewness, and kurtosis. While the mean of the SDF is observable (via the risk-free rate), the variance, skewness, and kurtosis of the SDF are not directly observable. We should expect that conditional SDF moments, somehow, pick information about risk factors embedded in the SDF. Thus, to improve our understanding of the relation between SDF moment risks and return in lights of a modern SDF approach, one may ask few questions. First, what is the function that maps SDF moments into risk factors? Alternatively, what is the function that maps risk factors into SDF moments? Second, should this function always exist? Third, why should moments of the SDF be priced in the cross-section of equity data? Throughout the paper, I maintain an arbitrage-free economy assumption and provide a theoretical motivation that is aimed to answer those questions. I show that stocks with high sensitivity to the SDF variance (kurtosis) exhibit, on average, high return, while stocks with high sensitivity to the SDF skewness have on average low return.

I theoretically derive closed-form expressions of the conditional physical moments of the SDF in a model-free environment, provided that the relative risk aversion under Constant Relative Risk Aversion (CRRA) preferences is known. The model-free environment refers to not making time-series assumptions about economic fundamentals, returns, or the SDF's distribution. I further derive conditional moments of the SDF when preferences depart from CRRA preferences. The first advantage of the conditional moment expressions is that the moments can be estimated in real-time without relying on a time-series of past return observations. The conditional moments of the SDF can be obtained at any time provided that there is a cross-section of option prices. The second advantage is that it provides a framework to investigate the implications of the real-time distribution of the SDF for the market return and equities. The third advantage is that the framework can be used to understand real-time distribution of the SDF across different maturities. I also derive the real-time risk-neutral distribution of the SDF by providing closed-form expressions of the risk-neutral variance, skewness, and kurtosis of the SDF in a model-free environment. This is important since the wedge between the real-time physical distribution of the SDF and the risk-neutral distribution of the SDF can be interpreted as a risk premium. The approach in this paper allows for computing SDF-based moments risk premium. I further use the real-time distribution of the SDF to derive closed-form expressions of conditional expected excess returns and conditional Sharpe ratios of hedging strategies that generate returns that are positively correlated with the SDF.

Second, I use option prices to quantify at each date the real-time conditional physical distribution of the SDF across different maturities. The conditional moments of the SDF are forward-looking. They are computed every day using one day of option price data. The maturities of interest range from 30 days to 365 days. For a reasonable relative risk aversion parameter of 2, I find that the conditional physical variance of the SDF is time-varying, is highly volatile during crisis periods, is skewed, and exhibits fat tails at each maturity. The mean of the 365-day conditional variance of the SDF is 0.45, with a standard deviation of 0.28. This translates into an unconditional SDF variance of 0.53 and approximate volatility (square root of the variance) of 72.69%.

Further, the conditional skewness of the SDF under the physical measure is time-varying, is often positive, and exhibits fat tails, regardless of the option maturity used. While investment assets are often negatively skewed, one should expect insurance assets, such as the SDF, to be positively skewed. Also, the conditional kurtosis of the SDF under the physical measure is time-varying and is more pronounced in crisis periods than in normal times. I also find that the variance of the SDF, under the risk-neutral measure, is time-varying and exhibits peaks during crisis periods. The mean of the 365-day risk-neutral variance of the SDF is 0.57, with a standard deviation of 0.43. This translates into an unconditional variance of the

SDF of 0.75 and an approximate unconditional volatility of 86.88%. Thus, the volatility of the SDF under the risk-neutral measure is often higher than the physical volatility of the SDF. While the skewness of the SDF under the physical measure is often positive, the skewness of the SDF under the risk-neutral measure is often negative. I explore why the SDF risk-neutral skewness is often negative by decomposing the SDF risk-neutral skewness as a function of the SDF physical moments. I find that the negative sign of the SDF risk neutral-skewness can be explained by the physical volatility of the SDF. I further find that the kurtosis of the SDF under the physical measure is often higher than the kurtosis of the SDF under the risk-neutral measure.

Next, I investigate whether the real-time distribution of the SDF predicts the excess market return in-sample and out-of-sample. I find that the variance, skewness, and kurtosis of the SDF predict the excess market return when options with maturity equal to or higher than 122 days are used to compute the conditional moments. The results are robust after controlling for risk-neutral moments of simple market returns. The wedge between the physical distribution of the SDF and the risk-neutral distribution of the SDF also significantly predicts the market excess return when options with maturity equal to or more than 122 days are used to characterize the distribution of the SDF. The adjusted in-sample R-square and out-of-sample R-square are very similar and range from 3% to a maximum of 14% when the maturity of option used varies from 122 days to 365 days. Results are robust after controlling for both the market variance risk premium and the Left Risk-Neutral Jump Variation measures. I further investigate the pricing implications of SDF variance, skewness, and kurtosis for the cross-section of stock returns, by using estimates of the moments of the SDF extracted from index options. I use the two-pass and three-pass methodologies to infer the price of risk of the SDF moments. I focus on portfolios formed on stock characteristics. These portfolios are available on Kenneth French's website.

I first use the two-pass regression methodology to infer the price of risks, and I report the Fama and MacBeth (1973) t-ratio under correctly specified models, the Shanken (1992) t-ratio, the Jagannathan and Wang (1998) t-ratio under correctly specified models that account for the EIV problem, and the Kan, Robotti, and Shanken (2013) misspecification robust t-ratios. Results indicate that the prices of the SDF variance, skewness, and kurtosis are significant after controlling for the Fama and French (2016) factors, high order moments of the market returns, variance risk premium, and Left Risk-Neutral Jump Variation measures. When I use 100 portfolios formed on size and book-to-market, the price of the SDF variance and SDF kurtosis is positive, while the price of the SDF skewness is negative. This is consistent with the theoretical predictions. This shows that book-to-market portfolios that have high exposure to the SDF kurtosis and variance exhibit high return on average, while book-to-market portfolios that have high exposure to the SDF skewness have a low return on average. However, when I use 100 portfolios formed on size and operating profitability, the price of the variance of the SDF is positive at short maturities (from 30 days to 90 days) and negative for higher maturities (from 122 days to 365 days). The price of the SDF skewness is positive, regardless of the option maturity used. The sign of the price of SDF skewness (kurtosis) is not consistent with the theoretical predictions. I further use the two-pass regression methodology to investigate the implications of the SDF-based moment risk premium, defined as the difference between the SDF under the physical and risk-neutral measures, for the cross-section of equities. I find that the price of SDF-based moment risk premium is significant, regardless of the option maturity used to compute the conditional moments.

To further explore the pricing of the SDF-based moments and SDF-based moment risk premium in the cross section of returns, I use the recent three-pass regression methodology proposed by Giglio and Xiu (2017) to infer the price of risk by combining many portfolios based on various characteristics. The advantage of the three-pass approach is that it permits inferring the price of risk when the number of assets is large. Giglio and Xiu (2017) argue that standard methods to estimate risk premia are biased in the presence of omitted priced factors correlated with the observed factors. Their methodology accounts for potential measurement error in the observed factors and detects when observed factors are spurious or even useless. I find that the prices of the SDF variance, skewness, and kurtosis are all statistically significant and positive after controlling for the Fama and French (2016) factors. Stocks with high exposure to the conditional moments of the SDF exhibit high return on average. With the three-pass regression, the price of the SDF variance (kurtosis) is consistent with the theoretical prediction, while the price of the SDF skewness is not. I also use the three-pass regression to investigate the implications of the SDF-based moment risk premium for the cross-section of returns. The price of risks of the SDF variance premium is negative and highly significant at all maturities. The price of risks of the SDF skewness premium and SDF kurtosis premium factors is positive, and significant at all maturities. These findings are robust to controlling for the Fama and French (2016) factors.

Overall, the sign of the price of the SDF variance (kurtosis) is consistent with the theoretical predictions while the sign of the price of the SDF skewness is not.

The remainder of the paper is organized as follows. Section 2 theoretically motivates the use of SDF moments in asset pricing, Section 3 presents the methodological framework to derive SDF moments in a model-free economy. Section 4 estimates the conditional moments of the SDF. Section 5 shows that the real-time distribution of the SDF predicts the excess market return. Section 6 uses the two-pass methodology to infer the price of SDF moments. Section 7 uses the three-pass regression methodology to estimate the price of risks of SDF moments. Section 8 concludes the paper.

2. Theoretical Motivation for Using SDF Moments in Asset Pricing

A growing number of articles have used the market return variance, skewness, and kurtosis to predict and explain the cross-section of returns (Bali and Murray (2013), Bali, Cakici, and Whitelaw (2011), Chang, Christoffersen, and Jacobs (2013), and Amaya, Christoffersen, Jacobs, and Vasquez (2015)). Kelly and Jiang (2014) show that tail risk has strong predictive power for aggregate market returns and also explain the cross-section of returns. Bollerslev and Todorov (2011) and Bollerslev, Todorov, and Xu (2015) show how information extracted from options can be used to forecast future market return. An, Bali, Ang, and Cakici (2014) show that option-based factors are priced in the cross-section of stocks.

I take an alternative route and argue that because the SDF is important in pricing assets, its distribution contains a rich set of information that could advance our quest of better understanding risk and returns. Thus, we should expect that conditional SDF moments will have to pick, somehow, information about risk factors embedded in the SDF. In light of this, one may ask key questions. First, what is the function that maps SDF moments into risk factors? Alternatively, what is the function that maps risk factors into SDF moments? Second, should this function always exist? Third, why should moments of the SDF be priced in the cross-section of equity data? I attempt to provide answers to those questions, and also improve our understanding of the relation between SDF moment risks and return in lights of a modern SDF approach. Throughout the paper, I maintain an arbitrage-free economy assumption.

Assumption 1 *Assume an arbitrage-free economy that guaranties the existence of a SDF and the existence of the risk-neutral measure.*

Definition 1 *The model-free environment is defined as an environment where no time-series assumptions are made about economic fundamentals, returns, or the SDF's distribution.*

In a one-period model, I consider a representative agent who maximizes expected utility subject to his or her budget constraint. In this section, I omit the time subscript to allow for simplicity. The SDF (up to a constant) has the form $m = u^{'}[R_M]$, where $u[.]$ represents the utility function and $u^{'}[x] > 0$. A SDF uniquely function of the market return can be interpreted as the projection of the true SDF on the market return. I also assume that the second-, third-, and fourth-order derivatives of the utility exist with $u^{''}[x] < 0$, $u^{'''}[x] > 0$, and $u^{''''}[x] < 0$ (see Dittmar (2002) and Harvey and Siddique (2000)). Further, I denote by $\mathcal{M}^{(2)}$, $\mathcal{M}^{(3)}$, $\mathcal{M}^{(4)}$, the second, third, and fourth physical moments of the SDF. Physical moments of the SDF are defined as

$$\mathcal{M}^{(n)} = \mathbb{E}\left(\left(m - \mathbb{E}(m)\right)^n\right), \tag{1}$$

where $\mathbb{E}(.)$ is the expectation operator under the physical measure. Similarly, I denote by $\mathcal{M}^{*(2)}$, $\mathcal{M}^{*(3)}$, $\mathcal{M}^{*(4)}$, the second, third, and fourth risk-neutral moments of the SDF,

$$\mathcal{M}^{*(n)} = \mathbb{E}^{*}\left(\left(m - \mathbb{E}^{*}(m)\right)^n\right), \tag{2}$$

where $\mathbb{E}^{*}(.)$ is the expectation operator under the risk neutral measure. The goal of this theoretical motivation is not to derived the exact closed-form solution of expected return on the market or individual stocks. The goal is to show via an approximation how moments of the SDF can be used as priced factors that explain expected excess return on assets. In this section, I use Taylor expansion-series to theoretically motivate the relation between the SDF moments and expected returns. While the approach of Taylor expansion-series is used to show the link between expected excess returns and SDF-based moments, it is important to notice that the closed-form solutions of SDF moments derived in the next section are well-defined and do not rely on Taylor expansion-series. They are computed using various specifications of investor preferences.

Result 1 *Denote by $\upsilon = u^{'-1}$ the inverse of $u^{'}$. The third-order Taylor expansion series of the inverse of* *the marginal utility, $\upsilon[.]$, allows approximating the expected excess market return as*

$$\mathbb{E}(R_M - R_f) = A_1 \mathcal{M}^{(2)} + A_2 \mathcal{M}^{(3)} + A_3 \mathcal{M}^{(4)}, \tag{3}$$

with

$$A_k = -R_f \frac{1}{k!} \left\{ \frac{\partial^k \upsilon[y]}{\partial^k y} \right\}_{y=\mathbb{E}(m)}, \tag{4}$$

where the terms $\frac{\partial^k \upsilon[y]}{\partial^k y}$ for $k = 1,2,3$, are given by expressions below

$$\frac{\partial \upsilon[y]}{\partial y} = \frac{1}{u''[\upsilon[y]]}, \; \frac{\partial^2 \upsilon[y]}{\partial^2 y} = -\frac{u'''[\upsilon[y]]}{(u''[\upsilon[y]])^3}, \; and \; \frac{\partial^3 \upsilon[y]}{\partial^3 y} = \frac{3\left(u'''[\upsilon[y]]\right)^2 - u''''[\upsilon[y]]\,u''[\upsilon[y]]}{(u''[\upsilon[y]])^5}, \tag{5}$$

where $\frac{\partial \upsilon[y]}{\partial y} < 0$, and $\frac{\partial^2 \upsilon[y]}{\partial^2 y} > 0$. Additionally, $\frac{\partial^3 \upsilon[y]}{\partial^3 y} < 0$ if the absolute prudence $-\frac{u'''[x]}{u''[x]}$ is a decreasing function of x.

Proof. See the Appendix. ■

Result 1 does not depend on a particular form of the utility function and it clearly shows that the expected excess market return is related to the moments of the SDF. The expected excess market return is a "weigthed" average of the SDF moments, where the weights are defined by the parameters $A_k s$. The parameters $A_k s$ can be interpreted as the price of the SDF-based moments risk factors. To illustrate implications of Result 1 in a context of specific forms of investor utilities, I consider three well-known utility specifications.

Example 1 Assume that investors have a linear marginal utility that corresponds to a linear SDF in terms of the market return. $u'[x] = a - bx$ where $u''[x] = -b$. Here, the coefficient b is positive. It can be shown that $\frac{\partial^k u[x]}{\partial^k x} = 0$ for $k > 2$. As a result, $\upsilon[y] = \frac{a}{b} - \frac{1}{b} y$ and $A_k = 0$ for $k > 1$. In this case, the expected excess market return is only related to the second moment of the SDF, where the second moment of the SDF (up to a constant term) is equal to the market variance. ♣

Example 2 Assume that investors have a quadratic marginal utility that corresponds to the quadratic model of Harvey and Siddique (2000) with $u'[x] = a - bx + cx^2$. In this model, it is assumed that $u''[x] < 0$ and $u'''[x] > 0$. Thus, $\frac{\partial^k u[x]}{\partial^k x} = 0$ for $k > 3$. The expected excess market return is of the form (3), where the coefficients A_ks are defined by (4), and $\upsilon[.]$ is the inverse of the marginal utility $u^{'}[x] = c + bx + ax^2$, with $a > 0$ and $b + 2ax < 0$. With the quadratic utility function, it follows from (5) that $\frac{\partial\upsilon[y]}{\partial y} < 0$, $\frac{\partial^2\upsilon[y]}{\partial^2 y} > 0$, and $\frac{\partial^3\upsilon[y]}{\partial^3 y} < 0$. Thus, in this model, $A_1 > 0$, $A_2 < 0$, and $A_3 > 0$. ♣

Example 3 Assume that investors have a CRRA utility. The marginal utility is $u^{'}[x] = x^{-\alpha}$ where α is the relative risk aversion parameter. It follows from (5) that $\frac{\partial\upsilon[y]}{\partial y} < 0$, $\frac{\partial^2\upsilon[y]}{\partial^2 y} > 0$, and $\frac{\partial^3\upsilon[y]}{\partial^3 y} < 0$. Thus, the expected excess market return has the form (3), where the coefficients $A_1 > 0$, $A_2 < 0$, and $A_3 > 0$. In Examples 2 and 3, the expected excess return on the market is a function of the SDF second, third and fourth moments. The key difference between the expected return specification in both examples is that moments of the SDFs and the coefficients A_k are specific to the marginal utility used. ♣

To further motivate why SDF moments are key determinants of the expected excess return on individual assets, I first show the following result.

Result 2 *Denote by $\upsilon = u^{'-1}$ the inverse of $u^{'}$. Assume that the first-, second- and third-order derivatives of $\upsilon[.]$ exist, and consider the following returns:*

$$R_M^{(i)} = \frac{R_M^i}{\mathbb{E}_t\left(mR_M^i\right)} \text{ for } i = 2, 3, \text{ and } 4, \tag{6}$$

where R_M is the market gross return. The third-order Taylor-expansion series of the inverse of the marginal utility, $\upsilon[.]$, allows approximating the expected excess return of (6) as

$$\mathbb{E}\left(R_M^i\right) - \mathbb{E}^*\left(R_M^i\right) = -R_f\left(\phi_1^{(i)}\mathcal{M}^{(2)} + \phi_2^{(i)}\mathcal{M}^{(3)} + \phi_3^{(i)}\mathcal{M}^{(4)}\right) \text{ for } i = 2, 3, 4.$$

The preference-based coefficients $\phi_1^{(i)}$, $\phi_2^{(i)}$, and $\phi_3^{(i)}$ for $i = 2$, 3, and 3 are defined below

$$\phi_1^{(i)} = \left\{\frac{\partial\left((\upsilon[y])^i\right)}{\partial y}\right\}_{y=\mathbb{E}(m)}, \; \phi_2^{(i)} = \left\{\frac{\partial^2\left((\upsilon[y])^i\right)}{\partial^2 y}\right\}_{y=\mathbb{E}(m)}, \; \phi_3^{(i)} = \left\{\frac{\partial^3\left((\upsilon[y])^i\right)}{\partial^3 y}\right\}_{y=\mathbb{E}(m)}. \tag{7}$$

Proof. See the Appendix. ■

Result 2 shows that the risk premium on the square and cubic market return is also a "weighted" average of SDF moments where the weights are also defined by investor preferences. $\phi_1^{(i)}$ is negative since

$\frac{\partial \upsilon[y]}{\partial y} < 0$. The parameter $\phi_2^{(i)}$ is positive since $\frac{\partial^2 \upsilon[y]}{\partial^2 y} > 0$. The parameter $\phi_3^{(i)}$ is negative if $\frac{\partial^3 \upsilon[y]}{\partial^3 y} < 0$. Since $\phi_1^{(i)}$ is negative, the second moment of the SDF is positively related to the risk premium $\mathbb{E}\left(R_M^2\right) - \mathbb{E}^*\left(R_M^2\right)$, $\mathbb{E}\left(R_M^3\right) - \mathbb{E}^*\left(R_M^3\right)$, and $\mathbb{E}\left(R_M^4\right) - \mathbb{E}^*\left(R_M^4\right)$. Christoffersen, Fournier, Jacobs, and Karoui (2017) provide, in the cross-section of return, empirical evidence that the price of coskewness risk, $\mathbb{E}\left(R_M^2\right) - \mathbb{E}^*\left(R_M^2\right)$, is significant and negative, while the price cokurtosis risk, $\mathbb{E}\left(R_M^3\right) - \mathbb{E}^*\left(R_M^3\right)$, is significant and positive. This confirms that the second moment of the SDF is positively priced in the cross-section of returns. Also, since $\phi_2^{(i)}$ is positive, the third moment of the SDF $\mathcal{M}^{(3)}$ is negatively related to the risk premium $\mathbb{E}\left(R_M^2\right) - \mathbb{E}^*\left(R_M^2\right)$, $\mathbb{E}\left(R_M^3\right) - \mathbb{E}^*\left(R_M^3\right)$, and $\mathbb{E}\left(R_M^4\right) - \mathbb{E}^*\left(R_M^4\right)$. This supports a negative price of risk of the third moment of the SDF. The fourth moment of the SDF is positively priced in the cross-section of return if $\phi_3^{(j)}$ is negative. Note that in Harvey and Siddique (2000) and Dittmar (2002), asset-pricing models, the square market return and the cubic market return have been used as priced factors to explain the cross-section of returns. Hence, Result 2 suggests that SDF moments can be used as key determinant of the expected excess return. Result 3 further shows that there is a function that maps SDF moments into priced factors.

Result 3 *Denote by $\upsilon = u^{'-1}$ the inverse of $u^{'}$. Assume that the first-, second- and third-order derivatives of $\upsilon[.]$ exist, and denote by $r_M = R_M - \mathbb{E}(R_M)$. Up to a third-order Taylor expansion series, the moments of the SDF can be approximated as*

$$\mathcal{M}^{(i)} = -\psi_1^{(i)}\mathbb{E}(m)\left(\left(\mathbb{E}(R_M) - R_f\right)\right) + \frac{1}{2}\psi_2^{(i)}\mathbb{E}(m)\left(\mathbb{E}^*\left(r_M^2\right) - \mathbb{E}\left(r_M^2\right)\right) + \frac{1}{3!}\psi_3^{(i)}\mathbb{E}(m)\left(\mathbb{E}^*\left(r_M^3\right) - \mathbb{E}\left(r_M^3\right)\right),$$

where

$$\psi_1^{(i)} = \left\{\frac{\partial \psi^{(i)}[x]}{\partial x}\right\}_{x=\mathbb{E}(m)}, \psi_2^{(i)} = \left\{\frac{\partial^2 \psi^{(i)}[x]}{\partial^2 x}\right\}_{x=\mathbb{E}(m)}, \psi_3^{(i)} = \left\{\frac{\partial^3 \psi^{(i)}[x]}{\partial^3 x}\right\}_{x=\mathbb{E}(m)}$$

and $\psi^{(i)}[x] = \left(u^{'}(x) - \mathbb{E}(m)\right)^{i-1}$.

Proof. See the Appendix. ■

Result 3 shows that the moments of the SDF contain information about various risk premia. The SDF moments are the weighted averages of different risk premia, where the weights are functions of preference parameters. The advantage on focusing on the SDF moments is that they summarize all risk premia into a single number. The relative importance of each risk premia is determined by the preference-based weights $\psi_k^{(i)}$ for $k = 1,2,3$. While Result 3 applies to any utility function, it is obtained by using a third-order Taylor expansion series. In the case of a CRRA utility, all high-order derivatives of $\psi^{(i)}[x]$ exist. This allows going beyond the third-order Taylor expansion series. Result 4 shows the function that maps SDF moments into various risk premium when investors have a CRRA utility.

Result 4 *When investors have a CRRA utility, the SDF moments can be decomposed as*

$$\mathcal{M}^{(i)} = \sum_{k=1}^{\infty} \frac{1}{k!}\psi_k^{(i)}\left(\mathbb{E}(m)\right)\left(\mathbb{E}^*\left(r_M^k\right) - \mathbb{E}\left(r_M^k\right)\right),$$

where $r_M = R_M - \mathbb{E}(R_M)$ *and*

$$\psi_k^{(i)} = \left\{\frac{\partial^k \psi^{(i)}[x]}{\partial^k x}\right\}_{x=\mathbb{E}(m)},$$

with

$$\psi^{(i)}[x] = \left(x^{-\alpha} - \mathbb{E}(m)\right)^{i-1}.$$

Proof. See the Appendix. ■

Result 4 shows that the moment of the SDF is a preference-based weighted average of all risk premia. SDF moments summarize into a single number the information contained in all risk premia. Moving to individual stocks, Result 5 shows how the expected excess return on any individual securities is also determined by the moments of the SDF.

Result 5 *Denote by* $\upsilon = u'^{-1}$ *the inverse of* u'. *Assume that the first-, second- and third-order derivatives of* $\upsilon[.]$ *exist, then, up to the third-order Taylor expansion series of* $\upsilon[.]$, *the expected excess return on individual securities can be approximated as*

$$\mathbb{E}(R_i - R_f) = \beta_i^{(2)}\mathcal{M}^{(2)} + \beta_i^{(3)}\mathcal{M}^{(3)} + \beta_i^{(4)}\mathcal{M}^{(4)},$$

where the beta coefficients $\beta_i^{(2)}$, $\beta_i^{(3)}$, *and* $\beta_i^{(3)}$ *represent the sensitivity of the return on the risky assets to the SDF moments.*

Proof. See the Appendix. ■

Since the expected excess return can also be written as

$$\mathbb{E}\left(R_i - R_f\right) = -\frac{1}{\mathbb{E}\left(m\right)}\text{cov}\left(R_i, m\right),$$

one could write

$$\mathbb{E}\left(R_i - R_f\right) = -\frac{1}{\mathbb{E}\left(m\right)}\frac{\text{cov}\left(R_i, m\right)}{\mathcal{M}^{(2)}}\mathcal{M}^{(2)}.$$

One could ask whether moments higher than the variance are related to the expected excess return. The covariance term discounted by the variance of the SDF, $\frac{\text{cov}(R_i,m)}{\mathcal{M}^{(2)}}$, is not independent of the variance of the SDF. Thus, it is difficult to discuss how any increase in the variance of the SDF affects the expected excess return. The same concern arises if I write the expected excess return as follows

$$\mathbb{E}\left(R_i - R_f\right) = -\frac{1}{\mathbb{E}\left(m\right)}\frac{\text{cov}\left(R_i, m\right)}{\mathcal{M}^{(3)}}\mathcal{M}^{(3)} = -\frac{1}{\mathbb{E}\left(m\right)}\frac{\text{cov}\left(R_i, m\right)}{\mathcal{M}^{(4)}}\mathcal{M}^{(4)}. \tag{8}$$

Expression (8) cannot be used to assess whether an increase (decrease) in the third and fourth moments of the SDF affects the expected excess return: Both risk quantities, $\frac{\text{cov}(R_i,m)}{\mathcal{M}^{(3)}}$ and $\frac{\text{cov}(R_i,m)}{\mathcal{M}^{(4)}}$, depend on the third and fourth moments of the SDF respectively. Further, by choosing a factor f that has no link with the expected excess return, I could write $\mathbb{E}\left(R_i - R_f\right) = -\frac{1}{\mathbb{E}(m)}\frac{\text{cov}(R_i,m)}{f}f$. This could be misleading because it allows to link the expected excess to a factor that might not be related to expected excess return. My approach has the advantage, in an arbitrage-free economy that guaranties the existence of a SDF and the existence of the risk-neutral measure, to link the moments of the SDFs to the expected excess return, and it shows whether SDF moments positively or negatively affect the expected excess return via preference parameters.

My paper is among a set of few papers that have shown how to recover, from option prices, time-varying physical moments. Schneider and Trojani (2015, 2016) use the Bregman divergence theory to trade higher-moment risks based on S&P 500 options and futures. Schneider and Trojani (2015) relate skewness risk to fear indexes similar to the one proposed by Bollerslev and Todorov (2011). Schneider and Trojani (2017b) propose a family of divergence swaps and obtain prices of risk for the variance, skewness and, the kurtosis of S&P 500 returns in an arbitrage-free economy. Martin and Wagner (2016) derive a formula for the expected return on individual stock as functions of the risk-neutral variance of the market and the stock's excess risk-neutral variance relative to the average stock. Gormsen and Jensen (2018) use the methodology in Martin (2017) to show how the market's higher order moments can be estimated ex ante. Kadan and Tang (2018) derives a bound on the expected individual stock returns. The closest paper to mine is Schneider and Trojani (2017a). They recover, under mild assumptions, the model-free conditional minimum variance projection of the pricing kernel on various trade-able realized moments of market returns. Similar to higher-order tradeable risks presented in Schneider and Trojani (2015, 2016, 2017), Results (1)-(4) show that there is a function that maps SDF moments into high-order tradeable risks.

In section 3.1, I provide a methodology to recover all physical and risk-neutral moments of the SDF when the marginal utility is known. Section 3.2 also extends the methodology to compute conditional moments when preferences depart from CRRA preferences. In Sections 5 and 7, I assess how the information content of the SDF moments can be used to forecast the excess market return, and, also explain the cross-section of returns.

3. A Theory for Computing SDF Moments

I assume that the distribution of the SDF is characterized by the mean, variance, skewness, and kurtosis.[1] The mean of the SDF is the inverse of the return on the risk-free asset, which is observable from the return on Treasury bills. While the mean of the SDF is observable, the variance, skewness, and kurtosis of the SDF are not observable. The information contained in the distribution of the SDF is embedded in the variance, skewness, and kurtosis of the SDF. I define $m_{t\to T}$ as the SDF from time t to T, and physical conditional moments of the SDF as

$$\mathcal{M}_t^{(n)}[T] = \mathbb{E}_t\left(\left(m_{t\to T} - \mathbb{E}_t\left(m_{t\to T}\right)\right)^n\right), \tag{9}$$

where $n > 1$. $\mathbb{E}_t(.)$ stands for the conditional expectation operator under the physical measure. $\mathcal{M}_t^{(2)}[T]$, $\mathcal{M}_t^{(3)}[T]$, and $\mathcal{M}_t^{(4)}[T]$ represent the second, third, and fourth moment of the SDF. I refer to the moments as the variance, skewness, and kurtosis of the SDF.

[1] I focus on the second, third, and fourth moment of the SDF. However, the methodology presented here applies to any moment of the SDF.

Result 6 *Under Assumption 1, the conditional physical moments of the SDF can be computed in a model-free environment by using the relationship*

$$\mathcal{M}_t^{(n+1)}[T] = \frac{1}{R_{f,t\to T}}\left(\mathcal{N}_t^{*(n)}[T] - \mathcal{M}_t^{(n)}[T]\right), \tag{10}$$

with

$$\mathcal{N}_t^{*(n)}[T] = \mathbb{E}_t^*\left(\left(m_{t\to T} - \mathbb{E}_t\left(m_{t\to T}\right)\right)^n\right), \tag{11}$$

where the asterisks denote quantities calculated with risk-neutral probabilities. $R_{f,t\to T}$ is the return on the risk-free asset from time t to T.

Proof. See Appendix A. ■

Result 6 shows that, when the SDF is a nonlinear function of the return of the market portfolio, the conditional physical moments of the SDF can be computed from option prices. The recursive expression (10) shows that the n+1 *th* conditional moment of the SDF is the discounted value of the difference between physical and a risk-neutral conditional moments of the SDF. The conditional physical moments of the SDF can be computed, at any time t, using option prices without making any assumption about the distribution of the return on the market portfolio. It has the advantage that it relates the SDF conditional moments to the risk-neutral distribution. The conditional physical moments of the SDF can be measured in real-time without making any time-series assumption about the underlying market process or relying on past observations. I look further at the risk-neutral distribution of the SDF by providing closed-form expressions of the risk-neutral moments of the SDF. To do so, I define the risk-neutral moments as

$$\mathcal{M}_t^{*(n)}[T] = \mathbb{E}_t^*\left(\left(m_{t\to T} - \mathbb{E}_t^*\left(m_{t\to T}\right)\right)^n\right). \tag{12}$$

This allows gauging whether the physical distribution of the SDF and the risk-neutral distribution of SDF contain a different set of information that can be used to understand asset prices. The wedge between the two distributions can be characterized by the difference between risk-neutral moments of the SDF and physical moments of the SDF. This difference is interpreted in this paper as the SDF-based moment risk premium. More precisely, I define the SDF-based moment risk premium as

$$\mathcal{MRP}^{(n)}[T] = \mathcal{M}_t^{(n)}[T] - \mathcal{M}_t^{*(n)}[T]. \tag{13}$$

In an unlikely case in which the SDF is linear in terms of the market return, this difference is proportional (up to a constant) to the market variance risk premium, market skewness risk premium, and market kurtosis risk premium when $n = 2$, 3, or 4, respectively. Further, one can note that the difference between the non-central physical and risk-neutral moments of the SDF is an important determinant of the expected excess return of the SDF-based moment returns defined as

$$\mathcal{R}_{t\to T}^{(n)} = \frac{m_{t\to T}^{n-1}}{\mathbb{E}_t\left(m_{t\to T} \times m_{t\to T}^{n-1}\right)} \text{ for } n > 0, \tag{14}$$

where $\mathcal{R}_{t\to T}^{(1)}$ corresponds to the return on the risk-free asset, $\mathcal{R}_{t\to T}^{(2)}$ corresponds to the return on the SDF security (defined in Hansen and Jagannathan (1991, page 234)). The returns defined in equation (14) are positively correlated with the SDF. The positive correlation of the return suggests that the non-central risk-neutral moments of SDFs, $\mathbb{E}_t^*\left[m_{t\to T}^{n-1}\right]$, are always higher than their corresponding physical ones, $\mathbb{E}_t\left[m_{t\to T}^{n-1}\right]$.[2] Thus, the returns defined in (14) correspond to returns of hedging strategies. These returns pay more in bad times (e.g, crisis periods). As a result of their positive correlation with the SDF, the expected values of returns defined in (14) are lower than the return on the risk-free asset. Hence, the corresponding expected excess returns are negative.

Result 7 *The expected excess returns on SDF moments defined in (14) are*

$$\mathbb{E}_t\left(\mathcal{R}_{t\to T}^{(n)} - R_{f,t\to T}\right) = R_{f,t\to T}\frac{\mathbb{E}_t\left(m_{t\to T}^{n-1}\right) - \mathbb{E}_t^*\left(m_{t\to T}^{n-1}\right)}{\mathbb{E}_t^*\left(m_{t\to T}^{n-1}\right)} \textit{ for } n > 1, \tag{15}$$

where

$$\mathbb{E}_t\left(m_{t\to T}^{n-1}\right) = \frac{1}{R_{f,t\to T}}\mathbb{E}_t^*\left(m_{t\to T}^{n-2}\right). \tag{16}$$

[2]This is verified empirically in Figure 5, where the conditional expected excess return defined in expression (15) is always negative.

The conditional Sharpe ratios are

$$\mathbb{SR}_t\left[T\right] = \frac{\mathbb{E}_t\left(\mathcal{R}^{(n)}_{t\to T} - R_{f,t\to T}\right)}{\sqrt{\mathbb{VAR}_t\left(\mathcal{R}^{(n)}_{t\to T} - R_{f,t\to T}\right)}} \text{ for } n > 1 \tag{17}$$

where

$$\mathbb{VAR}_t\left(\mathcal{R}^{(n)}_{t\to T} - R_{f,t\to T}\right) = \frac{R_{f,t\to T}\mathbb{E}^*_t\left(m^{2n-3}_{t\to T}\right) - \left(\mathbb{E}^*_t\left(m^{n-2}_{t\to T}\right)\right)^2}{\left(\mathbb{E}^*_t\left(m^{n-1}_{t\to T}\right)\right)^2}. \tag{18}$$

Proof. See Appendix A. ■

In Sections 3.1 and 3.2, I derive closed-form expressions of the SDF conditional moments, conditional expected excess return on SDF moments, and conditional Sharpe ratios. I first start by employing the CRRA preference. To mitigate concerns about the choice of the utility function, I thereafter follow a more general approach in Section 3.2 and extend my analysis to the case in which the representative investor has a general utility function that departs from the CRRA utility. The methodological approach proposed here can be applied to any utility function. The closed-form solutions of SDF moments are computed using various specifications of investor preferences, hence they do not rely on Taylor expansion-series.

3.1. Moments Under CRRA Preferences

I consider a representative agent with a CRRA utility and a risk aversion parameter α

$$u\left[x\right] = \frac{x^{1-\alpha} - 1}{1-\alpha} \tag{19}$$

and show that, with the knowledge of the relative risk aversion, I can recover the real-time distribution of the SDF. In this section, the representative agent maximizes her expected utility subject to her budget constraint,

$$\max_{W_{t\to T} = W_t\left(R_{f,t\to T} + \omega_t^{\perp}\left(\mathbf{R}_{t\to T} - R_{f,t\to T}\right)\right)} \mathbb{E}_t\left(u\left[W_T\right]\right),$$

where W_t is the initial wealth and $W_{t\to T}$ represents the representative agent terminal wealth. $\mathbf{R}_{t\to T}$ is the return vector on risky assets. With a CRRA utility, $u^{'}[x] = x^{-\alpha}$, the SDF has the form

$$m_{t\to T} = \frac{1}{R_{f,t\to T}} \frac{(W_{t\to T}/W_t)^{-\alpha}}{\mathbb{E}_t\left((W_{t\to T}/W_t)^{-\alpha}\right)}.$$

In the rest of the paper, I make a common assumption that the return on aggregate wealth $W_{t\to T}/W_t$ is the return on the market portfolio that is, $R_{M,t\to T} = W_{t\to T}/W_t$. I do not impose any restriction on the initial wealth W_t, in particular, W_t needs not to be equal to 1. I denote

$$\mu_t = \frac{1}{R_{f,t\to T}\mathbb{E}_t\left[(W_{t\to T}/W_t)^{-\alpha}\right]}$$

and express the SDF as

$$m_{t\to T} = \mu_t \left(R_{Mt\to T}\right)^{-\alpha}. \tag{20}$$

Next, I show how to obtain a closed-form expression of the constant parameter μ_t. From the identity (20), the constant parameter μ_t can alternatively be expressed as $\mu_t = m_t R^{\alpha}_{Mt\to T}$. I then apply the conditional expectation operator to μ_t under the physical measure,

$$\mu_t = \mathbb{E}_t\left(m_{t\to T} R^{\alpha}_{Mt\to T}\right) = \left(\mathbb{E}_t\left(m_{t\to T}\right)\right)\mathbb{E}_t\left(\frac{m_{t\to T}}{\mathbb{E}_t\left(m_{t\to T}\right)} R^{\alpha}_{Mt\to T}\right), \tag{21}$$

and show that, provided that $\mathbb{E}_t\left(m_{t\to T}\right) = \frac{1}{R_{f,t\to T}}$, the constant simplifies to

$$\mu_t = \frac{\delta_t}{R_{f,t\to T}} \text{ with } \delta_t = \mathbb{E}^*_t\left(R^{\alpha}_{Mt\to T}\right), \tag{22}$$

where $R_{f,t\to T}$ is the return for holding a government bond from time t to T. I consider the case in which the market return is the return on the S&P 500 index. Thus, I define the market return as $R_{Mt\to T} = \frac{S_T}{S_t}$, where S_t is the price of the S&P 500 index. Result 8 provides a closed-form expression of $\mathcal{N}^{*(n)}_t[T]$ which allows to compute the physical conditional moments of the SDF (see equation (10)).

Result 8 *The n th conditional physical moment of the SDF is given by (10), with*

$$\mathcal{N}^{*(n)}_t[T] = \frac{(\delta_t - 1)^n}{R^n_{f,t\to T}} + \frac{n\alpha(\delta_t - 1)^{n-1}\delta_t(1 - R_{f,t\to T})}{R^n_{f,t\to T}} + \frac{1}{R^{n-1}_{f,t\to T}}\left(\int_{S_t}^{\infty} h_{SS}[K]C_t[K]\,dK + \int_0^{S_t} h_{SS}[K]P_t[K]\,dK\right),$$

where

$$h_{SS}[K] = n(n-1)\delta_t^2\alpha^2\frac{1}{S_t^2}\left(\delta_t\left(\frac{K}{S_t}\right)^{-\alpha}-1\right)^{n-2}\left(\frac{K}{S_t}\right)^{-2(1+\alpha)} + n\delta_t\alpha(\alpha+1)\frac{1}{S_t^2}\left(\delta_t\left(\frac{K}{S_t}\right)^{-\alpha}-1\right)^{n-1}\left(\frac{K}{S_t}\right)^{-(\alpha+2)}$$

and

$$\delta_t = 1+\alpha(R_{f,t\to T}-1)+\alpha(\alpha-1)R_{f,t\to T}\frac{1}{S_t^2}\left(\int_{S_t}^{\infty}\left(\frac{K}{S_t}\right)^{\alpha-2}C_t[K]\,dK+\int_0^{S_t}\left(\frac{K}{S_t}\right)^{\alpha-2}P_t[K]\,dK\right).$$

Proof. See Appendix A. ■

Result 8 shows that, at any time t, the conditional physical moments of the SDF (see equation (10)) can be computed using option prices without making any time-series assumption about the distribution of the return on the market portfolio. It has the advantage of relating the SDF conditional moments to a directly observable quantity, but the disadvantage is that it requires the knowledge of the risk aversion. Provided that the risk aversion parameter is known, conditional physical moments of the SDF can be measured in real time without relying on past observations. I further derive the conditional risk-neutral moments of the SDF.

Result 9 *The n th conditional risk-neutral moment of the SDF is*

$$\mathcal{M}_t^{*(n)}[T] = \frac{\delta_t^n}{R_{f,t\to T}^n}\mathbb{E}_t^*(f[S_T]), \tag{23}$$

where

$$\mathbb{E}_t^*(f[S_T]) = (1-\zeta_t)^n - n\alpha(1-\zeta_t)^{n-1}(R_{f,t\to T}-1)+R_{f,t\to T}\left(\int_{S_t}^{\infty}f_{SS}[K]C_t[K]\,dK+\int_0^{S_t}f_{SS}[K]P_t[K]\,dK\right),$$

with

$$f_{SS}[K] = \frac{n\alpha}{S_t^2}(\alpha+1)\left(\frac{K}{S_t}\right)^{-\alpha-2}\left(\left(\frac{K}{S_t}\right)^{-\alpha}-\zeta_t\right)^{n-1}+\frac{n(n-1)(\alpha^2)}{S_t^2}\left(\frac{K}{S_t}\right)^{-2\alpha-2}\left(\left(\frac{K}{S_t}\right)^{-\alpha}-\zeta_t\right)^{n-2}$$

and

$$\zeta_t = 1 - \alpha\left(R_{f,t\to T} - 1\right) + R_{f,t\to T}\frac{\alpha(1+\alpha)}{S_t^2}\left(\int_{S_t}^{\infty}\left(\frac{K}{S_t}\right)^{-\alpha-2} C_t[K]\,dK + \int_0^{S_t}\left(\frac{K}{S_t}\right)^{-\alpha-2} P_t[K]\,dK\right) \quad (24)$$

Proof. See Appendix A. ■

Focusing on the time-varying conditional moments allows us to learn about the time-varying distribution of the SDF. In an unconditional setting, Hansen and Jagannathan (1991) provide a simple methodology to derive a lower bound on the variance of the SDF. This bound has become a key reference in evaluating asset-pricing models. Their approach requires an estimation of the minimum unconditional variance on the SDF, and it relies on the sample moments from historical returns. However, the computation of sample moments requires a choice of time window, making it difficult to estimate with accuracy the minimum variance of the SDF. Bekaert and Liu (2004) propose a way to estimate the Hansen and Jagannathan minimum variance on the SDF that takes into account conditioning information. Snow (1991) also derives restrictions on unconditional non-central moments of the SDF by relying on historical returns.

In asset pricing models, to obtain more reliable estimates of time-varying conditional moments of the SDF, a time-series model can be used. However, using a time-series model raises the question of whether the empirical results based on time-series assumptions on asset returns or economic fundamentals are robust to the choice of time-series assumptions. Nagel and Singleton (2011) use the GMM approach to show that many recently proposed consumption-based models of stock returns, when evaluated using an optimal set of managed portfolios and the associated model-implied conditional moment restrictions do not capture key features of risk premiums in equity markets. In my study, I instead estimate the time-series moments of the SDF from daily option prices. I use the formula in Results 6–9 to compute the SDF conditional moments. These conditional moments can be used to gauge asset pricing models, provided that the risk aversion is known. The moments can be computed using a single day of option data, and, hence, are forward-looking and can be computed in real time.

Next, Result 10 provides closed-form expressions of the non-central risk-neutral moments of the SDF. The non-central moments of the SDF are necessary to compute the conditional expected excess returns and conditional Sharpe ratios of the SDF-based moment returns defined in Result 7.

Result 10 *The conditional expected excess return on SDF moments, (15), and the corresponding conditional Sharpe ratios (17) can be computed by using the following identity:*

$$\begin{aligned}\mathbb{E}_t^*\left(m_{t\to T}^{n-2}\right) &= 1+\alpha(2-n)\left(R_{f,t\to T}-1\right) \\ &+\frac{\alpha(2-n)\left(\alpha(2-n)-1\right)R_{f,t\to T}}{S_t^2}\left(\begin{array}{c}\int_{S_t}^{\infty}\left(\frac{K}{S_t}\right)^{\alpha(2-n)-2}C_t[K]\,dK \\ +\int_0^{S_t}\left(\frac{K}{S_t}\right)^{\alpha(2-n)-2}P_t[K]\,dK\end{array}\right).\end{aligned} \tag{25}$$

Proof. See Appendix A. ■

3.2. Conditional Moments When Preferences Depart from CRRA Preferences

3.2.1. Departure from CRRA Preferences in Expected Utility Framework

I assume that the representative agent has a utility function $\vartheta[.]$ that departs from the reference utility $u[.]$. The functions $u[.]$ and $\vartheta[.]$ are continuous and concave, and have high-order derivatives that exist (see Eeckhoudt and Schlesinger (2006) and Deck and Schlesinger (2014) for a theoretical justification of high-order derivatives). I use the CRRA utility as a reference utility. In the expected utility framework, results in Result 11 hold for any reference utility $u[.]$. There exists a function $\upsilon=\vartheta\circ u^{-1}$ such that $\vartheta=\upsilon\circ u$. With this formulation, $\vartheta[.]$ departs from a reference utility function $u[.]$ because $\upsilon[.]$ is different from the identity function (since $\vartheta[.]$ departs from the reference utility $u[.]$). To derive moments of the SDF, in this setting, I show in Appendix B that the Taylor expansion-series of $\upsilon[u[x]]$ around $u[x]=u[S_t]$ leads to a new SDF as shown in Result 11.

Result 11 *In an expected utility framework, assume that the representative agent has a utility function $\vartheta[.]$ that departs from the utility $u[.]$. Using the Taylor expansion series of $\vartheta[.]$ around $u[x]=u[S_t]$, the SDF has the form*

$$m_{t\to T}^{SDF}=m_{t\to T}^{P}\times m_{t\to T}^{T}, \tag{26}$$

where $m_{t\to T}^{P}$ and $m_{t\to T}^{T}$ are uncorrelated with

$$m_{t\to T}^{P}=\frac{z_{Mt\to T}}{\mathbb{E}_t\left(z_{Mt\to T}\right)} \text{ and } m_{t\to T}^{T}=\frac{1}{R_{f,t\to T}}u'[S_T]\,\mathbb{E}_t^*\left(\frac{1}{u'[S_T]}\right),$$

with

$$z_{Mt\to T}=1+\sum_{k=2}^{\infty}(-1)^{k+1}\frac{\rho^{(k)}}{(k-1)!}\left(\frac{u\left[S_T\right]}{u\left[S_t\right]}-1\right)^{k-1} \text{ and } \mathbb{E}_t\left[z_{Mt\to T}\right]=\frac{\mathbb{E}_t^*\left(\frac{1}{u'\left[S_T\right]}\right)}{\mathbb{E}_t^*\left(\frac{1}{u'\left[S_T\right]}z_{Mt\to T}^{-1}\right)}$$

and

$$\rho^{(k)}=\left((-1)^{k+1}y^{k-1}\frac{\partial^k \upsilon\left[y\right]}{\partial^k y}/\frac{\partial \upsilon\left[y\right]}{\partial y}\right)_{y=u\left[S_t\right]}. \tag{27}$$

Proof. See the Online Appendix. ■

Result 11 shows that, when the utility function $\vartheta[.]$ departs from the reference utility $u[.]$, the SDF can be decomposed into two terms. The first component, $m_{t\to T}^{P}$, of the SDF is the " permanent " component of the SDF because its conditional mean equals to unity. The second component is the " transitory" component of the SDF. As shown in (27), $\rho^{(2)}$ can be interpreted as characterizing aversion of using the utility function $u[.]$. The "permanent" and "transitory" components of the SDF in this setting are different from the permanent and transitory components of the SDF in the recent literature.[3] The value $\rho^{(2)}=0$ indicates that investors use the CRRA utility. A high value of $\rho^{(2)}$ indicates that investors are willing to use a utility $\vartheta[.]$ that departs (and is far) from the CRRA utility. Result 11 specializes in any reference utility function $u[.]$.

To be consistent with Section 3.1, I now set $u[.]$ to be equal to the CRRA utility. When $u[.]$ is equal to the CRRA utility, the " transitory" component of the SDF is identical to (20). Under this specification, I set $k=2$ and use the SDF in (26) to provide closed-form formulas to compute the conditional moments and risk-neutral moments of the SDF. The closed-form expressions of conditional moments are found in the Internet Appendix.

[3] Borovicka, Hansen, and Scheinkman (2016) use the decomposition of the SDF into a permanent and transitory components (where the transitory component is the inverse of the return on the long-term bond) to highlight the implicit assumption in the Ross (2015) Recovery theorem.

3.2.2. Departure from CRRA Preferences in a Recursive Utility Framework

In Epstein and Zin (1989) recursive utility framework, provided that the market return is used as a proxy for the return on the aggregate consumption claim, the SDF has the form

$$m^{\mathrm{EZ}}_{t\to T} = \upsilon^{'}\left[c_{t\to T}\right] u^{'}\left[R_{Mt\to T}\right]. \tag{28}$$

Assume that the SDF correctly prices the market return and the risk-free rate,

$$\mathrm{E}_t\left[m^{\mathrm{EZ}}_{t\to T} R_{Mt\to T}\right] = 1 \text{ and } E_t\left[m^{\mathrm{EZ}}_{t\to T}\right] = \frac{1}{R_{f,t\mapsto T}}. \tag{29}$$

Since consumption does not co-move contemporaneously with the stock market return (e.g., see Hall (1978)), I assume that $c_{t\to T}$ and $R_{Mt\to T}$ are independent. Thus, expression (29) implies

$$\mathrm{E}_t\left[\upsilon^{'}\left[c_{t\to T}\right]\right]\mathrm{E}_t\left[u^{'}\left[R_{Mt\to T}\right]R_{Mt\to T}\right] = 1 \tag{30}$$

which simplifies to

$$\mathrm{E}_t\left[m_{t\to T} R_{Mt\to T}\right] = 1. \tag{31}$$

with

$$m_{t\to T} = \mathrm{E}_t\left[\upsilon^{'}\left[c_{t\to T}\right]\right] u^{'}\left[R_{Mt\to T}\right]. \tag{32}$$

Now, I show that $\mathrm{E}_t\left[\upsilon^{'}\left[c_{t\to T}\right]\right]$ can be recovered from option prices. (28) is equivalent to

$$m^{\mathrm{EZ}}_{t\to T}\left(u^{'}\left[R_{Mt\to T}\right]\right)^{-1} = \upsilon^{'}\left[c_{t\to T}\right]. \tag{33}$$

Thus, the expected value of expression (33) equals

$$\begin{aligned}
\mathrm{E}_t\left[\upsilon^{'}\left[c_{t\to T}\right]\right] &= \mathrm{E}_t\left[m^{\mathrm{EZ}}_{t\to T}\left(u^{'}\left[R_{Mt\to T}\right]\right)^{-1}\right] \\
&= \mathrm{E}_t\left[m^{\mathrm{EZ}}_{t\to T}\right]\mathrm{E}_t\left[\frac{m^{\mathrm{EZ}}_{t\to T}}{\mathrm{E}_t\left[m^{\mathrm{EZ}}_{t\to T}\right]}\left(u^{'}\left[R_{Mt\to T}\right]\right)^{-1}\right] \\
&= \frac{1}{R_{f,t\mapsto T}}\mathrm{E}^{*}_t\left[\left(u^{'}\left[R_{Mt\to T}\right]\right)^{-1}\right]
\end{aligned}$$

Expression (32) can, therefore, be written as

$$m_{t\to T} = \kappa u^{'}\left[R_{Mt\to T}\right]$$

with

$$\kappa = \frac{1}{R_{f,t\mapsto T}}\mathrm{E}_t^*\left[\left(u^{'}\left[R_{Mt\to T}\right]\right)^{-1}\right].$$

Recall that, in Epstein and Zin (1989), $u^{'}\left[R_{Mt\to T}\right]$ has the form

$$u^{'}\left[R_{Mt\to T}\right] = R_{Mt\to T}^{\theta-1}$$

with $\theta = \frac{1-\gamma}{1-\rho}$ where $\rho = \frac{1}{\psi} \geq 0$. Here, ψ is the Elasticity of Inter-temporal Substitution (EIS) and γ is the relative risk aversion. Under the assumption that consumption and return are contemporaneously independent, the SDF in Epstein and Zin recursive utility framework is of the form

$$m_{t\to T} = \kappa R_{Mt\to T}^{\theta-1} \text{ with } \kappa = \frac{1}{R_{f,t\mapsto T}}\mathrm{E}_t^*\left[R_{Mt\to T}^{1-\theta}\right] \tag{34}$$

Thus, under the assumption that consumption and return do not co-move (see Hall (1978)), the SDF (34) is similar to the SDF (20) which is obtained under CRRA preference with the exception that θ cannot be interpreted as a risk aversion parameter. It depends on both risk aversion and EIS. Although models comparison is not the purpose of this paper, the similarity of (34) and (20) can be used to derived all moments of the SDF under the physical measure for a fixed value θ. These moments can be used to discipline asset pricing models in a recursive utility setting when consumption and market return do not co-move contemporaneously.

4. Estimating the SDF Conditional Moments

4.1. Data

To quantify physical and risk-neutral moments of the SDF, I use the average of the bid-and-ask quotes on S&P 500 index options over the 1996–2015 period. The quotes are obtained from Option Metrics

IvyDB. I also obtain closing prices of the S&P 500 index over the same time period. I then exclude quotes that do not satisfy standard no-arbitrage conditions and follow the standard filtering approach by cleaning the quotes.

I need a continuum of option prices to compute the integrals that appear in the SDF moments. In practice, option prices are not observable at all strikes, so I use cubic splines to approximate the integrals. I generate a large number of implied volatilities. More specifically, I generate implied volatilities for moneyness levels ($\frac{K}{S_t}$) between 0.01% and 300%. Implied volatilities and moneyness levels are then used to generate call and put prices. I use moneyness smaller than 1 to compute put prices and moneyness larger than 1 to compute call prices. Together, call prices, put prices, and the cubic splines approximation approach are used to approximate integrals needed to compute the SDF moments.

4.2. Choosing the Relative Risk Aversion Coefficient

I set the risk aversion parameter $\alpha = 2$. The choice of this value is motivated by various studies in the empirical literature. Friend and Blume (1975) estimate the relative risk aversion to be approximately 2. Noussair, Trautmann, and VanDeKuilen (2014) use experimental methods to estimate a relative risk aversion of a representative individual between 0.88 and 1.43, depending on the specification of investors' utility functions. With CRRA utility, Noussair et al. (2014, Table II, page 347) estimate the relative risk aversion coefficient to be between 0.88 and 0.94. Recent studies have used option prices to estimate the relative risk aversion (Aït-Sahalia and Lo (2000), Jackwerth (2000), Rosenberg and Engle (2002), and Bliss and Panigirtzoglou (2004)). Both studies recover implied risk aversion from option prices. In particular, Rosenberg and Engle (2002) use one-month option prices, estimate the time-varying slope (risk aversion) of the CRRA pricing kernel, and show that it varies from a minimum of 2.36 to a maximum of 12.55. Further, Bliss and Panigirtzoglou (2004, Table V, page 429) use S&P500 option prices to estimate the relative risk aversion parameter for a CRRA utility when the option maturity ranges from 1 to 6 weeks. They find that the relative risk aversion estimate declines from 9.52 to 3.37 when the option maturity increases from 1 to 6 weeks. Bliss and Panigirtzoglou (2004, Table VII, page 432) also summarize in their table, estimates of risk aversion in various studies and show that estimates of risk aversion vary from 0 to 55. I focus on option maturities ranging from 30 days to 365 days. To my knowledge, estimates of the relative risk aversion from option prices for maturities ranging from 30 days to 365 days are not available in the literature.[4] Choosing different relative risk aversion values renders empirical results incomparable across option maturities. To avoid choosing a relative risk aversion coefficient for each maturity, I assume that the relative risk aversion is the same regardless of the option maturity used.[5]

4.3. Computing Conditional Physical Moments of the SDF

Figure 1 presents the conditional variance of the SDF for maturities ranging from 30 days to 365 days. The sample period is from January 4, 1996, to August 31, 2015. The variances are not annualized. Regardless of the investment horizon, the conditional variance varies significantly through time. It peaks during crisis periods. However, the peak is pronounced in November 2008. The 30-day SDF variance reaches its highest value in November 2008. The variance is approximately 0.45. This translates into 67.08% monthly volatility. The corresponding annual SDF volatility on November 20, 2008, is 232%. Table 1 reports the mean, standard deviation, skewness, kurtosis, and quantiles of the variance of the SDF, $\mathcal{M}_t^{(2)}[T]$, for horizons ranging from 30 days to 365 days. The mean of the conditional variance over the whole sample is 0.03 at the 30-day horizon and 0.45 at the 365-day horizon. The corresponding 30-day annualized volatility is 60%, while the 365-day volatility is 67%. Note that the conditional variance of the SDF increases with the investment horizon. The conditional variance is not only volatile but also exhibits skewness and fat tails. At the 30-day horizon, it varies from a minimum of 0.01 to a maximum of 0.45 over the sample period. At the 365-day horizon, it varies from a minimum of 0.13 to a maximum of 2.88 over the sample period. The mean of the 365-day conditional variance of the SDF is 0.45, with a standard deviation of 0.28. This translates into an unconditional variance of the SDF of 0.53 and approximate volatility (square root of the variance) of 72.69%.

Figure 1 also presents the conditional skewness of the SDF for maturities ranging from 30 days to 365 days. Regardless of the maturity chosen, the conditional skewness varies through time and is often positive. It is more stable in normal periods and highly volatile during crisis periods (e.g., 1997–1998 asian financial crisis, the 2002 recession, and the 2008 financial crisis). The peak is more pronounced during the 2008

[4] Jackwerth (2000) estimates absolute risk aversion for maturities up to 64 days. He finds that absolute risk aversion can be negative in some wealth states.

[5] In unreported results, I set the relative risk aversion to 3. Results are qualitatively similar and are available upon request.

crisis than in the remaining sample. The 30-day conditional skewness reaches its maximum value of 0.6 in November 2008, while the 365-day conditional skewness reaches 10 during the same month. A simple comparison of the conditional volatility dynamic to the conditional skewness dynamic in Figure 1 shows that the conditional skewness of the SDF is more pronounced during crisis periods than in normal periods. Table 1 also reports the mean, standard deviation, skewness, kurtosis, and quantiles of the skewness of the SDF, $\mathcal{M}_t^{(3)}[T]$, for maturities ranging from 30 to 365 days. The mean of the conditional skewness over the whole sample is 0.01 at the 30-day horizon and 0.40 at the 365-day horizon. The conditional skewness is more volatile than the conditional variance; it exhibits skewness and fat tails. At the 30-day horizon, the conditional skewness varies from a minimum of -0.07 to the highest value of 0.56 over the sample period. At the 365-day horizon, it varies from a minimum of 0.40 to a maximum value of 9.26 over the sample period.

Figure 1 also shows that the conditional kurtosis varies through time. Similar to the conditional skewness, the conditional kurtosis is more pronounced during crisis periods than in normal periods. Table 1 also reports the mean, standard deviation, skewness, kurtosis, and quantiles of the skewness of the SDF, $\mathcal{M}_t^{(4)}[T]$. The mean of the conditional kurtosis over the whole sample is 0 at the 30-day horizon and increases with the investment horizon. The mean is 0.65 at the 365-day horizon. At the 365-day horizon, it varies from the lowest value of 0.65 to the highest value of 33.95 over the sample period.

I further use the SDF obtained under preferences that depart from CRRA preferences to compute the conditional moments and check the robustness of moments obtained under CRRA preferences. To do so, I keep the risk aversion parameter $\alpha = 2$ and set the preference parameter $\rho^{(2)} = 5$ and $\rho^{(k)} = 0$ for $k > 2$. Figure 8 of the Internet Appendix displays the conditional moments of the SDF. Results are similar to those in Figure 1. Non-zero preference parameters ρ^k, when $k > 2$, could generate difference in the conditional moments. Viewed all, empirical results suggest that the SDF is highly volatile and highly skewed, and it exhibits more and fatter tails during crisis periods than in normal times.

4.4. Computing the Conditional Risk Neutral Moments of the SDF

Result 9 is used to compute the conditional risk-neutral moments of the SDF when investors have CRRA preferences. The conditional risk-neutral variance varies significantly through time and across maturities. It increases with the investment horizon. A comparison of Figures 1 and 2 shows that the conditional risk-neutral variance is often higher than the physical variance through time and across maturities. This implies, on average, a negative SDF variance risk premium, as defined in Equation (13). Table 2 displays the mean, standard deviation, skewness, kurtosis, and quantiles of the conditional risk-neutral variance of the SDF, $\mathcal{M}_t^{*(2)}[T]$. The mean of the 365-day risk-neutral variance of the SDF is 0.57, with a standard deviation of 0.43. This translates into an unconditional variance of the SDF of 0.75 and approximate unconditional volatility of 86.88%.

Next, the conditional risk-neutral skewness of the SDF is negative and varies over time and across maturities. It is more volatile during crisis periods. The table also reports the mean, standard deviation, skewness, kurtosis, and quantiles of the conditional risk-neutral skewness of the SDF, $\mathcal{M}_t^{*(3)}[T]$. The mean of the conditional risk-neutral skewness over the whole sample is -0.31 at the 365-day horizon. To explore why the risk-neutral moment of the SDF is often negative, I show the following result:

Result 12 *The skewness of the SDF under the risk-neutral measure can be decomposed as*

$$\mathcal{M}^{*(3)} = \frac{1}{\overline{m}} \left\{ \mathcal{A}_0 + \frac{1}{2!}\mathcal{A}_2 \mathcal{M}^{(2)} + \frac{1}{3!}\mathcal{A}_3 \mathcal{M}^{(3)} + \frac{1}{4!}\mathcal{A}_4 \mathcal{M}^{(4)} \right\}, \tag{35}$$

where

$$\begin{aligned} \mathcal{A}_0 &= -\left(\mathcal{M}^{(2)}\right)^3 < 0, &(36)\\ \frac{1}{2!}\mathcal{A}_2 &= \left(3\frac{1}{\overline{m}^2}\left(\mathcal{M}^{(2)}\right)^2 - 3\mathcal{M}^{(2)}\right) > 0, \\ \frac{1}{3!}\mathcal{A}_3 &= \left(-3\frac{1}{\overline{m}}\mathcal{M}^{(2)} + \overline{m}\right) < 0, and \\ \frac{1}{4!}\mathcal{A}_4 &= 1, &(37) \end{aligned}$$

where $\overline{m}$ is the mean of the SDF.

Proof. See the Appendix. ■

As shown in Result 12, the risk-neutral skewnesss of the SDF can be negative since the coefficients $\mathcal{A}_0$ and $\mathcal{A}_3$ are negative. These coefficients are uniquely determined by the mean and volatility of the SDF. Hence, the volatility of the SDF may potentially explain why the skewness of the SDF under the risk-neutral measure is often negative.

Figure 2 also displays the conditional risk-neutral kurtosis. The conditional risk-neutral kurtosis is highly volatile. In the same table, I report the mean, standard deviation, skewness, kurtosis, and quantiles of the conditional risk-neutral kurtosis of the SDF, $\mathcal{M}_t^{*(4)}[T]$. The conditional risk-neutral kurtosis is positively skewed and exhibits fat tails.

4.5. A Look at the SDF Moments Risk Premium Dynamic

This section explores the dynamic of the SDF-based moments risk premium. Table 3 reports summary statistics of the daily SDF variance risk premium at various horizons. The SDF variance risk premium is often negative, is negatively skewed, exhibits fat tails, and increases in absolute value when the option maturity increases from 30 days to 365 days. At the 30-day horizon, the mean of the SDF variance risk premium over the whole sample is -0.37%. At the 36-day horizon, the mean of the SDF variance risk premium is -11.91%. The SDF variance risk premium is on average negative. Thus, to compensate investors for exposure to SDF variance risk premium risk, one would expect the SDF variance risk premium to be negatively related to expected excess return.

The concept of SDF variance premium is new. Thus, one would expect a comparison between the market variance risk premium, commonly measured at monthly frequency, and the SDF variance risk premium. One may also expect a comparison between the Left risk-neutral Jump Variation (LJV) of Bollerslev, Todorov, and Xu (2015) and the SDF variance risk premium. How do they compare?

Since SDF moments risk premium are computed at daily frequency, within each month, I average daily estimates of the SDF variance risk premium to obtain monthly estimates of SDF variance risk premium. Figure 3 shows the monthly dynamic of the SDF moments risk premium at various horizons. To facilitate the comparison, I focus on the 30-day horizon risk premium (in Figure 4). The top graph in Figure 4 shows the dynamic of the SDF variance risk premium, the market variance risk premium, and the LJV measures.[6] At the 30-day horizon, the SDF variance risk premium and the market variance risk premium appear to be negatively correlated. The correlation between the two measures is -0.33. The SDF variance risk premium

[6]The LJV measure from Bollerslev, Todorov, and Xu (2015) is computed using only options with fewer than 45 days to expiration.

is also negatively correlated with the LJV measure, while the correlation between the LJV measure and the SDF variance risk premium is -0.83. Note that the correlation between the market variance risk premium and the LJV measure is 0.1. Here, the market variance risk premium is the difference between the market variance under the physical measure and the market variance under the risk-neutral measure.[7] At the 365-day horizon, the correlation between the SDF variance risk premium and the market variance risk premium is approximately 0, while the correlation between the SDF variance risk premium and the LJV measure is -0.52.

Table 3 also reports summary statistics of the daily SDF skewness risk premium at various horizons. In contrast to the SDF variance risk premium, the SDF skewness premium is always positive, is positively skewed, and increases with option maturity. Thus, to compensate investors for exposure to SDF skewness risk premium, one would expect the SDF to be positively related to expected return. The SDF skewness risk premium is more pronounced at long horizons (from 91 days to 365 days). To compare the dynamic of the SDF skewness premium to market variance risk premium and LJV dynamic, I average within each month, daily estimates of the SDF skewness risk premium to obtain monthly estimates of SDF skewness risk premium. LJV and variance risk premium measures are not available at daily frequency. The middle graph in Figure 4 also shows the dynamic of the SDF skewness risk premium. The SDF skewness risk premium is mostly pronounced during crisis or recession periods, and it is almost zero during normal times. At the 30-day horizon, the correlation between the monthly estimate of SDF skewness risk premium and the market variance risk premium is 0.34, while the correlation between the LJV measure and monthly estimates of SDF skewness risk premium is positive and equal to 0.83. Further, at the 365-day horizon, the correlation between the SDF skewness risk premium and the market variance risk premium is -0.04, while the correlation between the SDF skewness risk premium and the LJV measure is 0.81.

To end this section, I report in Table 3 summary statistics of the daily SDF kurtosis risk premium at various horizons. In contrast to the SDF variance risk premium, the SDF kurtosis premium is always positive, is positively skewed, and increases with option maturity. Thus, to compensate investors for exposure to SDF kurtosis risk premium, one would expect the SDF kurtosis risk premium to be positively related to expected return. The SDF kurtosis risk premium is pronounced during turbulence or recession periods, and is negligible during normal times. At the 30-day horizon, the correlation between the monthly esti-

[7]The monthly observations of the market variance risk premium are obtained from Hao Zhou's website.

mates of SDF kurtosis risk premium and monthly estimates of the variance risk premium is 0.45, while the correlation between the LJV measure and monthly estimates of SDF kurtosis risk premium is positive and equal to 0.80. I also find that the correlation between SDF kurtosis risk premium, market variance risk premium and LJV measures decrease when the maturity increases. For example, at the 365-day horizon, the correlation between the SDF kurtosis risk premium and the market variance risk premium is -0.02, while the correlation between the SDF kurtosis risk premium and the LJV measure is 0.11.

4.6. Conditional Expected Excess Return and Conditional Sharpe Ratios

I exploit non-central conditional moments of the SDF to compute the expected excess return on SDF moments and the Sharpe ratios. More specifically, I use closed-form expressions in Results 7 and 10 to compute these asset-pricing quantities. Both expected excess returns and Sharpe ratios are annualized to facilitate comparison over time and also across different maturities. Since strategies based on SDF moments are hedging strategies, the expected excess returns on SDF moments and the corresponding Sharpe ratios must be negative. Figure 5 presents the conditional expected excess returns $\mathbb{E}_t\left(\mathcal{R}^{(n)}_{t\to T} - R_{f,t\to T}\right)$, when $n = 2$, 3, and 4. Results are not in percentages. Across all maturities, the expected excess returns vary significantly through time and are pronounced during crisis periods. The conditional expected excess return reaches its maximum of -3, -5.5, and -6.5, when $n = 2$, 3, and 4, respectively. Regardless of the SDF-based return used, the annualized expected excess market return at the 365-day horizon is lower (in absolute value) than the annualized expected excess return at the 30-day horizon. This features (in absolute value) a downward term structure of the conditional expected excess returns on SDF moments. The average of the conditional expected excess return on SDF-based moments shows little variation across maturities when $n = 2$, 3, and 4, respectively. Thus, the term structure of the unconditional expected excess return on SDF moments is almost flat.

To assess the trade-off between risk and return, I plot, over time and across maturities, the annualized conditional Sharpe ratio $\mathbb{SR}_t\,[T]$ that corresponds to SDF moments when $n = 2$, $n = 3$, and $n = 4$. As shown in Figure 6, the Sharpe ratios vary significantly over time and across maturities. The term structures of the conditional Sharpe ratios are (in absolute value) upward sloping. The Sharpe ratio (in absolute value) is high during crisis periods and low and stable during normal periods. However, the unconditional Sharpe ratio shows little variation across maturities. This features (almost) a flat term structure of the unconditional Sharpe ratio of the SDF-based moment returns. Figures 9 and 10, in the Internet Appendix, present similar results when investor preferences depart from the CRRA preferences.

5. The Real-Time Distribution of the SDF Predicts the Market Return

5.1. Conditional Moments of the SDF Predicts the Excess Market Return

I investigate whether the SDF variance, skewness, and kurtosis predict the market expected excess return at any investment horizon. More specifically, I first run univariate regressions,

$$R_{M,t\rightarrow T} - R_{f,t} = \alpha_0 + \beta \mathbb{X}_t + \varepsilon_{t\rightarrow T}, \tag{38}$$

where $\mathbb{X}_t$ is either the variance, skewness, or kurtosis of the SDF. Since the SDF moments are computed at daily frequency, I examine return predictability using daily market returns. More specifically, for a given maturity $T-t$, I compute each day, the return from holding the market portfolio from t to T. For example, the one-month moments are used to predict future monthly returns. The three-month moments are used to predict future three-month returns. The return on the risk free security is also consistent with the maturity used in each regression. Panel A of Table 4 reports the β coefficients for the predictive regressions. The physical variance, skewness, or kurtosis of the SDF significantly predicts the market expected excess return for horizons ranging from 122 days to 365 days. T-stats in brackets are computed using Hansen and Hodrick (1980) methodology, with the number of lags equal to the time to maturity (in days). The β coefficients are all positive. A positive slope coefficient in each predictive regression indicates that an increase in the conditional moments of the SDF magnifies uncertainty in financial markets and, hence, increases the market risk premium. The adjusted R-square of predictive regressions ranges from 4% (at 122 days to maturity) to approximately 10% at 365 days to maturity. The signs of the β coefficients for the SDF variance and SDF kurtosis are consistent with Result 1 when investors have CRRA preferences ($u'[x] = x^{-\alpha}$). However, the signs of slope coefficients when the SDF skewness is used are not consistent with economic theory because Result 1 theoretically predicts that the slope of the SDF skewness is negative, provided that $u''[x] < 0$ and $u'''[x] > 0$.

I further investigate why, in the predictive regression results, the slope of the skewness of the SDF is positive. I have used the CRRA utility to derive the SDF moments. With a CRRA utility, $u^{''}[x] = -\alpha x^{-\alpha-1}$ and $u^{'''}[x] = \alpha(\alpha+1)x^{-\alpha-2} > 0$. Hence, the slope coefficient A_1 associated with the SDF volatility must be positive, while the slope coefficient A_2 associated with the SDF skewness must be negative (see Result 1). The empirical results from the predictive regression show that the positive slope coefficients associated to both SDF volatility and SDF skewness imply that $\frac{1}{u^{''}[x]} \leq 0$ and $-\frac{u^{'''}[x]}{\left(u^{''}[x]\right)^3} \leq 0$, which implies that $u^{'''}[x] \leq 0$. This is inconsistent with the CRRA preference and is puzzling. These puzzling results can potentially be explained by the risk aversion puzzle documented in Jackwerth (2000) and Aït-Sahalia and Lo (2000). To explore this issue carefully, I observe that a decreasing absolute risk aversion function implies that the first derivative of the absolute risk aversion $AR[x] = -\frac{u^{''}[x]}{u^{'}[x]}$ must be negative. Thus, $AR'[x] = -\frac{u^{'''}[x]u^{'}[x]-\left(u^{''}[x]\right)^2}{\left(u^{'}[x]\right)^2} \leq 0$. Since the empirical results from the predictive regressions imply that $u^{''}[\upsilon[\mathbb{E}(m)]] \leq 0$ and $u^{'''}[\upsilon[\mathbb{E}(m)]] \leq 0$, $AR'[\upsilon[\mathbb{E}(m)]] = \frac{u^{'''}[\upsilon[\mathbb{E}(m)]]u^{'}[\upsilon[\mathbb{E}(m)]]-\left(u^{''}[\upsilon[\mathbb{E}(m)]]\right)^2}{\left(u^{'}[\upsilon[\mathbb{E}(m)]]\right)^2} \geq 0$ holds empirically, and, hence, the absolute risk aversion function is increasing in the neighborhood of $x = \upsilon[\mathbb{E}(m)]$. This is consistent with the risk aversion puzzle documented in Jackwerth (2000) and Aït-Sahalia and Lo (2000). Jackwerth (2000) and Aït-Sahalia and Lo (2000) document that the absolute risk aversion function is negative in some wealth states and is also increasing in some wealth states.

I further follow Goyal and Welch (2008) and Campbell and Thompson (2008) and compute the out-of-sample performance of the forecasts. I compare the out-of-sample R^2_{OOS} statistic to the in-sample R^2 statistic. I compute the out-of-sample R-square as

$$R^2_{OOS} = 1 - \frac{\sum_{t=1}^{T} \varepsilon_t^2}{\sum_{t=1}^{T} \eta_t^2}, \tag{39}$$

where ε_t is the residual of the predictive regression (38) and $\eta_t = r_t - \overline{r}_t$, where r_t is the dependent variable (left-hand side variable of (38)), and $\overline{r}_t$ is the historical average of the dependent variable estimated through period $t-1$. The out-of-sample R-squares are comparable with in-sample R-squares. This supports the prediction that the SDF variance significantly predicts the excess market return in-sample and out-of-sample.

The skewness of the SDF produces slightly higher out-of-sample R-squares than the variance or kurtosis of the SDF. For the purpose of comparison, Panel B of Table 4 displays similar results when investor preferences depart from CRRA preferences. To compare the predictive results to results obtained when simple risk-neutral moments of the market return are used as predictors, I run the univariate regression (38), where $\mathbb{X}_t$ is either the variance, skewness, or kurtosis of the simple return $R_{M,t\to T}$. More precisely, $\mathbb{X}_t = \mathbb{VAR}^*_t(R_{M,t\to T})$, $\mathbb{SKEW}^*_t(R_{M,t\to T})$, or $\mathbb{KURT}^*_t(R_{M,t\to T})$. Closed form expressions of these moments are available in Appendix C of the Internet Appendix. Results are reported in Table 5. Also shown in Table 5, the variance, skewness, and kurtosis of the simple return $R_{M,t\to T}$ are weak predictors of the excess market return. Note that, in his framework, Martin (2017) uses a sample of option prices from 1996 to 2012 and finds that the simple risk-neutral variance $\mathbb{VAR}^*_t(R_{M,t\to T})$ predicts the excess market return with the highest out-of-sample R-square of 4.86% obtained at 6 months horizon.[8] Bollerslev, Tauchen, and Zhou (2009) show that the market variance risk premium predicts the market excess return at short horizons. Further Bollerslev, Todorov, and Xu (2015) compute a Left Risk-Neutral Jump Variation (LJV) measure at weekly frequency and use it to assess return predictability at monthly frequency. Monthly LJV measures are computed by averaging the within-month weekly values of LJV estimates. At monthly frequency, they show that much of the return predictability attributed to the variance risk premium may be attributed to time variation in the part of the variance risk premium associated with their LJV measure. Recently, Martin (2017, Table A.3) excludes the period from August 1, 2008, to July 31, 2009, and finds that market variance risk premium does not significantly predict the market excess return at any horizon once this crisis period is excluded. Motivated by these findings, I use the moments of simple returns as control variables in the predictive regression. Table 6 displays the result of the bivariate regressions. Results suggest that the predictive results are robust to the inclusion of the risk-neutral variance, skewness, and kurtosis of simple returns.

5.2. SDF-Based Moments Risk Premium Predicts the Market Return

In this section, I investigate whether the SDF-based moment risk premium defined in (13) predicts the market return at daily frequency. I run the univariate regression (38), where $\mathbb{X}_t$ represents the SDF-based

[8] Gormsen and Jensen (2018) use Martin (2017) approach to estimate ex ante market higher order moments. Martin and Wagner (2016) derive a formula that expresses the expected return on a stock in terms of the risk-neutral variance of the market and the stock's excess risk-neutral variance relative to the average stock.

variance risk premium, the SDF-based skewness risk premium, or the SDF-based kurtosis risk premium. Results are displayed in Table 7. Panel A presents the results when the CRRA preference is used, while Panel B presents the results when preferences that depart from CRRA preferences are used. As shown in Panel A, the SDF-based variance premium significantly predicts the excess market return for horizons ranging from 122 to 365 days with a negative slope and an adjusted R-squared ranging from 3.91% to 13.67%. Since the SDF variance risk premium is on average negative (see Table 3), the negative slope coefficient indicates that the SDF variance risk premium positively contributes to the expected excess market return. The out-of-sample R-squares (in the range of 3.84% to 13.61%) are slightly lower but comparable to the adjusted in-sample R-squares. The SDF-based skewness premium significantly predicts the excess market return for the same horizons with a positive slope and an adjusted R-square that varies from a minimum of 2.86% to a maximum of 8.33%. Because the SDF skewness risk premium is, on average, positive (see Table 3), one should expect the SDF risk premium to be positively related to the expected excess market return. The SDF-based kurtosis premium also predicts the excess market return with a positive slope. Since the SDF kurtosis risk premium is on average positive (see Table 3), one must expect the SDF kurtosis risk premium to be positively related to expected return. Panel B presents similar results.

I further checked the robustness of the results by running a bivariate regression of the form (38), where the risk-neutral variance, skewness, and kurtosis of simple returns are used as control variables. Results are reported in Table 8. Three remarks are in order. First, when $\mathbb{X}_t = \left\{\mathcal{MRP}_t^{(2)}[T],\ \mathbb{VAR}_t^*(R_{M,t\to T})\right\}$, the SDF-based variance risk premium significantly predicts the excess return with a negative slope for horizons ranging from 152 days to 365 days, while the variance of the simple return is insignificant, regardless of the investment horizon. Second, when $\mathbb{X}_t = \left\{\mathcal{MRP}_t^{(3)}[T],\ \mathbb{SKEW}_t^*(R_{M,t\to T})\right\}$, the SDF-based skewness risk premium significantly predicts the excess return for horizons ranging from 122 days to 365 days. In contrast, the skewness of the simple return significantly predicts excess returns only at very short horizons (30 days and 60 days). Third, when $\mathbb{X}_t = \left\{\mathcal{MRP}_t^{(4)}[T],\ \mathbb{KURT}_t^*(R_{M,t\to T})\right\}$, the SDF-based kurtosis risk premium significantly predicts the excess return at all horizons except for the 91-day horizon. Viewed all, results are robust to the inclusion of risk-neutral moments of simple returns.

5.3. Controlling for Variance Risk Premium and the Left Risk-Neutral Jump Variation Measures

As mentioned in Section 4.5, the variance risk premium (VRP) and the Left Risk-Neutral Jump Variation (LJV) measures are available at monthly frequency. To control for these measures in the predictive regressions, I use the average of daily SDF moments risk premium within a month as a measure of monthly SDF moments risk premium. I then use the monthly estimates of SDF moments risk premium together with VRP (LJV) measure as predictor variables. The main goal of this exercise is to check whether SDF moments risk premium still predict the market excess return at different investment horizons after controlling for VRP and LJV measures. I examine return predictability using monthly market returns. For a given maturity $T-t$, each month, I compute the return for holding the market portfolio from t to T. For example, the one-month moments are used to predict future monthly returns. The three-month moments are used to predict future three-month returns.

Table 9 reports results of the multivariate regression when SDF moments risk premium and LJV measures are used. The SDF moments risk premium remain significant for most maturities. The SDF variance risk premium is significant at 5% level for 30 days maturity and also when the maturity varies from 152 days to 365 days (with t-stat, in absolute value, above 2.8). At the same time, LJV is only significant at 365 days to maturity (with a t-stat of 2). Further, the SDF skewness risk premium is significant at 60, 273 and 365 days to maturity (with t-stat, in absolute value, above 2), while the LJV measure is significant at 60 days and 91 days to maturity with t-stats (in absolute value) above 3. The SDF kurtosis risk premium is also significant for most maturities except for 122 and 152 days.

Table 10 (11) reports results of the multivariate regression when, together, SDF moments risk premium and VRP (VRP-LJV) measures are used as predictor variables. The SDF moments risk premium are significant for maturity ranging from 122 days to 365 days to maturity with minimum t-stats, in absolute value, above 2.

6. Real-Time Distribution of the SDF Explains the Cross-Section of Returns

In this section, I use SDF moments computed under CRRA preference with $\alpha = 2$ and examine the cross-sectional relation between the SDF moments and returns by using the Fama and MacBeth (1973) two-pass cross-sectional methodology. My focus is on daily returns since conditional moments are computed at daily frequency. To do so, I consider portfolio sets available on the Kenneth French website. I first use the 100 Fama and French portfolios formed on size and book-to-market. Second, I use the 100 Fama and French portfolios formed on size and operating profitability. Third, in the Internet Appendix, I use the 25 Fama and French portfolios formed on size and book-to-market. All returns under consideration are daily returns since conditional moments are computed at daily frequency. The sample period corresponds to the sample period used to compute the SDF moments (i.e., from January 1996 to August 2015).

I present the time-series averages of the slope coefficients from the regressions of portfolio returns on the SDF variance, skewness, and kurtosis. I use two sets of control variables. First, I use the Fama and French (2016) factors: MKT, SMB, HML, RMW, and CMA. MKT is the market excess return. SMB is the average return on nine small stock portfolios minus the average return on nine big stock portfolios. HML is the average return on two value portfolios minus the average return on two growth portfolios. RMW is the average return on two robust operating profitability portfolios minus the average return on two weak operating profitability portfolios. CMA is the average return on two conservative investment portfolios minus the average return on two aggressive investment portfolios. See Fama and French (2016) for a complete description of how the factors are constructed.

Second, since SDF moments are computed from option prices, I use well-know control variables computed using option prices: Risk neutral moments of the market returns, variance risk premium (VRP), and Left Risk-Neutral Jump Variation measures (LJV). The average slopes provide standard Fama-MacBeth tests for determining which explanatory variables, on average, have non-zero price of risk.

6.1. SDF Conditional Moments Explain the Cross-Section of Returns

I use the following specification to estimate the price of risks. I have omitted the time subscript for simplicity:

$$\begin{aligned} E[R_i] - R_f &= \lambda_0 + \lambda_{MKT}\beta^i_{MKT} + \lambda_{\mathcal{M}^{(2)}}\beta^i_{\mathcal{M}^{(2)}} + \lambda_{\mathcal{M}^{(3)}}\beta^i_{\mathcal{M}^{(3)}} + \lambda_{\mathcal{M}^{(4)}}\beta^i_{\mathcal{M}^{(4)}} \\ &\quad + \lambda_{SMB}\beta^i_{SMB} + \lambda_{HML}\beta^i_{HML} + \lambda_{RMW}\beta^i_{RMW} + \lambda_{CMA}\beta^i_{CMA}. \end{aligned} \tag{40}$$

Results 1-5 suggest that, in theory, the price of the second moment $\lambda_{\mathcal{M}^{(2)}}$ is positive, the price of the third moment $\lambda_{\mathcal{M}^{(3)}}$ is negative, and the price of the fourth moment $\lambda_{\mathcal{M}^{(4)}}$ is positive. Below, I use various sorted portfolios to assess the significance of the price of risk of SDF moments. I also investigate whether the sign of the price of risks are consistent with the theoretical results in Results 1–5 when investors have CRRA utility.

6.1.1. Results for 100 Portfolios Formed on Size and Book-to-Market

Table 12 presents the estimation results of the beta pricing model (40). The estimates of the price of SDF variance risk, $\lambda_{\mathcal{M}^{(2)}}$, are all positive and significant regardless to the t-ratio used. The price of SDF variance risk varies from 0.531 at 30-day maturity to 0.095 at 365-day maturity.

I report the Fama and MacBeth (1973) t-ratio under correctly specified models (t_{FM}), the Shanken (1992) t-ratio (t_S), the Jagannathan and Wang (1998) t-ratio under correctly specified models that account for the EIV problem (t_{JW}), and the Kan, Robotti, and Shanken (2013) misspecification-robust t-ratios (t_{KRS}). The table also presents the sample cross-sectional R^2 of the beta pricing model (40), the p-value for the test of H0 : $R^2 = 1$ (labeled p($R^2 = 1$)), the p-value for the test of H0 : $R^2 = 0$ (labeled p($R^2 = 0$)), the p-value of Wald test under the null hypothesis that all prices of risk are equal to zero (p(W)), and the standard error of $\widehat{R}^2$ under the assumption that $0 < R^2 < 1$ (labeled se($\widehat{R}^2$)).

The estimates of the price of SDF skewness risk, $\lambda_{\mathcal{M}^{(3)}}$, are all statistically significant and negative at all maturities. This is consistent with the theoretical predictions in Results 1–5. The price of risk varies from -0.275 at 30-day maturity to -0.085 at 365-day maturity. Also, the estimates of the price of SDF kurtosis risk, $\lambda_{\mathcal{M}^{(4)}}$, are all significant. The price of risks are all positive except for that of 30-day maturity. The price of risk of the Fama and French factors, SMB, HML, and CMA is not significant at all maturities. In contrast, the price of the RMW factor is positive and statistically significant. The adjusted R^2 ranges from 66.5% (at 30-day maturity) to 81.8% (at 365-day maturity). When I use only the Fama and French 5 factors, the adjusted R^2 is 1.1%. This also shows a significant improvement of the beta pricing model (40) over the Fama and French five factor model.

6.1.2. Results for 100 Portfolios Formed on Size and Operating Profitability

Table 13 presents the estimation results of the beta pricing model (40). Regardless of the t-ratio used to gauge the significance of the price of risks, the estimates of the price of SDF volatility risk, $\lambda_{\mathcal{M}^{(2)}}$ are all significant. At short maturities (from 30 days to 91 days), the price of the SDF volatility is positive. It is negative for maturities ranging from 122 days to 273 days. It becomes again positive at 365-day maturity. At short maturities, stocks with high exposure to the SDF volatility earn high return on average.

The estimates of the price of SDF skewness risk, $\lambda_{\mathcal{M}^{(3)}}$, are all statistically significant and positive at all maturities. The price of risks vary from 0.659 at 30-day maturity to 0.029 at 365-day maturity. Regardless of option maturity used, stocks with high exposure to the SDF skewness earn high return on average. The estimates of the price of SDF kurtosis risk, $\lambda_{\mathcal{M}^{(4)}}$, are all statistically significant and negative at all maturities. The price of risks vary from -0.773 at 30-day maturity to -0.008 at 365-day maturity. This features, in absolute value, a decreasing term structure of the price of SDF kurtosis risk. The price of risk of recent commonly used factors SMB, HML, RMW, and CMA is not significant at all maturities. Regarding the pricing performance of the beta pricing model (40), the adjusted R^2 ranges from 31.8% (at 30-day maturity) to 62.5% (at 365-day maturity). In contrast, when I use only the five Fama and French factors, the adjusted R^2 is 2.5%. This shows a significant improvement of the beta pricing model (40) over the Fama and French five factor model. Although, the price of risks are significant, their signs are not often consistent with the theoretical predictions in Results 1–5.

6.2. Conditional SDF-Based Moments Premium Explain the Cross-Section of Returns

Section 6.1.2 provides empirical evidence that SDF moments are priced in the cross-section of returns. While SDF moments are important in explaining expected excess returns, one may ask whether SDF moments risk premium also explain the cross-section of returns. Section 4.5 provides ample evidence that the SDF variance risk premium is often negative while the SDF skewness (kurtosis) risk premium is often positive. Thus, provided that investors receive compensation by investing in stocks with exposure to SDF moment risks, exposure to SDF moments risk premium positively contributes to the expected excess return on stocks if the price of the SDF variance risk premium is negative while the price of the SDF skewness (kurtosis) risk premium is positive. To empirically verify this argument, I estimate the price of risks by using the SDF-based moments risk premium in the following specification:

$$E[R_i] - R_f = \lambda_0 + \lambda_{MKT}\beta^i_{MKT} + \lambda_{\mathcal{MRP}^{(2)}}\beta^i_{\mathcal{MRP}^{(2)}} + \lambda_{\mathcal{MRP}^{(3)}}\beta^i_{\mathcal{MRP}^{(3)}} + \lambda_{\mathcal{MRP}^{(4)}}\beta^i_{\mathcal{MRP}^{(4)}} + \lambda_{SMB}\beta^i_{SMB} + \lambda_{HML}\beta^i_{HML} + \lambda_{RMW}\beta^i_{RMW} + \lambda_{CMA}\beta^i_{CMA}. \quad (41)$$

6.2.1. Results for 100 Portfolios Formed on Size and Book-to-Market

Table 14 displays the estimation results of the beta pricing model (41). The estimates of the price of the SDF second moment risk premium, $\lambda_{\mathcal{MRP}^{(2)}}$, are all significant, regardless of the t-ratio used. The price of risks are all positive except for the 30-day horizon. The estimates of the price of SDF third moment risk premium, $\lambda_{\mathcal{MRP}^{(3)}}$, are positive and statistically significant at all horizons. The price of risk decreases from 0.699 (at 30-day horizon) to 0.012 (at 365-day horizon). Regardless of the t-ratio used, the estimates of the price of SDF fourth moment risk premium $\lambda_{\mathcal{MRP}^{(4)}}$ are significant from 30-day horizon to 122-day horizon. The sign of the price of risk is negative at the 30-day and 60-day horizon and positive for the remaining maturities.

The price of risk of commonly used factors SMB, HML, and CMA is not significant at all maturities. However, RMW is marginally significant when the Shanken (1992) t-ratio is used. Regarding the pricing performance of the beta pricing model (41), the adjusted R^2 ranges from 44.6% (at 30-day maturity) to 81.6% (at 365-day maturity). When I use only the five Fama and French, the adjusted R^2 is 1.09%. This shows a significant improvement of the beta pricing model (41) over the Fama and French five factor model.

6.2.2. Results for 100 Portfolios Formed on Size and Operating Profitability

Table 15 presents the estimation results of the beta pricing model (41). The estimates of the price of the SDF second moment risk premium, $\lambda_{\mathcal{MRP}^{(2)}}$, are negative and significant at all horizons. At a short horizon (30 days), the price is -1.09. It decreases (in absolute value) when the maturity increases and reaches -0.019 at the 365-day horizon.

The estimates of the price of SDF third moment risk premium, $\lambda_{\mathcal{MRP}^{(3)}}$, are statistically significant. There are positive at short horizons and close to zero for horizons that are in the range of 152 days to 365 days. At the 30-day horizon, the price is 0.171. At the 365-day horizon, the price is 0.002. The estimates of the price of SDF fourth moment risk premium, $\lambda_{\mathcal{MRP}^{(4)}}$, are all statistically significant and negative at all maturities except for the 365-day to maturity. The price decreases in absolute value from -1.430 at 30-day to -0.017 at the 273-day and reaches 0.002 at the 365-day horizon.

The price of risk of commonly used factors SMB, HML, RMW, and CMA is not significant at all maturities. Regarding the pricing performance of the beta pricing model (41), the R^2 ranges from 27.4% (at 30-day maturity) to 61.9% (at 365-day maturity). In contrast, when I use only the Fama and French five factors, the adjusted R^2, 2.48%, shows a significant improvement of the beta pricing model (41) over the Fama and French five factor model.

6.3. Controlling for Variance Risk Premium, Left Risk-Neutral Jump Variation, and Risk-Neutral Moments of the Market Return

Sections 6.1 and 6.2 provide evidence that SDF moments and SDF moment risk premium are priced in the cross-section of returns after controlling for well-known stock characteristics. Since SDF moments are computed using option prices, one may ask whether the price of SDF moments are still significant after controlling for known measures such as variance risk premium (VRP), Left Risk-Neutral Jump Variation (LJV), and risk-neutral moments of the market return. I use the 100 portfolios formed on size and book-to-market to estimate the price of SDF moments risk. In this section, I use monthly returns since VRP and Left Risk-Neutral Jump Variation (LJV) are available at monthly frequency. Market return risk-neutral moments are computed using closed-form expressions that are available in Appendix C of the Internet Appendix. Monthly estimates of SDF moments and market risk-neutral moments are obtained by averaging within a month, daily estimates of SDF moments. I first estimate the price of SDF moments risk by running the specification (42)

$$E\left[R_i\right]-R_f \quad = \quad \lambda_0+\lambda_{\mathcal{M}^{(2)}}\beta^i_{\mathcal{M}^{(2)}}+\lambda_{\mathcal{M}^{(3)}}\beta^i_{\mathcal{M}^{(3)}}+\lambda_{\mathcal{M}^{(4)}}\beta^i_{\mathcal{M}^{(4)}}+\lambda_{VRP}\beta^i_{VRP}. \qquad (42)$$

Results are reported in Table 16. The price of SDF variance, SDF skewness, and SDF kurtosis are significant and consistent with the theoretical predictions in Results 1–5 after controlling for the market variance risk premium. The price of the SDF variance (kurtosis) is positive, while the price of the SDF skewness is negative. Note that the price of the market variance risk premium is not significant. Next, I control for the LJV measure by running the specification

$$E[R_i] - R_f = \lambda_0 + \lambda_{\mathcal{M}^{(2)}} \beta^i_{\mathcal{M}^{(2)}} + \lambda_{\mathcal{M}^{(3)}} \beta^i_{\mathcal{M}^{(3)}} + \lambda_{\mathcal{M}^{(4)}} \beta^i_{\mathcal{M}^{(4)}} + \lambda_{LJV} \beta^i_{LJV} \quad (43)$$

and report the estimates of the prices of risk in Table 17. The price of the SDF moments are significant and also consistent with theoretical predictions in Results 1-5. I further estimate the price of risk by controlling for innovations in the market risk-neutral variance, skewness and kurtosis by running specifications (44)–(46)

$$E[R_i] - R_f = \lambda_0 + \lambda_{\mathcal{M}^{(2)}} \beta^i_{\mathcal{M}^{(2)}} + \lambda_{\mathcal{M}^{(3)}} \beta^i_{\mathcal{M}^{(3)}} + \lambda_{\mathcal{M}^{(4)}} \beta^i_{\mathcal{M}^{(4)}} + \lambda_{\Delta VAR} \beta^i_{\Delta VAR}. \quad (44)$$

$$E[R_i] - R_f = \lambda_0 + \lambda_{\mathcal{M}^{(2)}} \beta^i_{\mathcal{M}^{(2)}} + \lambda_{\mathcal{M}^{(3)}} \beta^i_{\mathcal{M}^{(3)}} + \lambda_{\mathcal{M}^{(4)}} \beta^i_{\mathcal{M}^{(4)}} + \lambda_{\Delta SKEW} \beta^i_{\Delta SKEW}. \quad (45)$$

$$E[R_i] - R_f = \lambda_0 + \lambda_{\mathcal{M}^{(2)}} \beta^i_{\mathcal{M}^{(2)}} + \lambda_{\mathcal{M}^{(3)}} \beta^i_{\mathcal{M}^{(3)}} + \lambda_{\mathcal{M}^{(4)}} \beta^i_{\mathcal{M}^{(4)}} + \lambda_{\Delta KURT} \beta^i_{\Delta KURT}. \quad (46)$$

Results are displayed in Tables 18, 19, and 20 respectively. The price of SDF moments risk are significant while the price of risks of innovations in the market risk-neutral variance, market risk-neutral skewness, and market risk-neutral kurtosis are not significant.

7. Inferring the Price of Risk Using the Three-Pass Regression Approach

In this section, I use the three-pass regression approach of Giglio and Xiu (2017) to estimate the price of SDF moments. In contrast to existing approaches, their approach allows inferring the price of risk factors in a linear asset-pricing model, when the number of assets is large. They argue that standard methods to estimate risk premia are biased in the presence of omitted priced factors correlated with the observed factors. Their methodology accounts for potential measurement error in the observed factors and detects when observed factors are spurious or even useless. I use a large set of standard portfolios of U.S. equities. It includes 460 portfolios at daily frequency: 100 portfolios sorted by size and book-to-market ratio, 100 portfolios sorted by size and profitability, 100 portfolios sorted by size and investment, 25 portfolios sorted by size and short-term reversal, 25 portfolios sorted by size and long-term reversal, 25 portfolios sorted by size and momentum, 25 portfolios sorted by profitability and investment, 25 portfolios sorted by book and investment, 25 portfolios sorted by book-to-market and profitability, and 10 industry portfolios. See Kenneth French's website.

I use SDF moments computed under CRRA preference with $\alpha = 2$ and first estimate the price of risks of the beta pricing model (40). Table 21 presents the estimates of the price of risk for each factor. The Giglio and Xiu (2017) t-ratios are reported. The price of the variance, skewness, and kurtosis of the SDF is positive and statistically significant at any horizon with t-ratios higher than three. Further, the price of risk of all factors increases with the maturity. This features an upward sloping term structure of the price of risk of the SDF moments. I further estimate the price of SDF moments risk, in presence of the Hou, Xue, and Zhang (2015) factors by running the specification[9]

$$\begin{aligned} E\left[R_i\right] - R_f &= \lambda_0 + \lambda_{MKT}\beta^i_{MKT} + \lambda_{\mathcal{M}^{(2)}}\beta^i_{\mathcal{M}^{(2)}} + \lambda_{\mathcal{M}^{(3)}}\beta^i_{\mathcal{M}^{(3)}} + \lambda_{\mathcal{M}^{(4)}}\beta^i_{\mathcal{M}^{(4)}} \\ &\quad + \lambda_{ME}\beta^i_{ME} + \lambda_{I/A}\beta^i_{I/A} + \lambda_{ROE}\beta^i_{ROE}, \end{aligned} \tag{47}$$

where ME is the difference between the return on a portfolio of small size stocks and the return on a portfolio of big size stocks, I/A is the difference between the return on a portfolio of low investment stocks and the return on a portfolio of high investment stocks, and ROE is the difference between the return on a portfolio of high profitability stocks and the return on a portfolio of low profitability stocks. Table 22 reports the results. The prices of the SDF variance, SDF skewness, and SDF kurtosis are all significant and positive, with a t-ratio above three. All prices of risk increase with the option maturity used. This features an upward term structure of the price of the SDF moments. Second, I estimate the price of risks of the beta pricing model (41). Table 23 presents the estimates of the price of risk for each factor. The prices of risks of the SDF variance premium are negative and highly significant at all maturities with t-ratios higher than three. Note that the SDF variance premium, as opposed to the SDF variance, defined in (13) is often negative. See the summary statistics of both the physical and risk-neutral variances of the SDF in Tables 1 and 2, respectively. In absolute value, the price of risks are upward sloping. The prices of risks of the SDF skewness premium and SDF kurtosis premium factors are all positive, significant (with a t-stat above three) and increase with the investment horizon, featuring an upward slopping term structure of the price of risks. Note that the summary statistics of both the physical and risk-neutral skewness of the SDF in Tables

[9]I am grateful to Kewei Hou and Lu Zhang for providing the ME, I/A, and ROE factors.

1 and 2 indicate that the SDF-based skewness risk premium is often positive. Similarly, Table 3 shows that the SDF-based kurtosis risk premium is often positive as well. The Fama and French (2016) factors SMB, HML, and RMW are not significant, while the CMA factor is marginally significant with a t-ratio of approximately 1.74.

Overall the sign of the price of the SDF variance and kurtosis are consistent with the predictions of Results 1–5 while the price of the SDF skewness is not consistent with Results 1–5. Note that, I obtain a similar finding with the predictive regressions in Section 5.

I further estimate the price of risk by running specification

$$E[R_i] - R_f = \lambda_0 + \lambda_{MKT}\beta^i_{MKT} + \lambda_{\mathcal{MRP}^{(2)}}\beta^i_{\mathcal{MRP}^{(2)}} + \lambda_{\mathcal{MRP}^{(3)}}\beta^i_{\mathcal{MRP}^{(3)}} + \lambda_{\mathcal{MRP}^{(4)}}\beta^i_{\mathcal{MRP}^{(4)}} + \lambda_{ME}\beta^i_{ME} + \lambda_{I/A}\beta^i_{I/A} + \lambda_{ROE}\beta^i_{ROE}. \quad (48)$$

Results are presented in Table 24. The estimates of the price of the SDF moment risk premium are significant at all horizons and are comparable to those obtained in Table 23.

8. Conclusion

I investigate whether the real-time distribution of the SDF contains a rich set of information that helps understand risk and return in the stock market. First, I theoretically derive closed-form expressions of the conditional physical moments and the conditional risk-neutral moments of the SDF under CRRA preferences and also under preferences that depart from CRRA preferences. The conditional moments can be recovered in real time from a cross-section of option prices, provided that the risk aversion is known. I further use the moments of the SDF to derive conditional expected excess return and conditional Sharpe ratios of SDF-based hedging strategies that yield returns that are positively correlated with the SDF.

Second, I empirically estimate the conditional moments of the SDF. For maturities in range of one month up to one year, the conditional moments of the SDF are time-varying and highly volatile, and exhibit fat tails. The magnitude of the conditional expected excess return and conditional Sharpe ratios of SDF-based hedging strategies varies significantly over time and is economically large in crisis periods. I show that the conditional moments of the SDF strongly predict the excess market return in sample and out-of-sample when the maturity of options used varies from four months to twelve months.

I further investigate the implications of the real-time conditional distribution of the SDF for the cross-section of returns. The theory suggests that stocks with high sensitivity to the SDF variance (kurtosis) exhibit on average high return, while stocks with high sensitivity to the SDF skewness exhibit on average low return. To verify this empirically, I first use the two-pass cross-sectional methodology to infer the price of the SDF variance, SDF skewness, and SDF kurtosis. I use various Fama and French sorted portfolios based on characteristics, and show that SDF moments are priced in the cross-section of returns after controlling for the Fama and French five factor model, risk neutral moments of the market return, variance risk premium, and the Left Risk-Neutral Variation measures. The price of SDF variance, SDF skewness, and SDF kurtosis risks are highly significant with t-ratios that are often above three. I find that the price of the SDF variance (kurtosis) is often positive when the two-pass methodology is applied. This shows that stocks with high exposure to SDF variance exhibit high returns on average. Consistent with the theoretical predictions, I also find that the price of the SDF skewness is negative. In contrast, when I use portfolios formed on size and operating profitability, the sign of the SDF moments risks are not often consistent with the theory. I further show that the SDF-based moment risk premium, defined as the difference between the physical and risk-neutral moments of the SDF, are priced in the cross-section of returns.

Finally, I use the three-pass cross-sectional regression methodology to infer the price of SDF moments from a large set of standard portfolios of U.S. equities and find that the prices of SDF variance, skewness, and kurtosis are all positive. While the price of the SDF variance (kurtosis) is consistent with the theory, the price of the SDF skewness is not consistent with the theoretical predictions.

References

Aït-Sahalia, Y., Lo, A., 2000. Nonparametric risk management and implied risk aversion. Journal of Econometrics 94, 9–51.

Almeida, C., Garcia, R., 2017. Economic implications of nonlinear pricing kernels. Management Science 63, 3361–3380.

Amaya, D., Christoffersen, P., Jacobs, K., Vasquez, A., 2015. Does realized skewness predict the cross-section of equity returns?. Journal of Financial Economics 118, 135–167.

An, B.-J., Bali, T., Ang, A., Cakici, N., 2014. The joint cross section of stocks and options. Journal of Finance 69, 2179–2337.

Bakshi, G., Kapadia, N., Madan, D., 2003. Stock return characteristics, skew laws, and the differential pricing of individual equity options. Review of Financial Studies 16, 101–143.

Bali, T., Cakici, N., Whitelaw, R., 2011. Maxing out: Stocks as lotteries and the cross-section of expected returns. Journal of Financial Economics 99, 427–446.

Bali, T. G., Murray, S., 2013. Does risk-neutral skewness predict the cross-section of equity option portfolios returns?. Journal of Financial and Quantitative Analysis 48, 1145–1171.

Bekaert, G., Liu, J., 2004. Conditioning information and variance bounds on pricing kernels. Review of Financial Studies 17, 339–378.

Bliss, R. R., Panigirtzoglou, N., 2004. Option-implied risk aversion estimates. Journal of Finance 59, 407–446.

Bollerslev, T., Tauchen, G., Zhou, H., 2009. Expected stock returns and variance risk premia. Review of Financial Studies 22, 4463–4493.

Bollerslev, T., Todorov, V., 2011. Tails, fears and risk premia. Journal of Finance 66, 2165–2211.

Bollerslev, T., Todorov, V., Xu, L., 2015. Tail risk premia and return predictability. Journal of Financial Economics 118, 113–134.

Borovicka, J., Hansen, L. P., Scheinkman, J. A., 2016. Misspecified recovery. Journal of Finance 71, 2493–2544.

Campbell, J., Thompson, S., 2008. Predicting excess stock returns out of sample: Can anything beat the historical average?. Review of Financial Studies 21, 1509–1531.

Carr, P., Madan, D., 2001. Optimal positioning in derivative securities. Quantitative Finance 1, 19–37.

Chang, B. Y., Christoffersen, P., Jacobs, K., 2013. Market skewness risk and the cross section of stock returns. Journal of Financial Economics 107, 46–68.

Christoffersen, P., Fournier, M., Jacobs, K., Karoui, M., 2017. Option-based estimation of the price of co-skewness and co-kurtosis risk. Working paper. Rotman School of Management.

Deck, C., Schlesinger, H., 2014. Consistency of higher order risk preferences. Econometrica 82, 1913–1943.

Dittmar, R., 2002. Nonlinear pricing kernels, kurtosis preference, and evidence from the cross section of equity returns. Journal of Finance 57, 369–403.

Eeckhoudt, L., Schlesinger, H., 2006. Putting risk in its proper place. American Economic Review 96, 280–289.

Epstein, L., Zin, S., 1989. Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework. Econometrica 57, 937–969.

Fama, E., MacBeth, J., 1973. Risk, return, and equilibrium: Empirical tests. Journal of Political Economy 81, 607–636.

Fama, E. F., French, K., 2016. Dissecting anomalies with a five-factor model. Review of Financial Studies, forthcoming.

Friend, I., Blume, M., 1975. The demand for risky assets. American Economic Review 65(5), 900–922.

Giglio, S., Xiu, D., 2017. Inference on risk premia in the presence of omitted factors. Working paper, Booth School of Business.

Gormsen, N. J., Jensen, C. S., 2018. High moment risk. Working Paper.

Goyal, A., Welch, I., 2008. A comprehensive look at the empirical performance of equity premium prediction. Review of Financial Studies 21, 1455–1508.

Hall, R. E., 1978. Stochastic implications of the life cycle permanent income hypothesis: Theory and evidence. Journal of Political Economy 86, 971–987.

Hansen, L., Hodrick, R., 1980. Forward exchange rates as optimal predictors of future spot rates: An econometric analysis. Journal of Political Economy 88, 829–853.

Hansen, L., Jagannathan, R., 1991. Implications of security market data for dynamic economies. Journal of Political Economy 99, 225–261.

Harvey, C., Siddique, A., 2000. Conditional skewness in asset pricing tests. Journal of Finance 55, 1263–1295.

Hou, K., Xue, C., Zhang, L., 2015. Digesting anomalies: An investment approach. Review of Financial Studies 28, 650–705.

Jackwerth, J., 2000. Recovering risk aversion from option prices and realized returns. Review of Financial Studies 13, 433–451.

Jagannathan, R., Wang, Z., 1998. An asymptotic theory for estimating beta-pricing models using cross-sectional regression. Journal of Finance 53, 1285–1309.

Kadan, O., Tang, X., 2018. A bound on expected stock returns. Working Paper, University of University in St Louis.

Kan, R., Robotti, C., Shanken, J., 2013. Pricing model performance and the two-pass cross-sectional regression methodology. Journal of Finance 6, 2617–2649.

Kelly, B., Jiang, H., 2014. Tail risk and asset prices. Review of Financial Studies 27, 2841–2871.

Martin, I., 2017. What is the expected return of the market?. Quarterly Journal of Economics 132, 367–433.

Martin, I., Wagner, C., 2016. What is the expected return on a stock?. Working Paper.

Nagel, S., Singleton, K., 2011. Estimation and evaluation of conditional asset pricing models. Journal of Finance 66, 873–909.

Noussair, C. N., Trautmann, S. T., VanDeKuilen, G., 2014. Higher order risk attitudes, demographics, and financial decisions. Review of Economic Studies 81, 325–355.

Rosenberg, J., Engle, R., 2002. Empirical pricing kernels. Journal of Financial Economics 64, 341–372.

Ross, S., 2015. The recovery theorem. Journal of Finance 70, 615–648.

Schneider, P., Trojani, F., 2015. Fear trading. Working paper. University of Lugano.

Schneider, P., Trojani, F., 2017a. (almost) model-free recovery. Journal of Finance, forthcoming.

Schneider, P., Trojani, F., 2017b. Divergence and the price of risk. Journal of Financial Econometrics, forthcoming.

Shanken, J., 1992. On the estimation of beta-pricing models. Review of Financial Studies 5, 1–33.

Snow, K., 1991. Diagnosing asset pricing models using the distribution of asset returns. Journal of Finance 46, 955–983.

A. Appendix

Proof of Result 1. Denote by $\upsilon = u^{'-1}$ the inverse of $u^{'}$. Since υ is the inverse function of u, $\upsilon[m] = R_M$. The third-order Taylor expansion series of $\upsilon[m]$ around $\overline{m} = \mathbb{E}(m)$ produces

$$R_M = \upsilon[m] = \upsilon[\overline{m}] + \sum_{k=1}^{3} \frac{1}{k!}(m-\overline{m})^k \left\{ \frac{\partial^k \upsilon[y]}{\partial^k y} \right\}_{y=\overline{m}}. \tag{A1}$$

The expected excess market return is, therefore,

$$\mathbb{E}(R_M - R_f) = -R_f \mathbb{COV}(m, R_M). \tag{A2}$$

Thus, I replace (A1) in (A2) and show that the expected excess market return is given by

$$\mathbb{E}(R_M - R_f) = A_1 \mathcal{M}^{(2)} + A_2 \mathcal{M}^{(3)} + A_3 \mathcal{M}^{(4)}, \tag{A3}$$

where

$$A_k = -R_f \frac{1}{k!} \left\{ \frac{\partial^k \upsilon[y]}{\partial^k y} \right\}_{y=\overline{m}}. \tag{A4}$$

To derive the first-, second- and third-order derivative of υ, note that

$$\upsilon\left[u^{'}[x]\right] = x. \tag{A5}$$

The first-order derivative of (A5) with respect to x is

$$\frac{\partial \upsilon\left[u^{'}[x]\right]}{\partial(u^{'}[x])} \frac{\partial\left(u^{'}[x]\right)}{\partial x} = 1,$$

which simplifies to

$$\frac{\partial \upsilon\left[u^{'}[x]\right]}{\partial(u^{'}[x])} u^{''}[x] = 1.$$

Hence, denoting $y = u^{'}[x]$, it follows that

$$\frac{\partial \upsilon[y]}{\partial y} = \frac{1}{u^{''}[\upsilon[y]]} < 0. \tag{A6}$$

Now, take, the first derivative of (A6) with respect to y:

$$\frac{\partial^2 \upsilon[y]}{\partial^2 y} = -\frac{u'''(\upsilon[y])}{(u''(\upsilon[y]))^3} > 0. \tag{A7}$$

Next, take the first derivative of (A7) with respect to y:

$$\frac{\partial^3 \upsilon[y]}{\partial^3 y} = \frac{3\left(u'''[\upsilon[y]]\right)^2 - u''''(\upsilon[y])\,u''[\upsilon[y]]}{(u''[\upsilon[y]])^5}.$$

The absolute prudence is defined as

$$-\frac{u'''[x]}{u''[x]},$$

A decreasing absolute prudence implies

$$\frac{\left(u'''[x]\right)^2 - u''''[x]\,u''[x]}{(u''[x])^2} \leq 0,$$

Observe that

$$\begin{aligned}\frac{\partial^3 \upsilon[y]}{\partial^3 y} &= \frac{3\left(u'''[\upsilon[y]]\right)^2 - u''''[\upsilon[y]]\,u''[\upsilon[y]]}{(u''[\upsilon[y]])^5} \\ &= \underbrace{\frac{2\left(u'''[\upsilon[y]]\right)^2}{(u''[\upsilon[y]])^5}}_{\leq 0} - \underbrace{\frac{1}{(u''[\upsilon[y]])^3}}_{\leq 0}\underbrace{\frac{\left(u''''[\upsilon[y]]\,u''[\upsilon[y]] - \left(u'''[\upsilon[y]]\right)^2\right)}{(u''[\upsilon[y]])^2}}_{\leq 0}.\end{aligned}$$

Hence,

$$\frac{\partial^3 \upsilon[y]}{\partial^3 y} \leq 0,$$

■ **Proof of Result 2.** The expected excess return on an individual asset is

$$\mathbb{E}_t(R_i - R_f) = -R_f \mathbb{COV}(R_i, m). \tag{A8}$$

A third-order Taylor expansion series of the SDF, $m = u^{'}[R_M]$, around $\mathbb{E}(R_M)$ gives

$$m = B_0 + B_1 R_M + B_2 R_M^2 + B_3 R_M^3.$$

Next, I replace the SDF in (A8) and decompose the expected excess return as

$$\mathbb{E}(R_i - R_f) = \widetilde{B}_1 \mathbb{COV}(R_i, R_M) + \widetilde{B}_2 \mathbb{COV}\left(R_i, R_M^2\right) + \widetilde{B}_3 \mathbb{COV}\left(R_i, R_M^3\right), \tag{A9}$$

where $\widetilde{B}_i = -R_f B_i$ for $i = 1$, 2, and 3. This equation holds for any return. Consider the following return:

$$R_M^{(j)} = \frac{R_M^j}{\mathbb{E}_t\left(m R_M^j\right)} \text{ for } j = 2,\ 3 \text{ and } 4. \tag{A10}$$

Observe that

$$R_M^{(j)} = \frac{R_M^j}{\left(\mathbb{E}(m)\right)\mathbb{E}\left(\frac{m}{\mathbb{E}(m)} R_M^j\right)} = R_f \frac{R_M^j}{\mathbb{E}^*\left(R_M^j\right)}. \tag{A11}$$

Thus,

$$R_M^{(j)} - R_f = R_f \left(\frac{R_M^j}{\mathbb{E}^*\left(R_M^j\right)} - 1 \right). \tag{A12}$$

Next, the expected value of (A12) when $j = 2, 3$ and 4 produces the following:

$$\mathbb{E}\left(R_M^{(j)} - R_f\right) = R_f \frac{\left(\mathbb{E}\left(R_M^j\right) - \mathbb{E}^*\left(R_M^j\right)\right)}{\mathbb{E}^*\left(R_M^j\right)}. \tag{A13}$$

Recall that $R_M^j = (\upsilon[m])^j$. Thus, the Taylor expansion series of this expression around the SDF mean $m = \overline{m}$ gives

$$(\upsilon[m])^j \simeq (\upsilon[\overline{m}])^j + \phi_1^{(j)}(m - \overline{m}) + \phi_2^{(j)}(m - \overline{m})^2 + \phi_3^{(j)}(m - \overline{m})^3, \tag{A14}$$

where $\phi_1^{(j)}$, $\phi_2^{(j)}$, and $\phi_3^{(j)}$ are defined in (7). Now, I apply the expectation operator under the physical measure to (A14):

$$\mathbb{E}\left[(\upsilon[m])^j\right] \simeq (\upsilon[\overline{m}])^j + \phi_2^{(j)}\mathbb{E}\left((m - \overline{m})^2\right) + \phi_3^{(j)}\mathbb{E}\left((m - \overline{m})^3\right). \tag{A15}$$

Second, I apply the expectation operator under the risk-neutral measure to (A14):

$$\mathbb{E}^*\left((\upsilon[m])^j\right) \simeq (\upsilon[\overline{m}])^j + \phi_1^{(j)}\mathbb{E}^*((m-\overline{m})) + \phi_2^{(j)}\mathbb{E}^*\left((m-\overline{m})^2\right) + \phi_3^{(j)}\mathbb{E}^*\left((m-\overline{m})^3\right). \tag{A16}$$

Since $R_M = \upsilon[m]$, the difference between (A15) and (A16) yields

$$\begin{aligned}\mathbb{E}\left(R_M^j\right) - \mathbb{E}^*\left(R_M^j\right) \quad = \quad & \phi_1^{(j)}\left(\mathbb{E}((m-\overline{m})) - \mathbb{E}^*((m-\overline{m}))\right) \\ & +\phi_2^{(j)}\left(\mathbb{E}\left((m-\overline{m})^2\right) - \mathbb{E}^*\left((m-\overline{m})^2\right)\right) \\ & +\phi_3^{(j)}\left(\mathbb{E}_t\left((m-\overline{m})^2\right) - \mathbb{E}^*\left((m-\overline{m})^2\right)\right).\end{aligned} \tag{A17}$$

According to Result 1,

$$\left(\mathbb{E}((m-\overline{m})) - \mathbb{E}^*(m-\overline{m})\right) \quad = \quad -\frac{1}{\overline{m}}\mathcal{M}^{(2)}, \tag{A18}$$

$$\left(\mathbb{E}\left((m-\overline{m})^2\right) - \mathbb{E}^*\left((m-\overline{m})^2\right)\right) \quad = \quad -\frac{1}{\overline{m}_t}\mathcal{M}^{(3)}, \text{ and} \tag{A19}$$

$$\left(\mathbb{E}\left((m-\overline{m})^3\right) - \mathbb{E}^*\left((m-\overline{m})^3\right)\right) \quad = \quad -\frac{1}{\overline{m}}\mathcal{M}^{(4)}. \tag{A20}$$

I then replace (A18)-(A20) in (A17):

$$\mathbb{E}^*\left(R_M^j\right) - \mathbb{E}\left(R_M^j\right) = \phi_1^{(j)}\frac{1}{\overline{m}}\mathcal{M}^{(2)} + \phi_2^{(j)}\frac{1}{\overline{m}}\mathcal{M}^{(3)} + \phi_3^{(j)}\frac{1}{\overline{m}}\mathcal{M}^{(4)}. \tag{A21}$$

This ends the proof. ■

Proof of Result 3. I observe that moments of the SDF $\mathcal{M}^{(i)}$ for $i = 2,3,4$ can be expressed as

$$\mathcal{M}^{(i)} = \mathbb{COV}\left(m, (m-\overline{m})^{i-1}\right),$$

where $\overline{m}$ is the mean of the SDF. Define

$$\psi^{(i)}[x] = \left(u^{'}[x] - \overline{m}\right)^{i-1}.$$

Since $m = u^{'}(R_M)$, it follows that

$$\mathcal{M}^{(i)} = \mathbb{COV}\left(m, \psi^{(i)}[R_M]\right). \tag{A22}$$

The third-order Taylor expansion series of $\psi^i[x]$ around $x_0 = \mathbb{E}(R_M)$ produces the following:

$$\psi^{(i)}[x] = \psi^{(i)}[x_0] + \frac{1}{1!}\psi_1^{(i)}(x - x_0) + \frac{1}{2!}\psi_2^{(i)}(x - x_0)^2 + \frac{1}{3!}\psi_3^{(i)}(x - x_0)^3, \tag{A23}$$

with

$$\psi_1^{(i)} = \left\{\frac{\partial\psi^{(i)}[x]}{\partial x}\right\}_{x=x_0}, \psi_2^{(i)} = \left\{\frac{\partial^2\psi^{(i)}[x]}{\partial^2 x}\right\}_{x=x_0}, \psi_3^{(i)} = \left\{\frac{\partial^3\psi^{(i)}[x]}{\partial^3 x}\right\}_{x=x_0}.$$

Together, (A23) and (A22) produce

$$\begin{aligned} \mathcal{M}^{(i)} &= \mathbb{COV}\left(m, \frac{1}{1!}\psi_1^{(i)}(x - x_0) + \frac{1}{2!}\psi_2^{(i)}(x - x_0)^2 + \frac{1}{3!}\psi_3^{(i)}(x - x_0)^3\right) \\ &= \psi_1^{(i)}\mathbb{COV}(m, (x - x_0)) + \frac{1}{2}\psi_2^{(i)}\mathbb{COV}\left(m, (x - x_0)^2\right) + \frac{1}{3!}\psi_3^{(i)}\mathbb{COV}\left(m, (x - x_0)^3\right). \end{aligned}$$

Note that

$$x - x_0 = R_M - \mathbb{E}[R_M] = r_M.$$

Hence,

$$\begin{aligned} \mathcal{M}^{(i)} &= \psi_1^{(i)}\mathbb{COV}(m, r_M) + \frac{1}{2}\psi_2^{(i)}\mathbb{COV}\left(m, r_M^2\right) + \frac{1}{3!}\psi_3^{(i)}\mathbb{COV}\left(m, r_M^3\right) \\ &= \psi_1^{(i)}\left(\mathbb{E}(mr_M) - \mathbb{E}(m)\mathbb{E}(r_M)\right) + \frac{1}{2}\psi_2^{(i)}\left(\mathbb{E}\left(mr_M^2\right) - \mathbb{E}(m)\mathbb{E}\left(r_M^2\right)\right) \\ &\quad + \frac{1}{3!}\psi_3^{(i)}\left(\mathbb{E}\left(mr_M^3\right) - \mathbb{E}(m)\mathbb{E}\left(r_M^3\right)\right) \\ &= \psi_1^{(i)}\mathbb{E}[m]\left(\mathbb{E}^*(r_M) - \mathbb{E}(r_M)\right) + \frac{1}{2}\psi_2^{(i)}\mathbb{E}(m)\left(\mathbb{E}^*\left(r_M^2\right) - \mathbb{E}\left(r_M^2\right)\right) \\ &\quad + \frac{1}{3!}\psi_3^{(i)}\mathbb{E}(m)\left(\mathbb{E}^*\left(r_M^3\right) - \mathbb{E}\left(r_M^3\right)\right). \end{aligned}$$

■

Proof of Result 4. In case of a CRRA utility,

$u^{'}[x] = x^{-\alpha}$ and $\psi^{(i)}[x] = (x^{-\alpha} - \overline{m})^{i-1}$. The SDF (up to a constant) is $m = R_M^{-\alpha}$, and

$$\begin{aligned} \mathcal{M}^{(i)} &= \mathbb{COV}\left(m, (m - \overline{m})^{i-1}\right) \\ &= \mathbb{COV}\left(m, \left(R_M^{-\alpha} - \overline{m}\right)^{i-1}\right) \\ &= \mathbb{COV}\left(m, \psi^{(i)}[R_M]\right). \end{aligned} \tag{A24}$$

The third-order Taylor expansion series of $\psi^i[x]$ around $x_0 = \mathbb{E}(R_M)$ produces the following:

$$\psi^{(i)}[x] = \psi^{(i)}[x_0] + \sum_{k=1}^{\infty} \frac{1}{k!} \psi_k^{(i)} (x - x_0)^k, \tag{A25}$$

with

$$\psi_k^{(i)} = \left\{ \frac{\partial^k \psi^{(i)}[x]}{\partial^k x} \right\}_{x=x_0}.$$

Together (A24) and (A25) produce

$$\mathcal{M}^{(i)} = \mathbb{COV}\left(m, \sum_{k=1}^{\infty} \frac{1}{k!} \psi_k^{(i)} (x - x_0)^k\right)$$

Note that

$$x - x_0 = R_M - \mathbb{E}[R_M] = r_M,$$

Hence,

$$\mathcal{M}^{(i)} = \sum_{k=1}^{\infty} \psi_k^{(i)} \mathbb{COV}\left(m, r_M^k\right),$$

I denote by $r_M = R_M - \mathbb{E}(R_M)$, thus, the moments of the SDF can be decomposed as

$$\begin{aligned} \mathcal{M}^{(i)} &= \sum_{k=1}^{\infty} \psi_k^{(i)} \left(\mathbb{E}\left(m r_M^k\right) - \mathbb{E}(m)\,\mathbb{E}\left(r_M^k\right)\right) \\ &= \sum_{k=1}^{\infty} \psi_k^{(i)} (\mathbb{E}(m)) \left(\mathbb{E}\left(\frac{m}{\mathbb{E}(m)} r_M^k\right) - \mathbb{E}\left(r_M^k\right)\right) \\ &= \sum_{k=1}^{\infty} \psi_k^{(i)} (\mathbb{E}(m)) \left(\mathbb{E}^*\left(r_M^k\right) - \mathbb{E}\left(r_M^k\right)\right). \end{aligned}$$

This ends the proof. ■

Proof of Result 5. By applying (A9) to the market return and also to the two returns in (A10), the expected excess return on any individual security can alternatively be written as

$$\mathbb{E}(R_i - R_f) = \beta_{i,1}(\mathbb{E}(R_M - R_f)) + \beta_{i,2}\left(\mathbb{E}\left(R_M^{(2)} - R_f\right)\right) + \beta_{i,3}\left(\mathbb{E}\left(R_M^{(3)} - R_f\right)\right). \tag{A26}$$

From Result 1, recall that

$$\mathbb{E}(R_M - R_f) = A_1\mathcal{M}^{(2)} + A_2\mathcal{M}^{(2)} + A_3\mathcal{M}^{(4)}. \tag{A27}$$

Together, (A21) and (A13) allow for writing the following:

$$\begin{aligned}
\mathbb{E}\left(R_M^{(2)} - R_f\right) &= \gamma_1\left(-\phi_1^{(2)}\frac{1}{\overline{m}}\mathcal{M}^{(2)} - \phi_2^{(2)}\frac{1}{\overline{m}}\mathcal{M}^{(3)} - \phi_3^{(2)}\frac{1}{\overline{m}}\mathcal{M}^{(4)}\right) \text{ and} && \text{(A28)}\\
\mathbb{E}\left(R_M^{(3)} - R_f\right) &= \gamma_{2t}\left(-\phi_1^{(3)}\frac{1}{\overline{m}}\mathcal{M}^{(2)} - \phi_2^{(3)}\frac{1}{\overline{m}}\mathcal{M}^{(3)} - \phi_3^{(3)}\frac{1}{\overline{m}}\mathcal{M}^{(4)}\right), && \text{(A29)}
\end{aligned}$$

with

$$\gamma_1 = \frac{R_f}{\mathbb{E}^*\left(R_M^2\right)} \text{ and } \gamma_2 = \frac{R_f}{\mathbb{E}^*\left(R_M^3\right)}.$$

Now, I replace (A27), (A28), and (A29) in the expected excess return decomposition (A26) and show that

$$\mathbb{E}(R_i - R_f) = \beta_i^{(2)}\mathcal{M}^{(2)} + \beta_i^{(3)}\mathcal{M}^{(3)} + \beta_i^{(4)}\mathcal{M}^{(4)},$$

where

$$\begin{aligned}
\beta_i^{(2)} &= \beta_{i,1}A_1 - \frac{\gamma_1}{\overline{m}}\beta_{i,2}\phi_1^{(2)} - \frac{\gamma_2}{\overline{m}}\beta_{i,3}\phi_1^{(3)}\\
\beta_i^{(3)} &= \beta_{i,1}A_2 - \frac{\gamma_1}{\overline{m}}\beta_{i,2}\phi_2^{(2)} - \frac{\gamma_2}{\overline{m}}\beta_{i,3}\phi_2^{(3)}, \text{ and}\\
\beta_i^{(4)} &= \beta_{i,1}A_3 - \frac{\gamma_1}{\overline{m}}\beta_{i,2}\phi_3^{(2)} - \frac{\gamma_2}{\overline{m}}\beta_{i,3}\phi_3^{(3)}.
\end{aligned}$$

■ **Proof of Result 12.** I first observe that

$$\begin{aligned}
\mathcal{M}^{*(3)} &= \mathbb{E}^*\left((m - \mathbb{E}^*(m))^3\right),\\
&= \mathbb{E}\left(\frac{m}{\mathbb{E}[m]}(m - \mathbb{E}^*(m))^3\right),\\
&= \frac{1}{\mathbb{E}(m)}\mathbb{E}\left(m(m - \mathbb{E}^*(m))^3\right).
\end{aligned}$$

Taylor expansion series of $m(m-\mathbb{E}^*(m))^3$ around $m=\mathbb{E}(m)=\overline{m}$ gives

$$m(m-\mathbb{E}^*(m))^3 = \mathcal{A}_0+\frac{1}{1!}\mathcal{A}_1(m-\overline{m})+\frac{1}{2!}\mathcal{A}_2(m-\overline{m})^2+\frac{1}{3!}\mathcal{A}_3(m-\overline{m})^3+\frac{1}{4!}\mathcal{A}_4(m-\overline{m})^4,$$

where

$$\begin{aligned}
\mathcal{A}_0 &= \overline{m}(\overline{m}-\mathbb{E}^*(m))^3, && \text{(A30)}\\
\mathcal{A}_1 &= (\overline{m}-\mathbb{E}^*(m))^3+3\overline{m}(\overline{m}-\mathbb{E}^*(m))^2,\\
\mathcal{A}_2 &= 6(\overline{m}-\mathbb{E}^*(m))^2+6\overline{m}(\overline{m}-\mathbb{E}^*(m)), \text{ and}\\
\mathcal{A}_3 &= 18(\overline{m}-\mathbb{E}^*(m))+6\overline{m},\\
\mathcal{A}_4 &= 24. && \text{(A31)}
\end{aligned}$$

Hence,

$$\mathbb{E}\left(m(m-\mathbb{E}^*(m))^3\right)=\mathcal{A}_0+\frac{1}{2!}\mathcal{A}_2\mathcal{M}^{(2)}+\frac{1}{3!}\mathcal{A}_3\mathcal{M}^{(3)}+\frac{1}{4!}\mathcal{A}_4\mathcal{M}^{(4)}.$$

Next, the following inequalities hold:

$$\begin{aligned}
\mathcal{A}_0 &= -\left(\mathcal{M}^{(2)}\right)^3<0, && \text{(A32)}\\
\frac{1}{2!}\mathcal{A}_2 &= \left(3\frac{1}{\overline{m}^2}\left(\mathcal{M}^{(2)}\right)^2-3\mathcal{M}^{(2)}\right)>0,\\
\frac{1}{3!}\mathcal{A}_3 &= \left(-3\frac{1}{\overline{m}}\mathcal{M}^{(2)}+\overline{m}\right)<0, \text{ and}\\
\frac{1}{4!}\mathcal{A}_4 &= 1. && \text{(A33)}
\end{aligned}$$

Finally, the risk-neutral skewness can be expressed as a function of all physical moments of the SDF:

$$\mathcal{M}^{*(3)}=\frac{1}{\overline{m}}\left\{\mathcal{A}_0+\frac{1}{2!}\mathcal{A}_2\mathcal{M}^{(2)}+\frac{1}{3!}\mathcal{A}_3\mathcal{M}^{(3)}+\frac{1}{4!}\mathcal{A}_4\mathcal{M}^{(4)}\right\}. \qquad \text{(A34)}$$

Assume that the skewness of the SDF is positive. Is the risk neutral skewness of the SDF also positive? Expression (A34) shows that the negative coefficients $\mathcal{A}_0$ and $\mathcal{A}_3$ are functions of the second moment of the SDF. Thus a volatile second moment of the SDF potentially explains why the skewness of the SDF can be negative. ■

B. Appendix

Proof of Result 6. I start by recognizing the following identity:

$$\begin{aligned}
\mathcal{M}_t^{(n+1)}[T] &= \mathbb{E}_t\left(\left(m_{t\to T}-\mathbb{E}_t\left(m_{t\to T}\right)\right)\left(m_{t\to T}-\mathbb{E}_t\left(m_{t\to T}\right)\right)^n\right) \\
&= \left(\mathbb{E}_t\left(m_{t\to T}\right)\right)\mathbb{E}_t\left(\frac{m_{t\to T}}{\mathbb{E}_t\left(m_{t\to T}\right)}\left(m_{t\to T}-\mathbb{E}_t\left(m_{t\to T}\right)\right)^n\right)-\left(\mathbb{E}_t\left(m_{t\to T}\right)\right)\mathbb{E}_t\left(\left(m_{t\to T}-\mathbb{E}_t\left(m_{t\to T}\right)\right)^n\right) \\
&= \left(\mathbb{E}_t\left(m_{t\to T}\right)\right)\left(\mathcal{M}_t^{*(n)}[T]-\mathcal{M}_t^{(n)}[T]\right),
\end{aligned}$$

with

$$\mathcal{N}_t^{*(n)}[T]=\mathbb{E}_t\left(\frac{m_{t\to T}}{\left(\mathbb{E}_t\left(m_{t\to T}\right)\right)}\left(m_{t\to T}-\mathbb{E}_t\left(m_{t\to T}\right)\right)^n\right)=\mathbb{E}_t^*\left(\left(m_{t\to T}-\mathbb{E}_t\left[m_{t\to T}\right]\right)^n\right). \tag{B1}$$

■ **Proof of Result 7.** The expected return on the SDF moments are

$$\mathbb{E}_t\left(\mathcal{R}_{t\to T}^{(n)}\right)=\frac{\mathbb{E}_t\left(m_{t\to T}^{n-1}\right)}{\mathbb{E}_t\left(m_{t\to T}\times m_{t\to T}^{n-1}\right)}=\frac{\mathbb{E}_t\left(m_{t\to T}^{n-1}\right)}{\left(\mathbb{E}_t\left(m_{t\to T}\right)\right)\mathbb{E}_t^*\left(m_{t\to T}^{n-1}\right)},$$

and the expected excess returns are

$$\begin{aligned}
\mathbb{E}_t\left(\mathcal{R}_{t\to T}^{(n)}-R_{f,t\to T}\right) &= R_{f,t\to T}\left(\frac{\mathbb{E}_t\left(m_{t\to T}^{n-1}\right)}{\mathbb{E}_t^*\left(m_{t\to T}^{n-1}\right)}-1\right) \\
&= R_{f,t\to T}\frac{\mathbb{E}_t\left(m_{t\to T}^{n-1}\right)-\mathbb{E}_t^*\left(m_{t\to T}^{n-1}\right)}{\mathbb{E}_t^*\left(m_{t\to T}^{n-1}\right)} \\
&= R_{f,t\to T}\frac{\frac{1}{R_{f,t\to T}}\mathbb{E}_t^*\left(m_{t\to T}^{n-2}\right)-\mathbb{E}_t^*\left(m_{t\to T}^{n-1}\right)}{\mathbb{E}_t^*\left(m_{t\to T}^{n-1}\right)},
\end{aligned}$$

where

$$\mathbb{E}_t\left(m_{t\to T}^{n-1}\right)=\frac{1}{R_{f,t\to T}}\mathbb{E}_t^*\left(m_{t\to T}^{n-2}\right).$$

We have

$$\left(\mathcal{R}_{t\to T}^{(n)}\right)^2=\frac{\left(m_{t\to T}^{n-1}\right)^2}{\left(\mathbb{E}_t\left(m_{t\to T}\times m_{t\to T}^{n-1}\right)\right)^2}=\frac{\left(m_{t\to T}^{n-1}\right)^2}{\left(\mathbb{E}_t\left(m_{t\to T}\right)\right)^2\left(\mathbb{E}_t^*\left(m_{t\to T}^{n-1}\right)\right)^2} \tag{B2}$$

and

$$\left(\mathcal{R}_{t\to T}^{(n)}\right)^2=\frac{\left(m_{t\to T}^{n-1}\right)^2}{\left(\mathbb{E}_t\left(m_{t\to T}\times m_{t\to T}^{n-1}\right)\right)^2}=\frac{\left(m_{t\to T}^{n-1}\right)^2}{\left(\mathbb{E}_t\left(m_{t\to T}\right)\right)^2\left(\mathbb{E}_t^*\left(m_{t\to T}^{n-1}\right)\right)^2}. \tag{B3}$$

Thus,

$$\begin{aligned}
\mathbb{E}_t\left(\left(\mathcal{R}_{t\to T}^{(n)}\right)^2\right) &= \frac{\mathbb{E}_t\left(m_{t\to T}^{2(n-1)}\right)}{\left(\mathbb{E}_t\left(m_{t\to T}\right)\right)^2\left(\mathbb{E}_t^*\left(m_{t\to T}^{n-1}\right)\right)^2} \\
&= \frac{\mathbb{E}_t\left(m_{t\to T}m_{t\to T}^{2(n-1)-1}\right)}{\left(\mathbb{E}_t\left(m_{t\to T}\right)\right)^2\left(\mathbb{E}_t^*\left(m_{t\to T}^{n-1}\right)\right)^2} \\
&= \frac{\left(\mathbb{E}_t\left(m_{t\to T}\right)\right)\mathbb{E}_t\left(\frac{m_{t\to T}}{\left(\mathbb{E}_t\left(m_{t\to T}\right)\right)}m_{t\to T}^{2(n-1)-1}\right)}{\left(\mathbb{E}_t\left(m_{t\to T}\right)\right)^2\left(\mathbb{E}_t^*\left(m_{t\to T}^{n-1}\right)\right)^2} \\
&= R_{f,t\to T}\frac{\mathbb{E}_t^*\left(m_{t\to T}^{2n-3}\right)}{\left(\mathbb{E}_t^*\left(m_{t\to T}^{n-1}\right)\right)^2}.
\end{aligned}$$

and

$$\mathbb{E}_t\left(\left(\mathcal{R}_{t\to T}^{(n)}\right)^2\right) = R_{f,t\to T}\frac{\mathbb{E}_t^*\left(m_{t\to T}^{2n-3}\right)}{\left(\mathbb{E}_t^*\left(m_{t\to T}^{n-1}\right)\right)^2},$$

since

$$\mathbb{E}_t\left(\mathcal{R}_{t\to T}^{(n)}\right) = \frac{\mathbb{E}_t^*\left(m_{t\to T}^{n-2}\right)}{\left(\mathbb{E}_t^*\left(m_{t\to T}^{n-1}\right)\right)^2}.$$

Replace both expressions in the Sharpe ratio expression to yield the result. ■

Proof of Result 8. I apply (I-A5) to $g\left[R_{Mt\to T}\right] = R_{Mt\to T}^{\alpha}$ with $x = R_{Mt\to T}$ and take the expectation under the risk-neutral measure to get

$$\mathbb{E}_t^*\left(R_{Mt\to T}^{\alpha}\right) = 1 + \alpha\left(R_{f,t\to T} - 1\right) + \frac{\alpha\left(\alpha-1\right)}{S_t^2}R_{f,t\to T}\left(\int_{S_t}^{\infty}\left(\frac{K}{S_t}\right)^{\alpha-2}C_t\left[K\right]dK + \int_0^{S_t}\left(\frac{K}{S_t}\right)^{\alpha-2}P_t\left[K\right]dK\right).$$

The SDF is, therefore,

$$m_{t\to T} = \frac{\delta_t}{R_{f,t}}R_{Mt\to T}^{-\alpha} \text{ with } \delta_t = \mathbb{E}_t^*\left(R_{Mt\to T}^{\alpha}\right).$$

Note that

$$\begin{aligned}
\frac{1}{R_{f,t\to T}}\mathbb{E}_t^*\left(\left(m_{t\to T} - \mathbb{E}_t\left(m_{t\to T}\right)\right)^n\right) &= \frac{1}{R_{f,t\to T}}\mathbb{E}_t^*\left(\left(\frac{\mathbb{E}_t^*\left(R_{Mt\to T}^{\alpha}\right)}{R_{f,t\to T}}R_{Mt\to T}^{-\alpha} - \frac{1}{R_{f,t\to T}}\right)^n\right) \\
&= \frac{1}{R_{f,t\to T}^{n+1}}\mathbb{E}_t^*\left(\left(\frac{\mathbb{E}_t^*\left(R_{Mt\to T}^{\alpha}\right)}{R_{Mt\to T}^{\alpha}} - 1\right)^n\right).
\end{aligned}$$

Since $R_{Mt\to T}=\frac{S_T}{S_t}$, I denote

$$h[S_T]=\left(\delta_t\left(\frac{S_T}{S_t}\right)^{-\alpha}-1\right)^n \text{ with } \delta_t=\mathbb{E}_t^*\left(R_{Mt\to T}^{\alpha}\right)$$

and derive the following expressions:

$$h[S_T]=\left(\delta_t\left(\frac{S_T}{S_t}\right)^{-\alpha}-1\right)^n,\ h_S[S_t]=\left(\frac{\partial h[S_T]}{\partial S_T}\right)_{S_T=S_t},\ \text{and } h_{SS}[K]=\left(\frac{\partial^2 h[S_T]}{\partial^2 S_T}\right)_{S_T=K}. \tag{B4}$$

Thus,

$$\begin{aligned}
h[S_t] &= (\delta_t-1)^n,\\
h_S[S_t] &= -n\alpha(\delta_t-1)^{n-1}\delta_t\frac{1}{S_t},\ \text{and}\\
h_{SS}[K] &= n(n-1)\delta_t^2\alpha^2\frac{1}{S_t^2}\left(\delta_t\left(\frac{K}{S_t}\right)^{-\alpha}-1\right)^{n-2}\left(\frac{K}{S_t}\right)^{-2(1+\alpha)}+n\delta_t\alpha(\alpha+1)\frac{1}{S_t^2}\left(\delta_t\left(\frac{K}{S_t}\right)^{-\alpha}-1\right)^{n-1}\left(\frac{K}{S_t}\right)^{-(\alpha+2)}.
\end{aligned}$$

Since

$$h[S_T]=h[S_t]+(S_T-S_t)h_S[S_t]+\int_{S_t}^{\infty}h_{SS}[K](S_T-K)^+dK+\int_0^{S_t}h_{SS}[K](K-S_T)^+dK,$$

I take the expected value under the risk-neutral measure of $h[S_T]$ and obtain the following expression:

$$\mathbb{E}_t^*(h[S_T]) \;=\; h[S_t]+(R_{f,t\to T}-1)S_t h_S[S_t]+R_{f,t\to T}\left\{\int_{S_t}^{\infty}h_{SS}[K]C_t[K]dK+\int_0^{S_t}h_{SS}[K]P_t[K]dK\right\}.$$

Hence,

$$\mathcal{N}_t^{*(n)}[T]=\mathbb{E}_t^*\left((m_{t\to T}-\mathbb{E}_t(m_T))^n\right)=\frac{1}{R_{f,t\to T}^n}\mathbb{E}_t^*(h[S_T]). \tag{B5}$$

This ends the proof. ■

Proof of Result 9. I recall that the SDF is of the form

$$m_{t\to T}=\frac{\delta_t}{R_{f,t\to T}}\left(\frac{S_T}{S_t}\right)^{-\alpha}.$$

The n *th* risk-neutral moment is $\mathcal{M}_t^{*(n)}[T]=\mathbb{E}_t^*\left((m_{t\to T}-\mathbb{E}_t^*(m_{t\to T}))^n\right)$. This risk-neutral moment can be expanded as follows:

$$\mathcal{M}_t^{*(n)}[T] = \left(\frac{\delta_t}{R_{f,t\to T}}\right)^n \mathbb{E}_t^*\left(\left(\left(\frac{S_T}{S_t}\right)^{-\alpha} - \zeta_t\right)^n\right)$$
$$= \left(\frac{\delta_t}{R_{f,t\to T}}\right)^n f[S_T],$$

where

$$f[S_T] = \left(\left(\frac{S_T}{S_t}\right)^{-\alpha} - \zeta_t\right)^n$$
$$\zeta_t = \mathbb{E}_t^*\left(\left(\frac{S_T}{S_t}\right)^{-\alpha}\right).$$

I use the spanning formula (I-A5) in Internet Appendix B to decompose $f[S_T]$ as

$$f[S_T] = (1-\zeta_t)^n + (S_T - S_t) f_S[S_t] + \int_{S_t}^{\infty} f_{SS}[K](S_T - K)^+ dK + \int_0^{S_t} f_{SS}[K](K - S_T)^+ dK.$$

Hence,

$$\mathbb{E}_t^*(f[S_T]) = (1-\zeta_t)^n + (R_{f,t\to T} - 1) S_t f_S[S_t] + R_{f,t\to T}\left\{\int_{S_t}^{\infty} f_{SS}[K] C_t[K] dK + \int_0^{S_t} f_{SS}[K] P_t[K] dK\right\},$$

where

$$f_S[S_T] = -\frac{n\alpha}{S_t}\left(\frac{S_T}{S_t}\right)^{-\alpha-1}\left(\left(\frac{S_T}{S_t}\right)^{-\alpha} - \zeta_t\right)^{n-1}, \; f_S[S_t] = -\frac{n\alpha}{S_t}(1-\zeta_t)^{n-1},$$

and

$$f_{SS}[S_T] = \frac{n\alpha}{S_t^2}(\alpha+1)\left(\frac{S_T}{S_t}\right)^{-\alpha-2}\left(\left(\frac{S_T}{S_t}\right)^{-\alpha} - \zeta_t\right)^{n-1} + \frac{n(n-1)(\alpha^2)}{S_t^2}\left(\frac{S_T}{S_t}\right)^{-2\alpha-2}\left(\left(\frac{S_T}{S_t}\right)^{-\alpha} - \zeta_t\right)^{n-2}.$$

The same approach can be use to show that

$$\zeta_t = 1 - \alpha(R_{f,t\to T} - 1) + R_{f,t\to T}\frac{\alpha(1+\alpha)}{S_t^2}\left\{\int_{S_t}^{\infty}\left(\frac{K}{S_t}\right)^{-\alpha-2} C_t[K] dK + \int_0^{S_t}\left(\frac{K}{S_t}\right)^{-\alpha-2} P_t[K] dK\right\}.$$

■ **Proof of Result 10.** Note that

$$m_{t\to T}^{n-2} = \frac{\delta_t^{n-2}}{R_{f,t\to T}^{n-2}} R_{M,t\to T}^{\alpha(2-n)} = \frac{\delta_t^{n-2}}{R_{f,t\to T}^{n-2}} \left(\frac{S_T}{S_t}\right)^{\alpha(2-n)}. \tag{B6}$$

I use the spanning formula (I-A5) in the Internet Appendix B to show the following:

$$\left(\frac{S_T}{S_t}\right)^{\alpha(2-n)} = 1+\alpha(2-n)\left(\frac{S_T}{S_t}-1\right)+\frac{\alpha(2-n)(\alpha(2-n)-1)}{S_t^2}\left\{\begin{array}{c} \int_{S_t}^{\infty}\left(\frac{K}{S_t}\right)^{\alpha(2-n)-2}(S_T-K)^+\,dK \\ +\int_0^{S_t}\left(\frac{K}{S_t}\right)^{\alpha(2-n)-2}(K-S_T)^+\,dK \end{array}\right\}$$

I then take the conditional expectation under the risk-neutral measure to get

$$\begin{aligned} &\mathbb{E}_t^*\left(\left(\frac{S_T}{S_t}\right)^{\alpha(2-n)}\right) \\ = &\ 1+\alpha(2-n)(R_{f,t\to T}-1)+\frac{\alpha(2-n)(\alpha(2-n)-1)R_{f,t\to T}}{S_t^2}\left\{\begin{array}{c} \int_{S_t}^{\infty}\left(\frac{K}{S_t}\right)^{\alpha(2-n)-2}C_t[K]\,dK \\ +\int_0^{S_t}\left(\frac{K}{S_t}\right)^{\alpha(2-n)-2}P_t[K]\,dK \end{array}\right\}. \end{aligned}$$

■

Table 1

Conditional Moments of the SDF under CRRA Preferences This table reports the mean, standard deviation, skewness, kurtosis, and quantiles of the SDF variance, skewness, and kurtosis at various horizons (reported in days).

Maturities (days)	Mean (%)	Std dev	Skew	Kurt	Min	1%	10%	25%	50%	75%	99%	Max
					$\mathcal{M}_t^{(2)}[T]$							
30	3.28	0.03	4.64	37.88	0.01	0.01	0.01	0.01	0.02	0.04	0.06	0.45
60	6.58	0.06	3.96	28.30	0.01	0.02	0.02	0.03	0.05	0.08	0.12	0.79
91	10.10	0.08	3.40	20.92	0.02	0.03	0.04	0.05	0.08	0.12	0.17	0.99
122	13.71	0.10	3.23	19.19	0.03	0.04	0.05	0.07	0.11	0.17	0.23	1.23
152	17.29	0.12	3.04	17.55	0.04	0.05	0.07	0.09	0.15	0.21	0.29	1.47
182	20.95	0.14	2.80	14.96	0.06	0.07	0.09	0.12	0.18	0.25	0.35	1.56
273	32.60	0.20	2.55	12.61	0.09	0.11	0.14	0.19	0.29	0.38	0.53	2.07
365	45.28	0.28	2.51	12.38	0.13	0.15	0.20	0.27	0.40	0.53	0.74	2.88
					$\mathcal{M}_t^{(3)}[T]$							
30	0.59	0.02	13.32	246.03	-0.07	0.00	0.00	0.00	0.00	0.00	0.01	0.56
60	1.73	0.05	11.76	205.50	-0.03	0.00	0.00	0.00	0.01	0.02	0.03	1.25
91	3.36	0.08	9.11	129.83	-0.03	0.00	0.00	0.01	0.02	0.03	0.07	1.75
122	5.44	0.11	8.18	109.72	-0.04	0.00	0.00	0.01	0.03	0.05	0.11	2.46
152	7.86	0.15	7.40	92.70	-0.04	0.00	0.01	0.02	0.04	0.08	0.16	3.14
182	10.69	0.18	6.16	63.27	-0.05	0.00	0.01	0.03	0.06	0.11	0.22	3.43
273	22.47	0.35	4.84	38.54	-0.07	0.00	0.02	0.05	0.12	0.25	0.48	5.49
365	40.13	0.62	4.52	33.12	-0.08	0.00	0.03	0.09	0.22	0.44	0.87	9.29
					$\mathcal{M}_t^{(4)}[T]$							
30	0.36	0.02	18.93	452.02	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.70
60	1.28	0.06	18.14	449.60	0.00	0.00	0.00	0.00	0.00	0.01	0.02	2.08
91	2.82	0.11	14.62	313.87	0.00	0.00	0.00	0.00	0.01	0.02	0.05	3.34
122	5.09	0.17	13.83	295.76	0.00	0.00	0.00	0.01	0.02	0.04	0.09	5.35
152	7.97	0.24	12.78	261.59	0.00	0.00	0.01	0.01	0.03	0.06	0.14	7.36
182	11.66	0.32	10.33	172.17	0.00	0.00	0.01	0.02	0.04	0.09	0.21	8.38
273	30.31	0.73	8.07	106.74	0.01	0.01	0.02	0.06	0.12	0.26	0.58	16.58
365	65.33	1.54	7.44	90.97	0.02	0.02	0.04	0.12	0.25	0.55	1.29	33.95

Table 2

Conditional Risk-Neutral Moments of the SDF under CRRA Preferences This table reports the mean, standard deviation, skewness, kurtosis, and quantiles of the risk-neutral SDF variance, skewness, and kurtosis at various horizons (reported in days).

Maturities (days)	Mean (%)	Std dev	Skew	Kurt	Min	1%	10%	25%	50%	75%	99%	Max
					$\mathcal{M}_t^{*(2)}[T]$							
30	3.66	0.04	6.65	75.69	0.01	0.01	0.01	0.02	0.03	0.04	0.07	0.80
60	7.55	0.08	5.81	60.67	0.01	0.02	0.02	0.03	0.06	0.09	0.14	1.41
91	11.79	0.11	4.71	40.62	0.02	0.03	0.04	0.06	0.09	0.14	0.21	1.76
122	16.22	0.14	4.29	34.52	0.03	0.04	0.06	0.08	0.13	0.19	0.29	2.18
152	20.68	0.17	3.88	28.66	0.04	0.05	0.08	0.11	0.17	0.24	0.36	2.45
182	25.28	0.20	3.44	22.48	0.05	0.07	0.09	0.14	0.20	0.30	0.45	2.55
273	40.26	0.30	2.82	15.26	0.08	0.11	0.14	0.22	0.32	0.48	0.73	3.30
365	57.11	0.43	2.56	12.54	0.11	0.14	0.19	0.30	0.45	0.68	1.07	4.17
					$\mathcal{M}_t^{*(3)}[T]$							
30	0.16	0.01	7.88	223.95	-0.17	-0.02	0.00	0.00	0.00	0.00	0.01	0.28
60	0.04	0.02	-8.01	139.65	-0.49	-0.06	-0.01	0.00	0.00	0.00	0.01	0.23
91	-0.47	0.04	-10.15	161.09	-1.10	-0.17	-0.02	0.00	0.00	0.01	0.01	0.16
122	-1.41	0.08	-10.68	181.54	-2.11	-0.34	-0.03	-0.01	0.00	0.01	0.01	0.13
152	-2.74	0.12	-11.42	220.64	-3.46	-0.52	-0.06	-0.03	0.00	0.01	0.02	0.15
182	-4.53	0.16	-9.10	137.25	-3.96	-0.81	-0.09	-0.05	-0.01	0.00	0.02	0.16
273	-13.89	0.36	-6.96	74.49	-7.15	-1.95	-0.27	-0.14	-0.05	-0.01	0.02	0.16
365	-31.29	0.74	-6.53	65.30	-14.12	-4.08	-0.60	-0.28	-0.13	-0.04	0.00	0.13
					$\mathcal{M}_t^{*(4)}[T]$							
30	0.25	0.01	14.81	275.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.29
60	0.83	0.03	14.76	317.43	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.99
91	1.84	0.06	12.62	246.67	0.00	0.00	0.00	0.00	0.01	0.01	0.03	1.85
122	3.36	0.11	13.21	279.54	0.00	0.00	0.00	0.00	0.01	0.03	0.05	3.51
152	5.36	0.18	15.55	402.47	0.00	0.00	0.00	0.01	0.02	0.04	0.08	6.29
182	7.94	0.24	12.50	265.85	0.00	0.00	0.01	0.01	0.03	0.06	0.13	7.50
273	21.81	0.62	9.56	146.52	0.01	0.01	0.02	0.04	0.09	0.17	0.35	15.92
365	51.08	1.56	10.32	182.19	0.02	0.02	0.04	0.08	0.19	0.36	0.80	43.69

Table 3

Conditional SDF Moments Premium of the SDF under CRRA Preferences This table reports the mean, standard deviation, skewness, kurtosis, and quantiles of the SDF variance, skewness, and kurtosis at various horizons (reported in days).

Maturities (days)	Mean (%)	Std dev	Skew	Kurt	Min	1%	10%	25%	50%	75%	99%	Max
					$\mathcal{M}_t^{(2)}[T] - \mathcal{M}_t^{*(2)}[T]$							
30	-0.37	0.01	-12.07	228.77	-0.35	-0.05	-0.01	0.00	0.00	0.00	0.00	0.15
60	-0.97	0.03	-11.01	188.97	-0.63	-0.10	-0.02	-0.01	0.00	0.00	0.00	0.18
91	-1.69	0.04	-8.51	122.13	-0.77	-0.16	-0.04	-0.02	-0.01	0.00	0.00	0.26
122	-2.52	0.05	-7.14	92.70	-0.95	-0.22	-0.06	-0.03	-0.01	0.00	0.00	0.37
152	-3.39	0.06	-5.90	65.42	-0.98	-0.27	-0.08	-0.04	-0.02	0.00	0.00	0.42
182	-4.34	0.07	-4.92	48.67	-1.11	-0.34	-0.11	-0.06	-0.03	-0.01	0.00	0.46
273	-7.71	0.13	-3.00	21.40	-1.37	-0.56	-0.21	-0.11	-0.05	0.00	0.02	0.86
365	-11.91	0.20	-2.27	14.21	-2.05	-0.85	-0.35	-0.18	-0.06	0.00	0.04	1.33
					$\mathcal{M}_t^{(3)}[T] - \mathcal{M}_t^{*(3)}[T]$							
30	0.43	0.02	15.07	305.54	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.49
60	1.69	0.06	13.55	275.63	0.00	0.00	0.00	0.00	0.01	0.01	0.03	1.73
91	3.83	0.11	10.37	168.10	0.00	0.00	0.00	0.01	0.01	0.03	0.07	2.85
122	6.85	0.18	9.78	154.15	0.00	0.00	0.01	0.01	0.03	0.06	0.13	4.58
152	10.60	0.25	9.56	154.74	0.00	0.01	0.01	0.02	0.05	0.10	0.20	6.60
182	15.22	0.33	7.76	99.33	0.01	0.01	0.02	0.03	0.07	0.14	0.29	7.39
273	36.39	0.69	6.10	58.02	0.01	0.02	0.04	0.08	0.18	0.35	0.71	12.65
365	71.48	1.33	5.72	50.61	0.03	0.05	0.08	0.17	0.35	0.69	1.42	23.43
					$\mathcal{M}_t^{(4)}[T] - \mathcal{M}_t^{*(4)}[T]$							
30	0.11	0.01	22.48	641.03	-0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.40
60	0.45	0.03	20.33	525.73	-0.11	0.00	0.00	0.00	0.00	0.00	0.01	1.09
91	0.99	0.05	16.10	345.71	-0.23	0.00	0.00	0.00	0.00	0.01	0.02	1.49
122	1.73	0.07	13.89	273.08	-0.38	-0.01	0.00	0.00	0.00	0.01	0.03	1.87
152	2.62	0.09	10.19	153.12	-0.45	-0.01	0.00	0.00	0.01	0.02	0.06	1.89
182	3.73	0.11	9.24	137.86	-0.53	-0.02	0.00	0.00	0.01	0.03	0.09	2.25
273	8.53	0.19	5.03	51.50	-1.49	-0.06	0.00	0.00	0.03	0.09	0.24	2.93
365	14.32	0.36	-4.96	158.82	-9.76	-0.21	-0.02	0.00	0.06	0.18	0.48	3.71

Table 4

Regression Results This table reports the slopes and adjusted R^2 of the regression results. I run the regression

$$R_{M,t\to T} - R_{f,t} = b_0 + \beta \mathbb{X}_t + \varepsilon_{t\to T}, \tag{7}$$

where $\mathbb{X}_t$ represents the variance, skewness, or kurtosis of the SDF. Panel A presents the univariate results when CRRA preferences are used and the risk aversion parameter is set to $\alpha = 2$. Panel B presents the univariate results when preferences that depart from CRRA preferences are used with $\alpha = 2$, $\rho^{(2)} = 5$, and $\rho^{(k)} = 0$ for $k > 2$. T-statistics in brackets are computed using Hansen and Hodrick (1980), with the number of lags equal to the time to maturity in days.

Panel A

Maturity (days)	30	60	91	122	152	182	273	365
$\mathcal{M}_t^{(2)}[T]$	0.11	0.10	0.14	0.22	0.24	0.24	0.22	0.19
t-stat	[0.63]	[0.46]	[0.83]	[2.14]	[2.68]	[2.61]	[2.15]	[2.11]
$R^2(\%)$	0.39	0.50	1.38	3.87	4.88	5.36	5.53	5.50
$R^2_{OOS}(\%)$	0.30	0.43	1.30	3.80	4.82	5.30	5.46	5.44
$\mathcal{M}_t^{(3)}[T]$	0.12	0.07	0.12	0.21	0.22	0.23	0.17	0.11
t-stat	[0.78]	[0.38]	[0.71]	[2.15]	[3.21]	[4.64]	[3.60]	[2.77]
$R^2(\%)$	0.22	0.2	0.83	3.99	6.14	8.34	10.44	9.76
$R^2_{OOS}(\%)$	0.12	0.12	0.76	3.92	6.07	8.28	10.37	9.70
$\mathcal{M}_t^{(4)}[T]$	0.1	0.00	0.04	0.10	0.10	0.11	0.07	0.04
t-stat	[0.97]	[0.04]	[0.44]	[2.30]	[3.55]	[5.25]	[4.58]	[3.21]
$R^2(\%)$	0.15	0.00	0.16	2.12	3.61	5.81	8.22	6.75
$R^2_{OOS}(\%)$	0.06	-0.08	0.08	2.05	3.55	5.76	8.15	6.69

Panel B

Maturity (days)	30	60	91	122	152	182	273	365
$\mathcal{M}_t^{(2)}[T]$	0.11	0.10	0.14	0.22	0.24	0.24	0.22	0.19
t-stat	[0.63]	[0.46]	[0.84]	[2.15]	[2.70]	[2.64]	[2.19]	[2.14]
$R^2(\%)$	0.39	0.51	1.39	3.91	4.94	5.44	5.71	5.83
$R^2_{OOS}(\%)$	0.30	0.43	1.31	3.82	4.85	5.34	5.51	5.48
$\mathcal{M}_t^{(3)}[T]$	0.12	0.07	0.12	0.21	0.22	0.23	0.17	0.11
t-stat	[0.78]	[0.38]	[0.71]	[2.15]	[3.21]	[4.65]	[3.61]	[2.78]
$R^2(\%)$	0.22	0.20	0.84	4.00	6.14	8.35	10.45	9.75
$R^2_{OOS}(\%)$	0.13	0.12	0.76	3.93	6.08	8.29	10.38	9.69
$\mathcal{M}_t^{(4)}[T]$	0.10	0.00	0.04	0.10	0.10	0.11	0.07	0.04
t-stat	[0.98]	[0.04]	[0.44]	[2.31]	[3.56]	[5.26]	[4.59]	[3.22]
$R^2(\%)$	0.15	0.00	0.16	2.12	3.60	5.81	8.22	6.73
$R^2_{OOS}(\%)$	0.06	-0.08	0.08	2.05	3.54	5.76	8.16	6.67

Table 5

Regression Results This table reports the slopes and adjusted R^2 of the regression results. I run the regression

$$R_{M,t\to T} - R_{f,t} = b_0 + \beta \mathbb{X}_t + \varepsilon_{t\to T}, \tag{8}$$

where $\mathbb{X}_t$ represents the variance, skewness, or kurtosis of the simple return. T-statistics in brackets are computed using Hansen and Hodrick (1980), with the number of lags equal to the time to maturity in days.

Maturity (days)	30	60	91	122	152	182	273	365
$\mathbb{VAR}^*_t(R_{M,t\to T})$	0.46	0.39	0.60	0.94	0.98	0.96	0.78	0.56
t-stat	[0.60]	[0.41]	[0.79]	[1.94]	[2.17]	[1.88]	[1.42]	[1.09]
$R^2(\%)$	0.36	0.40	1.15	3.11	3.62	3.66	2.92	1.88
$R^2_{OOS}(\%)$	0.27	0.32	1.07	3.04	3.56	3.60	2.85	1.81
$\mathbb{SKEW}^*_t(R_{M,t\to T})$	-1.29	-3.84	-2.02	-0.37	-0.17	0.29	0.25	-0.34
t-stat	[-1.41]	[-3.31]	[-1.24]	[-0.22]	[-0.10]	[0.14]	[0.16]	[-0.26]
$R^2(\%)$	0.08	1.08	0.47	0.02	0.01	0.03	0.05	0.21
$R^2_{OOS}(\%)$	-0.01	1.00	0.39	-0.05	-0.06	-0.03	-0.02	0.14
$\mathbb{KURT}^*_t(R_{M,t\to T})$	3.11	-0.73	2.20	4.57	4.64	4.68	2.55	1.00
t-stat	[0.53]	[-0.10]	[0.47]	[2.53]	[4.67]	[5.65]	[3.37]	[1.60]
$R^2(\%)$	0.09	0.01	0.24	1.96	2.82	3.95	3.57	1.43
$R^2_{OOS}(\%)$	-0.01	-0.07	0.16	1.89	2.76	3.89	3.50	1.36

Table 6

Regression Results This table reports the slopes and adjusted R^2 of the regression results. I run the regression

$$R_{M,t\to T} - R_{f,t} = b_0 + \beta \mathbb{X}_t + \varepsilon_{t\to T}, \tag{9}$$

where $\mathbb{X}_t$ represents the variance, skewness or kurtosis of the simple return. T-statistics in brackets are computed using Hansen and Hodrick (1980), with the number of lags equal to the time to maturity in days.

Maturity (days)	30	60	91	122	152	182	273	365
$\mathcal{M}_t^{(2)}[T]$	0.21	0.46	0.54	0.92	1.19	1.32	1.24	1.10
t-stat	[0.94]	[1.74]	[1.09]	[1.41]	[1.83]	[2.12]	[2.21]	[2.18]
$\mathbb{VAR}_t^*(R_{M,t\to T})$	-0.47	-1.68	-1.85	-3.32	-4.57	-5.26	-5.13	-4.76
t-stat	[-0.40]	[-1.64]	[-1.00]	[-1.11]	[-1.45]	[-1.64]	[-1.84]	[-1.88]
$R^2(\%)$	0.40	0.70	1.66	4.95	7.29	9.13	12.11	15.74
$R^2_{OOS}(\%)$	0.31	0.62	1.59	4.88	7.22	9.07	12.04	15.67
$\mathcal{M}_t^{(3)}[T]$	0.12	0.02	0.11	0.22	0.24	0.25	0.20	0.15
t-stat	[0.66]	[0.09]	[0.69]	[2.14]	[3.06]	[4.12]	[3.09]	[2.58]
$\mathbb{SKEW}_t^*(R_{M,t\to T})$	-0.19	-3.70	-1.81	-0.90	-1.23	-1.13	-1.35	-1.70
t-stat	[-0.53]	[-2.56]	[-1.43]	[-0.65]	[-0.86]	[-0.68]	[-0.94]	[-1.34]
$R^2(\%)$	0.22	1.09	1.21	4.13	6.47	8.70	11.66	13.87
$R^2_{OOS}(\%)$	0.12	1.01	1.13	4.07	6.41	8.65	11.62	13.84
$\mathcal{M}_t^{(4)}[T]$	0.09	0.03	0.01	0.06	0.08	0.09	0.08	0.05
t-stat	[7.26]	[1.33]	[0.57]	[1.85]	[2.31]	[2.75]	[2.76]	[2.04]
$\mathbb{KURT}_t^*(R_{M,t\to T})$	0.57	-1.74	1.83	2.47	1.87	1.20	-0.59	-1.14
t-stat	[0.10]	[-0.22]	[0.53]	[2.38]	[2.20]	[0.82]	[-0.39]	[-0.81]
$R^2(\%)$	0.15	0.04	0.25	2.42	3.84	5.94	8.30	7.65
$R^2_{OOS}(\%)$	0.06	-0.04	0.17	2.35	3.77	5.88	8.24	7.60

Table 7

Regression Results This table reports the slopes and adjusted R^2 of the regression results. I run the regression

$$R_{M,t\to T} - R_{f,t} = b_0 + \beta \mathbb{X}_t + \varepsilon_{t\to T}, \tag{10}$$

where $\mathbb{X}_t$ represents the variance, skewness, or kurtosis of the SDF or all three moments together. Panel A presents the univariate results when CRRA preferences are used and the risk aversion parameter is set to $\alpha = 2$. Panel B presents the univariate results when preferences that depart from CRRA preferences are used with $\alpha = 2$, $\rho^{(2)} = 5$, and $\rho^{(k)} = 0$ for $k > 2$. T-statistics in brackets are computed using Hansen and Hodrick (1980), with the number of lags equal to the time to maturity in days.

Panel A

Maturity (days)	30	60	91	122	152	182	273	365
$\mathcal{MRP}^{(2)}$	-0.17	-0.20	-0.27	-0.46	-0.53	-0.56	-0.48	-0.41
t-stat	-0.83	-0.69	-0.90	-2.00	-2.94	-3.92	-2.79	-2.45
$R^2(\%)$	0.19	0.52	1.17	3.91	6.28	7.95	10.53	13.67
$R^2_{OOS}(\%)$	0.1	0.44	1.1	3.84	6.22	7.89	10.47	13.61
$\mathcal{MRP}^{(3)}$	0.14	0.01	0.05	0.11	0.11	0.11	0.08	0.04
t-stat	0.81	0.05	0.53	2.39	3.76	5.43	4.24	3.11
$R^2(\%)$	0.19	0.00	0.34	2.86	4.40	6.47	8.33	6.95
$\mathcal{MRP}^{(4)}$	0.22	0.04	0.08	0.22	0.30	0.31	0.26	0.13
t-stat	1.55	0.24	0.52	1.88	2.72	3.74	3.04	1.69
$R^2(\%)$	0.18	0.02	0.18	1.94	4.06	5.47	6.69	4.33
$R^2_{OOS}(\%)$	0.10	-0.08	0.27	2.79	4.33	6.42	8.27	6.89

Panel B

Maturity (days)	30	60	91	122	152	182	273	365
$\mathcal{MRP}^{(2)}$	-0.17	-0.20	-0.27	-0.45	-0.53	-0.55	-0.48	-0.41
t-stat	-0.83	-0.69	-0.90	-2.00	-2.94	-3.93	-2.80	-2.45
$R^2(\%)$	0.19	0.52	1.17	3.92	6.30	7.98	10.56	13.67
$R^2_{OOS}(\%)$	0.10	0.44	1.10	3.85	6.24	7.92	10.50	13.61
$\mathcal{MRP}^{(3)}$	0.14	0.01	0.05	0.11	0.11	0.11	0.08	0.04
t-stat	0.81	0.05	0.53	2.40	3.77	5.45	4.26	3.12
$R^2(\%)$	0.20	0.00	0.34	2.86	4.40	6.48	8.35	6.96
$R^2_{OOS}(\%)$	0.10	-0.08	0.27	2.79	4.34	6.43	8.29	6.89
$\mathcal{MRP}^{(4)}$	0.22	0.04	0.08	0.22	0.30	0.31	0.25	0.13
t-stat	1.56	0.24	0.52	1.88	2.73	3.75	3.03	1.63
$R^2(\%)$	0.18	0.02	0.18	1.94	4.07	5.49	6.66	4.10
$R^2_{OOS}(\%)$	0.08	-0.06	0.10	1.87	4.01	5.43	6.59	4.04

Table 8

Regression Results The table reports the slopes and adjusted R^2 of the regression results. I run the regression

$$R_{M,t\to T} - R_{f,t} = b_0 + \beta \mathbb{X}_t + \varepsilon_{t\to T}, \tag{11}$$

where $\mathbb{X}_t$ represents the variance, skewness, or kurtosis of the simple return. T-statistics in brackets are computed using Hansen and Hodrick (1980), with the number of lags equal to the time to maturity in days.

Maturity (days)	30	60	91	122	152	182	273	365
$\mathcal{MRP}^{(2)}$	-0.08	-0.15	-0.18	-0.33	-0.45	-0.50	-0.47	-0.42
t-stat	-0.73	-1.62	-0.98	-1.32	-1.74	-1.99	-2.09	-2.08
$\mathbb{VAR}_t^*(R_{M,t\to T})$	0.39	0.21	0.38	0.52	0.38	0.28	0.10	-0.05
t-stat	0.50	0.22	0.58	0.94	0.62	0.38	0.15	-0.08
$R^2(\%)$	0.39	0.60	1.49	4.55	6.67	8.18	10.58	13.69
$R^2_{OOS}(\%)$	0.30	0.52	1.41	4.48	6.61	8.12	10.51	13.62
$\mathcal{MRP}^{(3)}$	0.13	0.00	0.07	0.13	0.13	0.14	0.10	0.07
t-stat	0.80	0.02	0.64	2.06	2.81	3.75	3.18	2.52
$\mathbb{SKEW}_t^*(R_{M,t\to T})$	-1.06	-3.84	-2.42	-1.89	-2.20	-2.15	-2.01	-1.97
t-stat	-1.94	-3.45	-1.40	-0.93	-1.10	-1.04	-1.17	-1.33
$R^2(\%)$	0.25	1.08	0.99	3.43	5.35	7.62	10.66	11.88
$R^2_{OOS}(\%)$	0.16	1.00	0.91	3.36	5.28	7.57	10.59	11.82
$\mathcal{MRP}^{(4)}$	0.19	0.07	0.05	0.16	0.24	0.25	0.23	0.16
t-stat	5.73	1.65	0.69	1.64	2.30	2.91	2.72	2.23
$\mathbb{KURT}_t^*(R_{M,t\to T})$	1.38	-1.64	1.70	3.31	3.03	3.22	2.05	1.46
t-stat	0.24	-0.22	0.45	3.11	5.94	3.34	2.29	2.00
$R^2(\%)$	0.19	0.07	0.29	2.81	5.11	7.14	8.93	7.24
$R^2_{OOS}(\%)$	0.10	-0.01	0.21	2.74	5.04	7.08	8.86	7.17

Table 9

Regression Results The table reports the slopes and adjusted R^2 of the regression results. I run the regression

$$R_{M,t\to T} - R_{f,t} = b_0 + \beta \mathbb{X}_t + \varepsilon_{t\to T}, \tag{12}$$

where $\mathbb{X}_t$ represents the SDF variance risk premium and LJV, SDF skewness risk premium and LJV, or SDF kurtosis risk premium and LJV. The predictive regression is a monthly frequency, and the market return and return on the risk-free rate are consistent with the maturity used. T-statistics are computed using Hansen and Hodrick (1980), with the number of lags equal to the time to maturity in days.

			$\mathcal{MRP}^{(2)}$		LJV			
Maturity(days)	b_0	t-stat	β	t-stat	$\beta\times10^2$	t-stat	$R^2(\%)$	$R^2_{OOS}(\%)$
30	0.00	-0.09	2.38	2.82	2.93	1.90	4.23	2.43
60	0.00	-0.29	1.24	1.66	4.72	1.73	2.54	0.96
91	0.00	0.14	-0.07	-0.09	1.79	0.51	1.23	-0.33
122	-0.01	-0.48	-0.80	-1.86	0.24	0.07	5.97	4.56
152	-0.01	-0.62	-1.01	-3.42	-0.64	-0.20	10.07	8.76
182	-0.01	-0.75	-0.88	-4.30	0.42	0.15	11.90	10.63
273	-0.02	-0.85	-0.48	-4.69	4.78	1.86	12.70	11.26
365	-0.01	-0.40	-0.38	-6.00	4.98	2.00	13.17	11.77
			$\mathcal{MRP}^{(3)}$		LJV			
Maturity(days)	b_0	t-stat	β	t-stat	$\beta\times10^2$	t-stat	$R^2(\%)$	$R^2_{OOS}(\%)$
30	0.00	-0.15	-0.80	-1.62	1.90	1.11	1.81	0.00
60	-0.01	-1.35	-0.69	-3.58	6.88	3.38	6.42	4.86
91	-0.01	-0.59	-0.26	-1.84	6.34	2.50	3.07	1.53
122	-0.01	-0.63	-0.03	-0.32	5.45	1.73	4.38	2.96
152	-0.01	-0.44	0.04	0.48	5.00	1.29	6.17	4.84
182	0.00	-0.20	0.10	1.66	2.47	0.54	8.20	6.92
273	-0.01	-0.28	0.07	2.29	3.59	0.79	10.07	8.62
365	0.00	0.08	0.03	2.32	5.20	1.20	8.03	6.61
			$\mathcal{MRP}^{(4)}$		LJV			
Maturity(days)	b_0	t-stat	β	t-stat	$\beta\times10^2$	t-stat	$R^2(\%)$	$R^2_{OOS}(\%)$
30	0.00	-0.57	-2.80	-2.60	2.37	1.47	3.59	1.79
60	-0.02	-1.92	-2.14	-4.75	7.61	3.91	8.41	6.86
91	-0.02	-1.31	-1.35	-2.41	9.02	3.09	5.31	3.77
122	-0.01	-0.56	-0.03	-0.07	4.99	1.40	4.35	2.93
152	0.00	0.14	0.72	1.90	-0.35	-0.10	8.57	7.25
182	0.00	-0.01	0.71	3.00	-0.40	-0.16	11.61	10.34
273	-0.01	-0.73	0.32	5.08	5.45	2.19	12.20	10.76
365	-0.02	-0.98	0.20	5.53	10.72	3.89	11.90	10.49

Table 10

Regression Results The table reports the slopes and adjusted R^2 of the regression results. I run the regression

$$R_{M,t\to T} - R_{f,t} = b_0 + \beta \mathbb{X}_t + \varepsilon_{t\to T}, \tag{13}$$

where $\mathbb{X}_t$ represents the SDF variance risk premium and the variance risk premium (VRP), SDF skewness risk premium and VRP, or SDF kurtosis risk premium and VRP. The predictive regression is a monthly frequency and the market return and return on the risk-free rate are consistent with the maturity used. T-statistics are computed using Hansen and Hodrick (1980), with the number of lags equal to the time to maturity in days.

			$\mathcal{MRP}^{(2)}$		VRP			
Maturity(days)	b_0	t-stat	β	t-stat	$\beta \times 10^2$	t-stat	$R^2(\%)$	$R^2_{OOS}(\%)$
30	0.00	0.61	0.54	1.04	0.02	0.99	1.94	0.13
60	-0.01	-1.26	-0.31	-0.85	0.07	4.56	5.49	3.93
91	-0.02	-2.47	-0.60	-2.23	0.13	6.83	11.87	10.37
122	-0.03	-2.93	-0.92	-4.29	0.13	6.08	13.74	12.35
152	-0.04	-3.01	-1.00	-6.10	0.13	6.32	16.27	14.97
182	-0.04	-2.80	-0.94	-6.99	0.14	5.92	17.14	15.89
273	-0.02	-1.28	-0.61	-6.66	0.11	2.96	13.69	12.25
365	-0.01	-0.59	-0.45	-7.31	0.11	2.12	13.75	12.35
			$\mathcal{MRP}^{(3)}$		VRP			
Maturity(days)	b_0	t-stat	β	t-stat	$\beta \times 10^2$	t-stat	$R^2(\%)$	$R^2_{OOS}(\%)$
30	0.00	0.23	-0.13	-0.53	0.02	1.12	1.45	-0.37
60	-0.01	-0.78	0.01	0.07	0.07	3.79	5.05	3.49
91	-0.02	-1.82	0.09	1.04	0.13	6.96	10.38	8.86
122	-0.02	-1.99	0.15	2.28	0.13	5.56	10.62	9.23
152	-0.02	-1.80	0.15	2.74	0.13	4.91	11.63	10.32
182	-0.03	-1.96	0.15	4.21	0.14	5.51	13.54	12.28
273	-0.02	-1.10	0.09	5.90	0.12	2.83	12.05	10.61
365	-0.01	-0.37	0.05	5.62	0.12	2.08	9.26	7.84
			$\mathcal{MRP}^{(4)}$		VRP			
Maturity(days)	b_0	t-stat	β	t-stat	$\beta \times 10^2$	t-stat	$R^2(\%)$	$R^2_{OOS}(\%)$
30	0.00	0.44	-0.69	-1.00	0.01	0.83	1.80	-0.02
60	-0.01	-0.81	0.05	0.13	0.07	3.57	5.06	3.49
91	-0.02	-1.89	0.33	1.25	0.13	6.47	10.54	9.03
122	-0.02	-2.30	0.66	3.12	0.14	5.95	12.42	11.03
152	-0.03	-2.50	0.81	5.12	0.15	5.94	16.48	15.18
182	-0.03	-2.45	0.76	6.25	0.15	5.18	18.34	17.08
273	-0.02	-1.10	0.44	7.29	0.11	3.49	13.07	11.64
365	0.00	0.07	0.22	2.97	0.12	1.70	7.95	6.53

Table 11

Regression Results The table reports the slopes and adjusted R^2 of the regression results. I run the regression:

$$R_{M,t\to T} - R_{f,t} = b_0 + \beta \mathbb{X}_t + \varepsilon_{t\to T}, \tag{14}$$

where $\mathbb{X}_t$ represents the SDF variance risk premium and the difference between variance risk premium and the LJV measure (VRP-LJV), SDF skewness risk premium and VRP-LJV, or SDF kurtosis risk premium and VRP-LJV. The predictive regression is a monthly frequency and the market return and return on the risk-free rate are consistent with the maturity used. T-statistics are computed using Hansen and Hodrick (1980), with the number of lags equal to the time to maturity in days.

			$\mathcal{MRP}^{(2)}$		VRP-LJV			
Maturity(days)	b_0	t-stat	β	t-stat	$\beta \times 10^2$	t-stat	$R^2(\%)$	$R^2_{OOS}(\%)$
30	0.00	0.64	0.53	1.01	0.02	0.98	1.91	0.10
60	-0.01	-1.24	-0.33	-0.90	0.07	4.57	5.44	3.88
91	-0.02	-2.46	-0.62	-2.32	0.13	6.83	11.86	10.35
122	-0.03	-2.92	-0.93	-4.36	0.13	6.10	13.73	12.34
152	-0.04	-3.01	-1.01	-6.16	0.13	6.35	16.27	14.97
182	-0.04	-2.78	-0.95	-7.04	0.14	5.94	17.12	15.86
273	-0.02	-1.25	-0.62	-6.71	0.11	2.94	13.63	12.20
365	-0.01	-0.56	-0.45	-7.37	0.11	2.10	13.70	12.30
			$\mathcal{MRP}^{(3)}$		VRP-LJV			
Maturity(days)	b_0	t-stat	β	t-stat	$\beta \times 10^2$	t-stat	$R^2(\%)$	$R^2_{OOS}(\%)$
30	0.00	0.25	-0.12	-0.50	0.02	1.12	1.43	-0.38
60	-0.01	-0.74	0.01	0.11	0.07	3.77	4.96	3.39
91	-0.02	-1.78	0.10	1.10	0.13	6.95	10.29	8.78
122	-0.02	-1.96	0.15	2.33	0.13	5.54	10.55	9.16
152	-0.02	-1.77	0.15	2.78	0.13	4.89	11.58	10.27
182	-0.03	-1.94	0.15	4.26	0.14	5.50	13.51	12.25
273	-0.02	-1.08	0.09	5.97	0.11	2.81	12.02	10.58
365	-0.01	-0.35	0.05	5.69	0.12	2.07	9.23	7.81
			$\mathcal{MRP}^{(4)}$		VRP-LJV			
Maturity(days)	b_0	t-stat	β	t-stat	$\beta \times 10^2$	t-stat	$R^2(\%)$	$R^2_{OOS}(\%)$
30	0.00	0.46	-0.69	-0.98	0.01	0.82	1.77	-0.04
60	-0.01	-0.77	0.06	0.17	0.07	3.55	4.97	3.40
91	-0.02	-1.85	0.35	1.32	0.13	6.46	10.47	8.96
122	-0.02	-2.27	0.67	3.18	0.14	5.94	12.39	11.00
152	-0.03	-2.48	0.82	5.18	0.15	5.95	16.50	15.20
182	-0.03	-2.43	0.77	6.29	0.15	5.19	18.34	17.09
273	-0.02	-1.06	0.44	7.34	0.11	3.46	13.00	11.57
365	0.00	0.13	0.22	2.97	0.11	1.65	7.82	6.40

Table 12

Estimates of the Price of Risk: This table presents the estimation results of the beta pricing model (40). The model is estimated using daily returns on the 100 portfolios formed on size and book-to-market. The column Only FF5 presents results when only the five Fama French factors are used. The data are from January 1996 to August 2015. I report the Fama and MacBeth (1973) t-ratio under correctly specified models (t_{FM}), the Shanken (1992) t-ratio (t_S), the Jagannathan and Wang (1998) t-ratio under correctly specified models that account for the EIV problem (t_{JW}), and the Kan, Robotti, and Shanken (2013) misspecification-robust t-ratios (t_{KRS}). The table also presents the sample cross-sectional R^2 of the beta pricing model (40). p($R^2 = 1$) is the p-value for the test of H0 : $R^2 = 1$, p($R^2 = 0$) is the p-value for the test of H0 : $R^2 = 0$, and p(W) is the p-value of Wald test under the null hypothesis that all prices of risk are equal to zero. se($\widehat{R}^2$) is the standard error of $\widehat{R}^2$ under the assumption that $0 < R^2 < 1$.

Maturity	Only FF5	30	60	91	122	152	182	273	365
λ_0	0.091	0.142	0.148	0.147	0.146	0.145	0.145	0.141	0.138
t_{FM}	8.486	12.925	13.457	13.417	13.302	13.227	13.115	12.643	12.311
t_S	8.466	7.866	8.075	8.153	8.156	8.180	8.305	8.271	8.111
t_{JW}	7.636	6.921	6.757	6.859	7.107	7.192	7.280	7.545	7.496
t_{KRS}	6.747	6.755	6.687	6.939	7.240	7.321	7.301	7.126	6.677
λ_{MKT}	-0.031	-0.019	-0.037	-0.041	-0.046	-0.051	-0.055	-0.060	-0.063
t_{FM}	-2.850	-5.311	-5.596	-5.556	-5.484	-5.446	-5.426	-5.220	-5.082
t_S	-2.848	-4.351	-4.549	-4.549	-4.512	-4.499	-4.529	-4.423	-4.316
t_{JW}	-3.368	-4.691	-4.718	-4.730	-4.810	-4.814	-4.825	-4.858	-4.755
t_{KRS}	-3.033	-4.905	-4.915	-4.966	-5.027	-5.004	-4.935	-4.667	-4.332
$\lambda_{\mathcal{M}^{(2)}}$		0.531	0.379	0.299	0.257	0.225	0.191	0.127	0.095
t_{FM}		26.743	24.380	21.606	20.356	18.932	16.125	12.161	10.754
t_S		16.787	15.057	13.462	12.791	11.978	10.400	8.061	7.170
t_{JW}		6.434	5.850	5.495	5.354	5.098	4.849	4.283	3.908
t_{KRS}		7.140	6.366	5.770	5.484	5.018	4.386	3.195	2.685
$\lambda_{\mathcal{M}^{(3)}}$		-0.275	-0.453	-0.407	-0.338	-0.284	-0.239	-0.137	-0.085
t_{FM}		11.612	12.142	13.109	13.612	13.830	13.225	11.453	10.723
t_S		7.204	7.456	8.170	8.582	8.801	8.605	7.664	7.215
t_{JW}		4.047	3.993	3.936	3.962	3.915	3.858	3.558	3.327
t_{KRS}		3.699	3.970	4.003	4.028	3.992	3.837	3.205	2.814
$\lambda_{\mathcal{M}^{(4)}}$		-0.027	0.175	0.163	0.124	0.100	0.084	0.042	0.022
t_{FM}		10.993	10.726	12.280	12.927	13.702	14.649	15.292	15.299
t_S		6.829	6.586	7.657	8.151	8.728	9.574	10.334	10.410
t_{JW}		3.680	3.904	4.066	4.108	4.158	4.260	4.112	3.895
t_{KRS}		3.219	3.659	4.008	4.167	4.254	4.417	4.329	4.010

Table 12
Estimates of the Price of Risk, continued

Maturity	Only FF5	30	60	91	122	152	182	273	365
λ_{SMB}	0.053	0.102	0.110	0.112	0.111	0.109	0.113	0.106	0.103
t_{FM}	1.097	1.146	1.182	1.162	1.127	1.114	1.127	1.149	1.161
t_S	1.097	1.120	1.153	1.136	1.102	1.089	1.104	1.128	1.141
t_{JW}	1.047	1.069	1.098	1.075	1.038	1.020	1.031	1.052	1.063
t_{KRS}	1.040	1.078	1.103	1.078	1.039	1.019	1.028	1.047	1.058
λ_{HML}	0.031	0.155	0.176	0.183	0.185	0.183	0.182	0.177	0.174
t_{FM}	0.958	1.141	1.077	1.098	1.105	1.099	1.096	1.096	1.089
t_S	0.958	1.108	1.044	1.066	1.074	1.069	1.068	1.071	1.064
t_{JW}	0.931	1.080	1.027	1.050	1.058	1.052	1.052	1.053	1.045
t_{KRS}	0.926	1.066	1.014	1.038	1.047	1.043	1.044	1.046	1.038
λ_{RMW}	0.096	0.047	0.052	0.045	0.030	0.019	0.016	0.011	0.016
t_{FM}	2.464	2.696	3.268	3.299	3.090	2.919	2.789	2.514	2.386
t_S	2.461	2.005	2.407	2.450	2.308	2.192	2.129	1.961	1.869
t_{JW}	1.930	1.847	2.259	2.318	2.223	2.146	2.097	1.951	1.860
t_{KRS}	1.646	1.662	2.087	2.165	2.082	2.012	1.973	1.811	1.714
λ_{CMA}	-0.074	-0.249	-0.298	-0.316	-0.309	-0.303	-0.313	-0.326	-0.322
t_{FM}	0.281	-0.876	-1.184	-1.229	-1.130	-1.114	-1.229	-1.427	-1.497
t_S	0.280	-0.613	-0.819	-0.858	-0.795	-0.789	-0.887	-1.056	-1.114
t_{JW}	0.029	-0.634	-0.849	-0.907	-0.853	-0.854	-0.956	-1.145	-1.188
t_{KRS}	0.226	-0.575	-0.768	-0.817	-0.763	-0.759	-0.838	-0.963	-0.983
R^2	0.011	0.665	0.692	0.717	0.741	0.763	0.784	0.809	0.818
p($R^2 = 1$)	0.000	0.026	0.059	0.060	0.066	0.075	0.083	0.111	0.148
p($R^2 = 0$)	0.074	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
p(W)	0.015	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
se($\widehat{R}^2$)	0.006	0.079	0.078	0.074	0.072	0.068	0.063	0.061	0.063

Table 13

Estimates of the Price of Risk This table presents the estimation results of the beta pricing model (40). The model is estimated using daily returns on the 100 portfolios formed on size and operating profitability. The column Only FF5 presents results when only the five Fama French factors are used. The data are from January 1996 to August 2015. I report the Fama and MacBeth (1973) t-ratio under correctly specified models (t_{FM}), the Shanken (1992) t-ratio (t_S), the Jagannathan and Wang (1998) t-ratio under correctly specified models that account for the EIV problem (t_{JW}), and the Kan, Robotti, and Shanken (2013) misspecification-robust t-ratios (t_{KRS}). The table also presents the sample cross-sectional R^2 of the beta pricing model (40). p($R^2 = 1$) is the p-value for the test of H0 : $R^2 = 1$, p($R^2 = 0$) is the p-value for the test of H0 : $R^2 = 0$, and p(W) is the p-value of Wald test under the null hypothesis that all prices of risk are equal to zero. se($\widehat{R}^2$) is the standard error of $\widehat{R}^2$ under the assumption that $0 < R^2 < 1$.

Maturity	Only FF5	30	60	91	122	152	182	273	365
λ_0	0.046	0.089	0.087	0.079	0.077	0.074	0.072	0.070	0.071
t_{FM}	4.272	7.566	7.329	6.755	6.524	6.272	6.062	5.947	6.057
t_S	4.267	5.928	5.629	5.196	5.133	5.022	4.950	4.965	5.070
t_{JW}	3.912	5.271	4.920	4.586	4.594	4.655	4.765	4.871	4.984
t_{KRS}	3.619	4.506	5.009	4.739	4.647	4.579	4.527	4.483	4.623
λ_{MKT}	-0.000	-0.015	-0.015	-0.010	-0.013	-0.013	-0.015	-0.017	-0.020
t_{FM}	-0.543	-2.586	-2.451	-2.106	-1.981	-1.852	-1.761	-1.664	-1.711
t_S	-0.543	-2.365	-2.222	-1.911	-1.814	-1.708	-1.635	-1.561	-1.608
t_{JW}	-0.658	-2.516	-2.294	-1.981	-1.905	-1.840	-1.803	-1.768	-1.825
t_{KRS}	-0.643	-2.378	-2.408	-2.072	-1.969	-1.871	-1.794	-1.745	-1.811
$\lambda_{\mathcal{M}^{(2)}}$		0.223	0.054	0.003	-0.016	-0.017	-0.012	-0.002	0.002
t_{FM}		7.540	7.616	7.469	7.193	6.990	6.868	7.101	7.547
t_S		5.944	5.886	5.781	5.693	5.627	5.636	5.957	6.350
t_{JW}		4.256	4.077	3.986	4.039	4.108	4.263	4.557	4.836
t_{KRS}		2.578	3.280	3.611	3.686	3.739	3.816	4.125	4.475
$\lambda_{\mathcal{M}^{(3)}}$		0.659	0.489	0.371	0.257	0.186	0.136	0.056	0.029
t_{FM}		5.535	6.931	7.865	8.201	8.120	7.995	8.002	7.997
t_S		4.374	5.372	6.103	6.508	6.553	6.575	6.724	6.736
t_{JW}		3.527	3.860	4.021	4.186	4.268	4.452	4.728	4.826
t_{KRS}		2.509	3.037	3.403	3.669	3.766	3.936	4.398	4.568
$\lambda_{\mathcal{M}^{(4)}}$		-0.773	-0.361	-0.222	-0.128	-0.084	-0.058	-0.020	-0.008
t_{FM}		3.206	4.272	5.418	5.965	6.069	6.158	6.339	6.342
t_S		2.536	3.314	4.207	4.736	4.900	5.064	5.324	5.340
t_{JW}		2.340	2.805	3.173	3.427	3.514	3.694	3.990	4.077
t_{KRS}		1.563	1.823	2.252	2.590	2.759	2.986	3.475	3.638

Table 13

Estimates of the Price of Risk, continued

Maturity	Only FF5	30	60	91	122	152	182	273	365
λ_{SMB}	0.054	0.064	0.067	0.064	0.062	0.056	0.057	0.059	0.061
t_{FM}	1.163	1.263	1.279	1.286	1.250	1.235	1.230	1.199	1.194
t_S	1.163	1.253	1.269	1.275	1.241	1.227	1.223	1.193	1.189
t_{JW}	1.099	1.173	1.192	1.202	1.175	1.163	1.160	1.131	1.128
t_{KRS}	1.102	1.174	1.193	1.204	1.177	1.166	1.163	1.134	1.130
λ_{HML}	0.019	0.079	0.063	0.066	0.067	0.075	0.076	0.076	0.079
t_{FM}	0.193	-0.208	-0.158	-0.070	0.020	0.116	0.102	0.107	0.133
t_S	0.193	-0.185	-0.140	-0.062	0.018	0.105	0.093	0.099	0.123
t_{JW}	0.193	-0.182	-0.138	-0.060	0.017	0.102	0.091	0.097	0.123
t_{KRS}	0.184	-0.179	-0.134	-0.058	0.017	0.098	0.088	0.094	0.117
λ_{RMW}	0.095	0.077	0.102	0.118	0.123	0.121	0.117	0.107	0.101
t_{FM}	2,067	1.837	1.845	1.875	1.831	1.843	1.834	1.840	1.848
t_S	2.067	1.809	1.814	1.844	1.804	1.818	1.811	1.821	1.828
t_{JW}	1.701	1.509	1.513	1.534	1.499	1.510	1.502	1.506	1.513
t_{KRS}	1.701	1.518	1.522	1.542	1.503	1.513	1.505	1.506	1.513
λ_{CMA}	-0.078	-0.213	-0.199	-0.190	-0.184	-0.183	-0.181	-0.180	-0.182
t_{FM}	-0.530	-1.873	-1.854	-1.770	-1.667	-1.584	-1.523	-1.426	-1.440
t_S	-0.529	-1.582	-1.543	-1.473	-1.411	-1.358	-1.324	-1.258	-1.273
t_{JW}	-0.562	-1.528	-1.533	-1.470	-1.439	-1.415	-1.401	-1.354	-1.381
t_{KRS}	-0.524	-1.486	-1.455	-1.352	-1.271	-1.260	-1.256	-1.207	-1.238
R^2	0.025	0.318	0.429	0.525	0.569	0.606	0.627	0.629	0.625
p($R^2 = 1$)	0.000	0.002	0.024	0.147	0.235	0.312	0.288	0.148	0.102
p($R^2 = 0$)	0.122	0.007	0.001	0.000	0.000	0.000	0.000	0.000	0.000
p(W)	0.134	0.031	0.004	0.001	0.001	0.000	0.000	0.000	0.000
se($\widehat{R}^2$)	0.018	0.186	0.164	0.130	0.111	0.096	0.088	0.085	0.085

Table 14

Estimates of the Price of Risk: This table presents the estimation results of the beta pricing model (41). The model is estimated using daily returns on the 100 portfolios formed on size and book-to-market. The data are from January 1996 to August 2015. I report the Fama and MacBeth (1973) t-ratio under correctly specified models (t_{FM}), the Shanken (1992) t-ratio (t_S), the Jagannathan and Wang (1998) t-ratio under correctly specified models that account for the EIV problem (t_{JW}), and the Kan, Robotti, and Shanken (2013) misspecification-robust t-ratios (t_{KRS}). The table also presents the sample cross-sectional R^2 of the beta pricing model ((41)). p($R^2 = 1$) is the p-value for the test of H0 : $R^2 = 1$, p($R^2 = 0$) is the p-value for the test of H0 : $R^2 = 0$, and p(W) is the p-value of Wald test under the null hypothesis that all prices of risk are equal to zero. se($\widehat{R}^2$) is the standard error of $\widehat{R}^2$ under the assumption that $0 < R^2 < 1$.

Maturity	Only FF5	30	60	91	122	152	182	273	365
λ_0	0.091	0.118	0.129	0.129	0.126	0.124	0.118	0.122	0.134
t_{FM}	8.486	10.862	11.751	11.822	11.548	11.294	10.578	11.180	12.389
t_S	8.465	7.247	8.097	7.925	7.419	7.408	7.081	7.667	8.607
t_{JW}	7.636	6.825	7.397	7.182	6.879	6.724	6.779	6.913	7.799
t_{KRS}	6.747	5.861	6.727	7.044	6.985	6.745	6.441	7.129	7.754
λ_{MKT}	-0.031	-0.019	-0.054	-0.064	-0.072	-0.070	-0.063	-0.059	-0.047
t_{FM}	-2.850	-4.235	-4.703	-4.663	-4.494	-4.348	-4.050	-4.305	-4.826
t_S	-2.848	-3.639	-4.097	-4.014	-3.790	-3.702	-3.470	-3.744	-4.227
t_{JW}	-3.368	-4.266	-4.792	-4.623	-4.342	-4.213	-4.224	-4.109	-4.798
t_{KRS}	-3.034	-3.993	-4.638	-4.707	-4.478	-4.321	-4.174	-4.242	-4.910
$\lambda_{\mathcal{MRP}^{(2)}}$		-0.568	0.107	0.454	0.511	0.467	0.373	0.212	0.094
t_{FM}		-12.655	-8.614	-3.524	-0.451	1.889	2.833	4.647	5.363
t_S		-8.572	-6.016	-2.394	-0.294	1.257	1.924	3.235	3.778
t_{JW}		-4.417	-3.454	-1.501	-0.221	1.092	1.746	3.094	3.164
t_{KRS}		-3.318	-2.526	-1.041	-0.140	0.707	1.163	2.165	2.079
$\lambda_{\mathcal{MRP}^{(3)}}$		0.699	0.255	0.119	0.061	0.042	0.032	0.017	0.012
t_{FM}		21.586	21.230	18.069	15.053	10.891	9.792	8.465	9.361
t_S		14.757	15.005	12.444	9.956	7.330	6.726	5.956	6.674
t_{JW}		5.022	5.294	4.285	3.582	3.971	4.059	4.131	4.043
t_{KRS}		5.405	6.346	4.907	4.016	3.433	3.323	3.394	3.067
$\lambda_{\mathcal{MRP}^{(4)}}$		-0.897	-0.098	0.215	0.267	0.242	0.181	0.090	0.023
t_{FM}		13.020	11.152	8.522	6.364	3.163	1.942	-1.007	-4.455
t_S		8.841	7.814	5.809	4.159	2.110	1.321	-0.701	-3.127
t_{JW}		3.932	4.455	3.584	2.811	1.785	1.170	-0.694	-3.037
t_{KRS}		2.999	3.257	2.873	2.163	1.196	0.793	-0.490	-1.647

Table 14

Estimates of the Price of Risk, continued

Maturity	Only FF5	30	60	91	122	152	182	273	365
λ_{SMB}	0.053	0.111	0.113	0.117	0.116	0.141	0.146	0.145	0.127
t_{FM}	1.097	1.253	1.208	1.109	1.050	1.069	1.116	1.157	1.115
t_S	1.097	1.232	1.189	1.090	1.030	1.050	1.097	1.139	1.098
t_{JW}	1.047	1.184	1.144	1.045	0.981	0.996	1.040	1.077	1.036
t_{KRS}	1.040	1.188	1.142	1.040	0.972	0.992	1.037	1.073	1.028
λ_{HML}	0.031	0.118	0.165	0.185	0.182	0.141	0.114	0.115	0.148
t_{FM}	0.958	1.123	1.074	1.189	1.229	1.259	1.122	1.152	1.050
t_S	0.958	1.099	1.054	1.164	1.199	1.230	1.098	1.130	1.030
t_{JW}	0.931	1.067	1.016	1.119	1.157	1.197	1.072	1.111	1.010
t_{KRS}	0.926	1.060	1.002	1.105	1.149	1.190	1.062	1.105	1.004
λ_{RMW}	0.096	0.100	0.061	0.027	-0.021	0.006	0.042	0.040	0.072
t_{FM}	2.464	2.244	2.431	2.433	2.160	2.383	2.769	2.938	3.326
t_S	2.461	1.779	1.968	1.934	1.668	1.867	2.198	2.370	2.696
t_{JW}	1.930	1.528	1.681	1.648	1.392	1.603	1.889	2.135	2.404
t_{KRS}	1.646	1.609	1.479	1.484	1.282	1.520	1.811	2.070	2.269
λ_{CMA}	-0.072	-0.188	-0.265	-0.319	-0.331	-0.336	-0.335	-0.302	-0.349
t_{FM}	0.281	-0.808	-1.379	-1.915	-2.035	-2.403	-2.511	-1.957	-2.237
t_S	0.281	-0.608	-1.064	-1.445	-1.486	-1.783	-1.893	-1.504	-1.736
t_{JW}	0.290	-0.648	-1.078	-1.404	-1.424	-1.828	-2.001	-1.624	-1.960
t_{KRS}	0.226	-0.566	-0.946	-1.274	-1.306	-1.728	-1.906	-1.557	-1.837
R^2	0.011	0.446	0.475	0.558	0.643	0.740	0.790	0.857	0.816
p($R^2 = 1$)	0.000	0.000	0.002	0.008	0.032	0.032	0.071	0.377	0.140
p($R^2 = 0$)	0.074	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
p(W)	0.015	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
se($\widehat{R}^2$)	0.006	0.077	0.086	0.097	0.104	0.067	0.055	0.038	0.062

Table 15

Estimates of the Price of Risk: This table presents the estimation results of the beta pricing model (41). The model is estimated using daily returns on the 100 portfolios formed on size and operating profitability. The data are from January 1996 to August 2015. I report the Fama and MacBeth (1973) t-ratio under correctly specified models (t_{FM}), the Shanken (1992) t-ratio (t_S), the Jagannathan and Wang (1998) t-ratio under correctly specified models that account for the EIV problem (t_{JW}), and the Kan, Robotti, and Shanken (2013) misspecification-robust t-ratios (t_{KRS}). The table also presents the sample cross-sectional R^2 of the beta pricing model ((41)). p($R^2 = 1$) is the p-value for the test of H0 : $R^2 = 1$, p($R^2 = 0$) is the p-value for the test of H0 : $R^2 = 0$, and p(W) is the p-value of Wald test under the null hypothesis that all prices of risk are equal to zero. se($\widehat{R}^2$) is the standard error of $\widehat{R}^2$ under the assumption that $0 < R^2 < 1$.

Maturity	Only FF5	30	60	91	122	152	182	273	365
λ_0	0.046	0.070	0.073	0.066	0.061	0.046	0.047	0.049	0.060
t_{FM}	4.272	6.176	6.381	5.919	5.478	4.230	4.245	4.224	5.025
t_S	4.267	5.055	4.941	4.636	4.471	3.567	3.628	3.602	4.201
t_{JW}	3.912	4.592	4.465	4.337	4.397	3.213	3.263	3.127	3.991
t_{KRS}	3.619	4.135	4.673	4.414	4.337	2.943	2.950	2.742	3.615
λ_{MKT}	-0.000	-0.005	-0.009	-0.004	-0.009	0.000	-0.004	-0.014	-0.021
t_{FM}	-0.543	-1.684	-1.814	-1.499	-1.243	-0.549	-0.585	-0.656	-1.178
t_S	-0.543	-1.574	-1.658	-1.379	-1.163	-0.520	-0.556	-0.622	-1.105
t_{JW}	-0.658	-1.717	-1.772	-1.542	-1.355	-0.600	-0.626	-0.673	-1.234
t_{KRS}	-0.643	-1.716	-1.845	-1.560	-1.336	-0.576	-0.600	-0.636	-1.209
$\lambda_{\mathcal{MRP}^{(2)}}$		-1.091	-0.712	-0.500	-0.343	-0.263	-0.191	-0.074	-0.019
t_{FM}		-6.131	-7.667	-8.271	-8.143	-7.428	-7.522	-10.236	-12.774
t_S		-5.054	-5.990	-6.536	-6.700	-6.308	-6.469	-8.802	-10.815
t_{JW}		-3.801	-4.179	-4.588	-5.128	-5.473	-5.792	-6.646	-5.735
t_{KRS}		-2.670	-3.337	-3.965	-4.074	-4.241	-4.076	-5.327	-6.283
$\lambda_{\mathcal{MRP}^{(3)}}$		0.171	0.045	0.024	0.009	-0.003	-0.003	-0.001	0.002
t_{FM}		2.730	4.403	5.135	4.853	3.661	3.664	4.634	6.235
t_S		2.250	3.442	4.057	3.992	3.107	3.148	3.968	5.237
t_{JW}		2.069	2.808	3.131	3.334	2.962	3.012	3.524	3.773
t_{KRS}		1.169	2.112	2.624	2.795	2.063	1.895	2.271	3.208
$\lambda_{\mathcal{MRP}^{(4)}}$		-1.430	-0.560	-0.332	-0.195	-0.131	-0.082	-0.017	0.004
t_{FM}		2.513	3.756	4.650	4.876	4.477	4.661	7.456	10.033
t_S		2.074	2.936	3.674	4.009	3.798	4.002	6.396	8.540
t_{JW}		1.834	2.477	2.975	3.419	3.508	3.829	5.210	5.608
t_{KRS}		1.154	1.528	2.162	2.348	2.504	2.477	3.600	6.087

Table 15
Estimates of the Price of Risk, continued

Maturity	Only FF5	30	60	91	122	152	182	273	365
λ_{SMB}	0.054	0.053	0.061	0.061	0.057	0.036	0.040	0.052	0.058
t_{FM}	1.163	1.253	1.331	1.311	1.316	1.282	1.284	1.301	1.307
t_S	1.163	1.245	1.321	1.301	1.309	1.276	1.279	1.295	1.301
t_{JW}	1.099	1.167	1.247	1.229	1.244	1.212	1.217	1.231	1.237
t_{KRS}	1.102	1.171	1.251	1.230	1.244	1.213	1.218	1.233	1.240
λ_{HML}	0.019	0.055	0.048	0.062	0.053	0.065	0.074	0.064	0.058
t_{FM}	0.193	0.288	0.047	0.094	-0.072	0.250	0.228	0.088	0.004
t_S	0.193	0.263	0.042	0.084	-0.066	0.232	0.213	0.082	0.004
t_{JW}	0.193	0.254	0.040	0.080	-0.061	0.219	0.200	0.076	0.003
t_{KRS}	0.184	0.241	0.039	0.077	-0.060	0.212	0.194	0.073	0.003
λ_{RMW}	0.095	0.103	0.127	0.143	0.150	0.149	0.143	0.144	0.123
t_{FM}	2.067	1.951	1.939	1.928	1.870	1.921	1.872	1.888	1.926
t_S	2.067	1.928	1.908]	1.898	1.847	1.902	1.855	1.871]	1.906
t_{JW}	1.701	1.605	1.588	1.578	1.535	1.576	1.540	1.553	1.574
t_{KRS}	1.701	1.609	1.595	1.584	1.538	1.574	1.538	1.552	1.572
λ_{CMA}	-0.078	-0.169	-0.175	-0.181	-0.172	-0.145	-0.162	-0.192	-0.191
t_{FM}	-0.530	-1.395	-1.560	-1.648	-1.677	-1.235	-1.415	-1.881	-1.840
t_S	-0.530	-1.216	-1.305	-1.390	-1.458	-1.099	-1.271	-1.687	-1.627
t_{JW}	-0.562	-1.204	-1.267	-1.373	-1.446	-1.167	-1.332	-1.687	-1.654
t_{KRS}	-0.524	-1.120	-1.182	-1.300	-1.371	-1.095	-1.236	-1.580	-1.563
R^2	0.025	0.274	0.469	0.583	0.624	0.645	0.637	0.619	0.619
p($R^2 = 1$)	0.000	0.001	0.063	0.283	0.317	0.369	0.313	0.274	0.288
p($R^2 = 0$)	0.122	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000
p(W)	0.134	0.027	0.002	0.000	0.000	0.000	0.000	0.000	0.000
se($\widehat{R}^2$)	0.018	0.156	0.141	0.101	0.087	0.083	0.086	0.087	0.088

Table 16

Estimates of the Price of Risk: Controlling for VRP This table presents the estimation results of the beta pricing model (42). The model is estimated using monthly returns on the 100 portfolios formed on size and operating profitability. The data are from January 1996 to August 2015. I report the Fama and MacBeth (1973) t-ratio under correctly specified models (t_{FM}), the Shanken (1992) t-ratio (t_S), the Jagannathan and Wang (1998) t-ratio under correctly specified models that account for the EIV problem (t_{JW}), and the Kan, Robotti, and Shanken (2013) misspecification-robust t-ratios (t_{KRS}). The table also presents the sample cross-sectional R^2 of the beta pricing model ((41)). p($R^2 = 1$) is the p-value for the test of H0 : $R^2 = 1$, p($R^2 = 0$) is the p-value for the test of H0 : $R^2 = 0$, and p(W) is the p-value of Wald test under the null hypothesis that all prices of risk are equal to zero. se($\widehat{R}^2$) is the standard error of $\widehat{R}^2$ under the assumption that $0 < R^2 < 1$.

Maturity	30	60	91	122	152	182	273	365
λ_0	0.069	0.067	0.067	0.067	0.068	0.068	0.068	0.068
t_{FM}	11.581	11.449	11.436	11.473	11.568	11.639	11.667	11.687
t_S	10.327	10.320	10.254	10.194	10.156	10.139	10.077	10.041
t_{JW}	10.292	10.824	11.297	11.537	11.665	11.733	11.812	11.847
t_{KRS}	6.985	7.025	7.121	7.216	7.294	7.369	7.421	7.444
$\lambda_{\mathcal{M}^{(2)}}$	0.380	0.251	0.189	0.154	0.129	0.105	0.068	0.050
t_{FM}	5.258	4.486	3.896	3.459	3.106	2.792	2.256	1.860
t_S	5.002	4.278	3.701	3.270	2.916	2.610	2.099	1.726
t_{JW}	2.560	2.073	1.735	1.494	1.322	1.179	0.941	0.772
t_{KRS}	2.294	1.784	1.464	1.256	1.119	1.017	0.828	0.685
$\lambda_{\mathcal{M}^{(3)}}$	-0.708	-0.629	-0.471	-0.351	-0.275	-0.216	-0.109	-0.065
t_{FM}	3.797	3.342	2.847	2.449	2.043	1.659	1.057	0.738
t_S	3.630	3.201	2.714	2.323	1.925	1.556	0.987	0.687
t_{JW}	2.121	1.692	1.350	1.109	0.905	0.722	0.444	0.306
t_{KRS}	1.785	1.370	1.084	0.897	0.741	0.608	0.388	0.271
$\lambda_{\mathcal{M}^{(4)}}$	0.166	0.299	0.222	0.153	0.115	0.088	0.038	0.019
t_{FM}	3.474	3.307	3.100	2.930	2.708	2.461	2.045	1.772
t_S	3.348	3.192	2.977	2.803	2.574	2.327	1.924	1.663
t_{JW}	1.947	1.713	1.504	1.363	1.232	1.093	0.870	0.741
t_{KRS}	1.657	1.412	1.231	1.123	1.025	0.934	0.772	0.667
λ_{VRP}	-0.002	0.000	0.002	0.002	0.003	0.003	0.003	0.003
t_{FM}	1.175	1.319	1.449	1.503	1.554	1.584	1.587	1.582
t_S	1.129	1.272	1.395	1.442	1.485	1.509	1.507	1.499
t_{JW}	0.798	0.903	0.998	1.045	1.091	1.115	1.107	1.098
t_{KRS}	0.644	0.726	0.809	0.847	0.898	0.938	0.953	0.958
R^2	0.077	0.074	0.082	0.091	0.101	0.108	0.114	0.118
p($R^2 = 1$)	0.002	0.001	0.001	0.002	0.002	0.002	0.003	0.003
p($R^2 = 0$)	0.067	0.132	0.136	0.126	0.103	0.089	0.081	0.075
p(W)	0.218	0.168	0.094	0.062	0.043	0.037	0.036	0.036
se($\widehat{R}^2$)	0.054	0.056	0.058	0.061	0.064	0.065	0.066	0.067

Table 17

Estimates of the Price of Risk: Controlling for LJV measure This table presents the estimation results of the beta pricing model (43). The model is estimated using monthly returns on the 100 portfolios formed on size and operating profitability. The data are from January 1996 to August 2015. I report the Fama and MacBeth (1973) t-ratio under correctly specified models (t_{FM}), the Shanken (1992) t-ratio (t_S), the Jagannathan and Wang (1998) t-ratio under correctly specified models that account for the EIV problem (t_{JW}), and the Kan, Robotti, and Shanken (2013) misspecification-robust t-ratios (t_{KRS}). The table also presents the sample cross-sectional R^2 of the beta pricing model ((41)). p($R^2 = 1$) is the p-value for the test of H0 : $R^2 = 1$, p($R^2 = 0$) is the p-value for the test of H0 : $R^2 = 0$, and p(W) is the p-value of Wald test under the null hypothesis that all prices of risk are equal to zero. se($\widehat{R}^2$) is the standard error of $\widehat{R}^2$ under the assumption that $0 < R^2 < 1$.

Maturity (days)	30	60	91	122	152	182	273	365
λ_0	0.074	0.073	0.073	0.073	0.073	0.073	0.073	0.073
t_{FM}	11.975	11.908	11.840	11.823	11.876	11.903	11.886	11.893
t_S	10.618	10.645	10.494	10.454	10.303	10.228	10.209	10.155
t_{JW}	10.389	10.812	11.053	11.097	11.249	11.342	11.406	11.476
t_{KRS}	5.404	5.362	5.348	5.336	5.380	5.405	5.393	5.422
$\lambda_{\mathcal{M}^{(2)}}$	0.348	0.199	0.151	0.119	0.100	0.081	0.049	0.036
t_{FM}	4.136	3.332	2.724	2.296	1.982	1.701	1.273	1.091
t_S	3.949	3.185	2.591	2.181	1.864	1.592	1.191	1.017
t_{JW}	1.933	1.482	1.179	0.963	0.822	0.699	0.513	0.435
t_{KRS}	1.309	1.013	0.822	0.678	0.589	0.517	0.390	0.336
$\lambda_{\mathcal{M}^{(3)}}$	-1.032	-0.614	-0.477	-0.340	-0.268	-0.217	-0.109	-0.065
t_{FM}	3.559	3.076	2.502	2.117	1.779	1.450	1.018	0.949
t_S	3.417	2.956	2.389	2.018	1.680	1.362	0.955	0.889
t_{JW}	1.879	1.491	1.148	0.927	0.766	0.614	0.415	0.380
t_{KRS}	1.216	0.970	0.767	0.631	0.531	0.443	0.314	0.295
$\lambda_{\mathcal{M}^{(4)}}$	0.876	0.439	0.297	0.187	0.143	0.112	0.047	0.023
t_{FM}	3.823	3.578	3.250	3.014	2.815	2.566	2.203	2.145
t_S	3.703	3.467	3.130	2.899	2.684	2.433	2.085	2.024
t_{JW}	2.106	1.822	1.556	1.379	1.265	1.127	0.924	0.882
t_{KRS}	1.393	1.204	1.050	0.946	0.881	0.813	0.702	0.688
λ_{LJV}	-1.044	-0.892	-0.688	-0.526	-0.669	-0.658	-0.547	-0.558
t_{FM}	1.549	1.316	1.119	1.045	0.894	0.843	0.831	0.800
t_S	1.477	1.259	1.066	0.995	0.844	0.793	0.781	0.751
t_{JW}	0.771	0.636	0.534	0.493	0.425	0.407	0.408	0.396
t_{KRS}	0.502	0.417	0.353	0.326	0.281	0.271	0.271	0.264
R^2	0.064	0.059	0.065	0.066	0.077	0.082	0.081	0.084
p($R^2 = 1$)	0.001	0.001	0.001	0.001	0.001	0.002	0.002	0.002
p($R^2 = 0$)	0.128	0.172	0.158	0.167	0.132	0.111	0.117	0.110
p(W)	0.091	0.042	0.022	0.016	0.014	0.016	0.017	0.016
se($\widehat{R}^2$)	0.037	0.035	0.036	0.038	0.043	0.046	0.048	0.050

Table 18

Estimates of the Price of Risk:Controlling for innovation in market risk neutral variance This table presents the estimation results of the beta pricing model (44). The model is estimated using monthly returns on the 100 portfolios formed on size and operating profitability. The data are from January 1996 to August 2015. I report the Fama and MacBeth (1973) t-ratio under correctly specified models (t_{FM}), the Shanken (1992) t-ratio (t_S), the Jagannathan and Wang (1998) t-ratio under correctly specified models that account for the EIV problem (t_{JW}), and the Kan, Robotti, and Shanken (2013) misspecification-robust t-ratios (t_{KRS}). The table also presents the sample cross-sectional R^2 of the beta pricing model ((41)). p($R^2 = 1$) is the p-value for the test of H0 : $R^2 = 1$, p($R^2 = 0$) is the p-value for the test of H0 : $R^2 = 0$, and p(W) is the p-value of Wald test under the null hypothesis that all prices of risk are equal to zero. se($\widehat{R}^2$) is the standard error of $\widehat{R}^2$ under the assumption that $0 < R^2 < 1$.

Maturity	30	60	91	122	152	182	273	365
λ_0	0.071	0.070	0.069	0.068	0.069	0.069	0.069	0.069
t_{FM}	11.925	11.685	11.544	11.465	11.483	11.480	11.494	11.535
t_S	10.522	10.446	10.307	10.172	10.071	9.993	9.914	9.887
t_{JW}	10.469	10.891	11.274	11.410	11.473	11.476	11.599	11.672
t_{KRS}	7.428	7.487	7.580	7.514	7.570	7.583	7.532	7.535
$\lambda_{\mathcal{M}^{(2)}}$	0.442	0.276	0.203	0.162	0.136	0.112	0.072	0.053
t_{FM}	5.166	4.443	3.883	3.378	2.983	2.631	2.061	1.655
t_S	4.891	4.221	3.682	3.191	2.798	2.458	1.916	1.533
t_{JW}	2.514	2.072	1.773	1.508	1.313	1.153	0.892	0.711
t_{KRS}	2.158	1.735	1.472	1.251	1.099	0.986	0.779	0.624
$\lambda_{\mathcal{M}^{(3)}}$	-1.102	-0.703	-0.480	-0.358	-0.283	-0.224	-0.114	-0.068
t_{FM}	3.701	3.278	2.823	2.371	1.931	1.515	0.876	0.538
t_S	3.523	3.130	2.686	2.248	1.819	1.420	0.817	0.500
t_{JW}	2.063	1.690	1.383	1.114	0.886	0.684	0.379	0.229
t_{KRS}	1.679	1.330	1.089	0.889	0.720	0.575	0.332	0.203
$\lambda_{\mathcal{M}^{(4)}}$	0.368	0.315	0.210	0.151	0.116	0.090	0.039	0.020
t_{FM}	3.384	3.244	3.076	2.871	2.624	2.350	1.895	1.601
t_S	3.249	3.123	2.951	2.746	2.494	2.222	1.783	1.502
t_{JW}	1.905	1.720	1.543	1.382	1.233	1.078	0.828	0.686
t_{KRS}	1.569	1.369	1.230	1.118	1.013	0.918	0.734	0.617
$\lambda_{\Delta VAR}$	0.628	0.282	0.136	0.045	0.002	-0.028	-0.028	-0.024
t_{FM}	1.866	1.499	1.513	1.360	1.268	1.144	1.264	1.227
t_S	1.777	1.436	1.447	1.301	1.208	1.086	1.200	1.164
t_{JW}	2.187	1.627	1.560	1.435	1.302	1.167	1.306	1.293
t_{KRS}	1.776	1.391	1.340	1.241	1.136	1.029	1.219	1.271
R^2	0.084	0.078	0.084	0.091	0.102	0.108	0.115	0.120
p($R^2 = 1$)	0.002	0.002	0.001	0.002	0.002	0.002	0.003	0.003
p($R^2 = 0$)	0.066	0.128	0.133	0.121	0.099	0.085	0.074	0.067
p(W)	0.048	0.135	0.107	0.087	0.074	0.072	0.059	0.059
se($\widehat{R}^2$)	0.057	0.059	0.060	0.062	0.065	0.066	0.068	0.069

Table 19

Estimates of the Price of Risk: Controlling for innovation in market risk neutral skewness This table presents the estimation results of the beta pricing model (45). The model is estimated using monthly returns on the 100 portfolios formed on size and operating profitability. The data are from January 1996 to August 2015. I report the Fama and MacBeth (1973) t-ratio under correctly specified models (t_{FM}), the Shanken (1992) t-ratio (t_S), the Jagannathan and Wang (1998) t-ratio under correctly specified models that account for the EIV problem (t_{JW}), and the Kan, Robotti, and Shanken (2013) misspecification-robust t-ratios (t_{KRS}). The table also presents the sample cross-sectional R^2 of the beta pricing model ((41)). p($R^2 = 1$) is the p-value for the test of H0 : $R^2 = 1$, p($R^2 = 0$) is the p-value for the test of H0 : $R^2 = 0$, and p(W) is the p-value of Wald test under the null hypothesis that all prices of risk are equal to zero. se($\widehat{R}^2$) is the standard error of $\widehat{R}^2$ under the assumption that $0 < R^2 < 1$.

Maturity	30	60	91	122	152	182	273	365
λ_0	0.069	0.067	0.067	0.067	0.067	0.068	0.068	0.068
t_{FM}	11.522	11.355	11.323	11.363	11.438	11.498	11.544	11.594
t_S	10.234	10.067	9.977	9.944	9.895	9.874	9.868	9.880
t_{JW}	10.380	10.815	11.075	11.227	11.316	11.326	11.401	11.482
t_{KRS}	7.080	7.139	7.253	7.343	7.408	7.461	7.470	7.486
$\lambda_{\mathcal{M}^{(2)}}$	0.408	0.285	0.211	0.167	0.138	0.114	0.073	0.054
t_{FM}	5.149	4.323	3.735	3.269	2.911	2.614	2.065	1.676
t_S	4.890	4.091	3.520	3.069	2.715	2.428	1.913	1.550
t_{JW}	2.544	2.056	1.730	1.462	1.277	1.129	0.867	0.697
t_{KRS}	2.354	1.872	1.538	1.285	1.128	1.008	0.778	0.623
$\lambda_{\mathcal{M}^{(3)}}$	-1.288	-0.981	-0.633	-0.435	-0.328	-0.252	-0.121	-0.071
t_{FM}	3.751	3.257	2.754	2.323	1.906	1.523	0.889	0.557
t_S	3.580	3.098	2.606	2.190	1.785	1.420	0.826	0.517
t_{JW}	2.158	1.731	1.382	1.101	0.875	0.680	0.376	0.231
t_{KRS}	1.900	1.503	1.186	0.944	0.759	0.603	0.340	0.209
$\lambda_{\mathcal{M}^{(4)}}$	0.720	0.567	0.329	0.205	0.146	0.106	0.042	0.020
t_{FM}	3.451	3.286	3.078	2.880	2.643	2.390	1.928	1.630
t_S	3.322	3.154	2.937	2.740	2.498	2.248	1.808	1.526
t_{JW}	1.994	1.782	1.571	1.391	1.237	1.081	0.823	0.683
t_{KRS}	1.756	1.562	1.370	1.212	1.087	0.972	0.755	0.626
$\lambda_{\Delta SKEW}$	-4.578	-3.348	-1.706	-0.934	-0.680	-0.490	-0.181	-0.081
t_{FM}	-0.899	-0.901	-0.596	-0.130	0.035	-0.002	0.092	0.267
t_S	-0.860	-0.870	-0.579	-0.126	0.034	-0.002	0.089	0.258
t_{JW}	-1.135	-1.278	-0.883	-0.186	0.050	-0.003	0.125	0.352
t_{KRS}	-0.840	-0.917	-0.700	-0.159	0.044	-0.003	0.117	0.344
R^2	0.082	0.091	0.099	0.105	0.115	0.122	0.124	0.125
p($R^2 = 1$)	0.002	0.002	0.002	0.002	0.003	0.003	0.004	0.004
p($R^2 = 0$)	0.041	0.062	0.069	0.072	0.061	0.053	0.055	0.055
p(W)	0.081	0.048	0.046	0.060	0.055	0.049	0.057	0.065
se($\widehat{R}^2$)	0.051	0.057	0.061	0.065	0.068	0.071	0.071	0.072

Table 20

Estimates of the Price of Risk: Controlling for innovation in market risk neutral kurtosis This table presents the estimation results of the beta pricing model (46). The model is estimated using monthly returns on the 100 portfolios formed on size and operating profitability. The data are from January 1996 to August 2015. I report the Fama and MacBeth (1973) t-ratio under correctly specified models (t_{FM}), the Shanken (1992) t-ratio (t_S), the Jagannathan and Wang (1998) t-ratio under correctly specified models that account for the EIV problem (t_{JW}), and the Kan, Robotti, and Shanken (2013) misspecification-robust t-ratios (t_{KRS}). The table also presents the sample cross-sectional R^2 of the beta pricing model ((41)). p($R^2 = 1$) is the p-value for the test of H0 : $R^2 = 1$, p($R^2 = 0$) is the p-value for the test of H0 : $R^2 = 0$, and p(W) is the p-value of Wald test under the null hypothesis that all prices of risk are equal to zero. se($\widehat{R}^2$) is the standard error of $\widehat{R}^2$ under the assumption that $0 < R^2 < 1$.

Maturity	30	60	91	122	152	182	273	365
λ_0	0.070	0.068	0.067	0.067	0.067	0.068	0.068	0.068
t_{FM}	11.747	11.506	11.355	11.345	11.412	11.453	11.531	11.599
t_S	10.500	10.381	10.167	10.029	9.949	9.886	9.878	9.890
t_{JW}	10.412	10.693	10.961	11.158	11.270	11.276	11.412	11.483
t_{KRS}	7.161	7.196	7.288	7.334	7.412	7.464	7.467	7.485
$\lambda_{\mathcal{M}^{(2)}}$	0.378	0.252	0.192	0.158	0.132	0.110	0.072	0.054
t_{FM}	5.165	4.349	3.715	3.216	2.843	2.528	2.019	1.650
t_S	4.919	4.150	3.528	3.034	2.661	2.353	1.872	1.526
t_{JW}	2.501	1.997	1.668	1.414	1.230	1.087	0.850	0.688
t_{KRS}	2.207	1.709	1.417	1.203	1.058	0.954	0.757	0.613
$\lambda_{\mathcal{M}^{(3)}}$	-0.758	-0.662	-0.505	-0.387	-0.303	-0.240	-0.119	-0.070
t_{FM}	3.721	3.217	2.694	2.248	1.824	1.436	0.843	0.531
t_S	3.561	3.083	2.567	2.129	1.714	1.341	0.784	0.493
t_{JW}	2.082	1.638	1.304	1.045	0.826	0.639	0.357	0.221
t_{KRS}	1.732	1.319	1.057	0.862	0.695	0.555	0.321	0.200
$\lambda_{\mathcal{M}^{(4)}}$	0.213	0.324	0.245	0.176	0.132	0.101	0.041	0.020
t_{FM}	3.414	3.217	2.996	2.799	2.561	2.309	1.886	1.608
t_S	3.293	3.107	2.877	2.673	2.428	2.177	1.770	1.505
t_{JM}	1.920	1.686	1.489	1.335	1.189	1.044	0.807	0.675
t_{KRS}	1.616	1.376	1.227	1.120	1.014	0.921	0.734	0.617
$\lambda_{\Delta KURT}$	1.660	-0.056	-0.714	-0.690	-0.591	-0.527	-0.199	-0.087
t_{FM}	-0.057	-0.217	-0.217	-0.033	0.034	-0.031	0.183	0.260
t_S	-0.055	-0.211	-0.211	-0.032	0.033	-0.030	0.178	0.252
t_{JW}	-0.075	-0.281	-0.277	-0.042	0.043	-0.039	0.234	0.335
t_{KRS}	-0.059	-0.231	-0.235	-0.038	0.039	-0.036	0.225	0.329
R^2	0.075	0.073	0.083	0.095	0.107	0.116	0.121	0.124
p($R^2 = 1$)	0.002	0.001	0.001	0.002	0.002	0.003	0.004	0.004
p($R^2 = 0$)	0.075	0.134	0.122	0.100	0.079	0.065	0.060	0.057
p(W)	0.274	0.236	0.155	0.112	0.084	0.070	0.065	0.063
se($\widehat{R}^2$)	0.054	0.055	0.057	0.061	0.065	0.068	0.070	0.071

Table 21

Estimates of the Price of Risk: This table presents the estimation results of the beta pricing model (40) using the Giglio and Xiu (2017) Three-pass approach. The model is estimated using 460 portfolios at daily frequency: 100 portfolios sorted by size and book-to-market ratio, 100 portfolios sorted by size and profitability, 100 portfolios sorted by size and investment, 25 portfolios sorted by size and short-term reversal, 25 portfolios sorted by size and long-term reversal, 25 portfolios sorted by size and momentum, 25 portfolios sorted by profitability and investment, 25 portfolios sorted by book and investment, 25 portfolios sorted by book-to-market and profitability, and 10 industry portfolios (see Kenneth French's website). It also presents the t-ratio computed using the standard errors of Giglio and Xiu (2017).

Maturity	Only FF5	30	60	91	122	152	182	273	365
λ_0	0.081	0.081	0.081	0.081	0.081	0.081	0.081	0.081	0.081
t_{GX}	10.762	10.762	10.762	10.762	10.762	10.762	10.762	10.762	10.762
λ_{MKT}	-0.047	-0.047	-0.047	-0.047	-0.047	-0.047	-0.047	-0.047	-0.047
t_{GX}	-2.545	-2.545	-2.545	-2.545	-2.545	-2.545	-2.545	-2.545	-2.545
λ_{SMB}	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010
t_{GX}	1.090	1.090	1.090	1.090	1.090	1.090	1.090	1.090	1.090
λ_{HML}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
t_{GX}	0.241	0.241	0.241	0.241	0.241	0.241	0.241	0.241	0.241
λ_{RMW}	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010
t_{GX}	1.323	1.323	1.323	1.323	1.323	1.323	1.323	1.323	1.323
λ_{CMA}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
t_{GX}	1.737	1.737	1.737	1.737	1.737	1.737	1.737	1.737	1.737
$\lambda_{\mathcal{M}^{(2)}}$		0.005	0.009	0.013	0.017	0.021	0.024	0.036	0.049
t_{GX}		6.208	6.313	6.484	6.633	6.750	6.886	6.996	6.961
$\lambda_{\mathcal{M}^{(3)}}$		0.002	0.005	0.009	0.015	0.021	0.027	0.055	0.099
t_{GX}		4.387	4.739	5.122	5.362	5.559	5.763	5.986	6.037
$\lambda_{\mathcal{M}^{(4)}}$		0.002	0.006	0.012	0.021	0.032	0.045	0.114	0.251
t_{GX}		3.920	4.227	4.716	4.975	5.236	5.572	5.958	6.059

Table 22

Estimates of the Price of Risk: This table presents the estimation results of the beta pricing model (47) using the Giglio and Xiu (2017) Three-pass approach. The model is estimated using 460 portfolios at daily frequency: 100 portfolios sorted by size and book-to-market ratio, 100 portfolios sorted by size and profitability, 100 portfolios sorted by size and investment, 25 portfolios sorted by size and short-term reversal, 25 portfolios sorted by size and long-term reversal, 25 portfolios sorted by size and momentum, 25 portfolios sorted by profitability and investment, 25 portfolios sorted by book and investment, 25 portfolios sorted by book-to-market and profitability, and 10 industry portfolios (see Kenneth French's website). It also presents the t-ratio computed using the standard errors of Giglio and Xiu (2017).

Maturity	Only XHZ	30	60	91	122	152	182	273	365
λ_0	0.061	0.061	0.061	0.061	0.061	0.061	0.061	0.061	0.061
t_{GX}	9.785	9.785	9.785	9.785	9.785	9.785	9.785	9.785	9.785
λ_{MKT}	-0.025	-0.025	-0.025	-0.025	-0.025	-0.025	-0.025	-0.025	-0.025
t_{GX}	-1.423	-1.423	-1.423	-1.423	-1.423	-1.423	-1.423	-1.423	-1.423
λ_{ME}	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009
t_{GX}	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963
$\lambda_{I/A}$	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007
t_{GX}	1.265	1.265	1.265	1.265	1.265	1.265	1.265	1.265	1.265
λ_{ROE}	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
t_{GX}	0.550	0.550	0.550	0.550	0.550	0.550	0.550	0.550	0.550
$\lambda_{\mathcal{M}^{(2)}}$		0.005	0.009	0.013	0.017	0.020	0.024	0.036	0.049
t_{GX}		6.152	6.259	6.436	6.587	6.708	6.847	6.962	6.927
$\lambda_{\mathcal{M}^{(3)}}$		0.002	0.005	0.009	0.015	0.020	0.027	0.054	0.099
t_{GX}		4.340	4.684	5.077	5.315	5.514	5.724	5.953	6.007
$\lambda_{\mathcal{M}^{(4)}}$		0.002	0.006	0.012	0.021	0.032	0.045	0.114	0.250
t_{GX}		3.878	4.172	4.669	4.924	5.185	5.530	5.923	6.028

Table 23

Estimates of the Price of Risk: This table presents the estimation results of the beta pricing model (41) using the Giglio and Xiu (2017) Three-pass approach. The model is estimated using 460 portfolios at daily frequency: 100 portfolios sorted by size and book-to-market ratio, 100 portfolios sorted by size and profitability, 100 portfolios sorted by size and investment, 25 portfolios sorted by size and short-term reversal, 25 portfolios sorted by size and long-term reversal, 25 portfolios sorted by size and momentum, 25 portfolios sorted by profitability and investment, 25 portfolios sorted by book and investment, 25 portfolios sorted by book-to-market and profitability, and 10 industry portfolios (see Kenneth French's website). It also presents the t-ratio computed using the standard errors of Giglio and Xiu (2017).

Maturity	Only FF5	30	60	91	122	152	182	273	365
λ_0	0.081	0.081	0.081	0.081	0.081	0.081	0.081	0.081	0.081
t_{GX}	10.762	10.762	10.762	10.762	10.762	10.762	10.762	10.762	10.762
λ_{MKT}	-0.047	-0.047	-0.047	-0.047	-0.047	-0.047	-0.047	-0.047	-0.0466
t_{GX}	-2.545	-2.545	-2.545	-2.545	-2.545	-2.545	-2.545	-2.545	-2.545
λ_{SMB}	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010
t_{GX}	1.090	1.090	1.090	1.090	1.090	1.090	1.090	1.090	1.090
λ_{HML}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
t_{GX}	0.241	0.241	0.241	0.241	0.241	0.241	0.241	0.241	0.241
λ_{RMW}	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010
t_{GX}	1.323	1.323	1.323	1.323	1.323	1.323	1.323	1.323	1.323
λ_{CMA}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
t_{GX}	1.737	1.737	1.737	1.737	1.737	1.737	1.737	1.737	1.737
$\lambda_{\mathcal{MRP}^{(2)}}$		-0.001	-0.002	-0.003	-0.005	-0.006	-0.007	-0.011	-0.016
t_{GX}		-4.023	-4.312	-4.466	-4.569	-4.552	-4.425	-3.890	-3.398
$\lambda_{\mathcal{MRP}^{(3)}}$		0.002	0.007	0.014	0.025	0.037	0.051	0.116	0.228
t_{GX}		4.252	4.617	5.069	5.291	5.504	5.807	6.190	6.288
$\lambda_{\mathcal{MRP}^{(4)}}$		0.001	0.002	0.004	0.007	0.009	0.012	0.020	0.015
t_{GX}		3.335	3.795	4.355	4.853	5.204	5.329	5.077	1.904

Table 24

Estimates of the Price of Risk: This table presents the estimation results of the beta pricing model (48) using the Giglio and Xiu (2017) Three-pass approach. The model is estimated using 460 portfolios at daily frequency: 100 portfolios sorted by size and book-to-market ratio, 100 portfolios sorted by size and profitability, 100 portfolios sorted by size and investment, 25 portfolios sorted by size and short-term reversal, 25 portfolios sorted by size and long-term reversal, 25 portfolios sorted by size and momentum, 25 portfolios sorted by profitability and investment, 25 portfolios sorted by book and investment, 25 portfolios sorted by book-to-market and profitability, and 10 industry portfolios (see Kenneth French's website). It also presents the t-ratio computed using the standard errors of Giglio and Xiu (2017).

Maturity	Only HXZ	30	60	91	122	152	182	273	365
λ_0	0.061	0.061	0.061	0.061	0.061	0.061	0.061	0.061	0.061
t_{GX}	9.785	9.785	9.785	9.785	9.785	9.785	9.785	9.785	9.785
λ_{MKT}	-0.025	-0.025	-0.025	-0.025	-0.025	-0.025	-0.025	-0.025	-0.025
t_{GX}	-1.423	-1.423	-1.423	-1.423	-1.423	-1.423	-1.423	-1.423	-1.423
λ_{ME}	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009
t_{GX}	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963
$\lambda_{I/A}$	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007
t_{GX}	1.265	1.265	1.265	1.265	1.265	1.265	1.265	1.265	1.265
λ_{ROE}	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
t_{GX}	0.550	0.550	0.550	0.550	0.550	0.550	0.550	0.550	0.550
$\lambda_{\mathcal{MRP}^{(2)}}$		-0.001	-0.002	-0.003	-0.004	-0.006	-0.007	-0.011	-0.015
t_{GX}		-3.980	-4.266	-4.428	-4.531	-4.518	-4.393	-3.861	-3.373
$\lambda_{\mathcal{MRP}^{(3)}}$		0.002	0.007	0.014	0.025	0.037	0.050	0.115	0.227
t_{GX}		4.209	4.555	5.019	5.238	5.453	5.766	6.157	6.257
$\lambda_{\mathcal{MRP}^{(4)}}$		0.001	0.002	0.004	0.007	0.009	0.012	0.020	0.014
t_{GX}		3.280	3.743	4.315	4.826	5.199	5.325	5.078	1.834

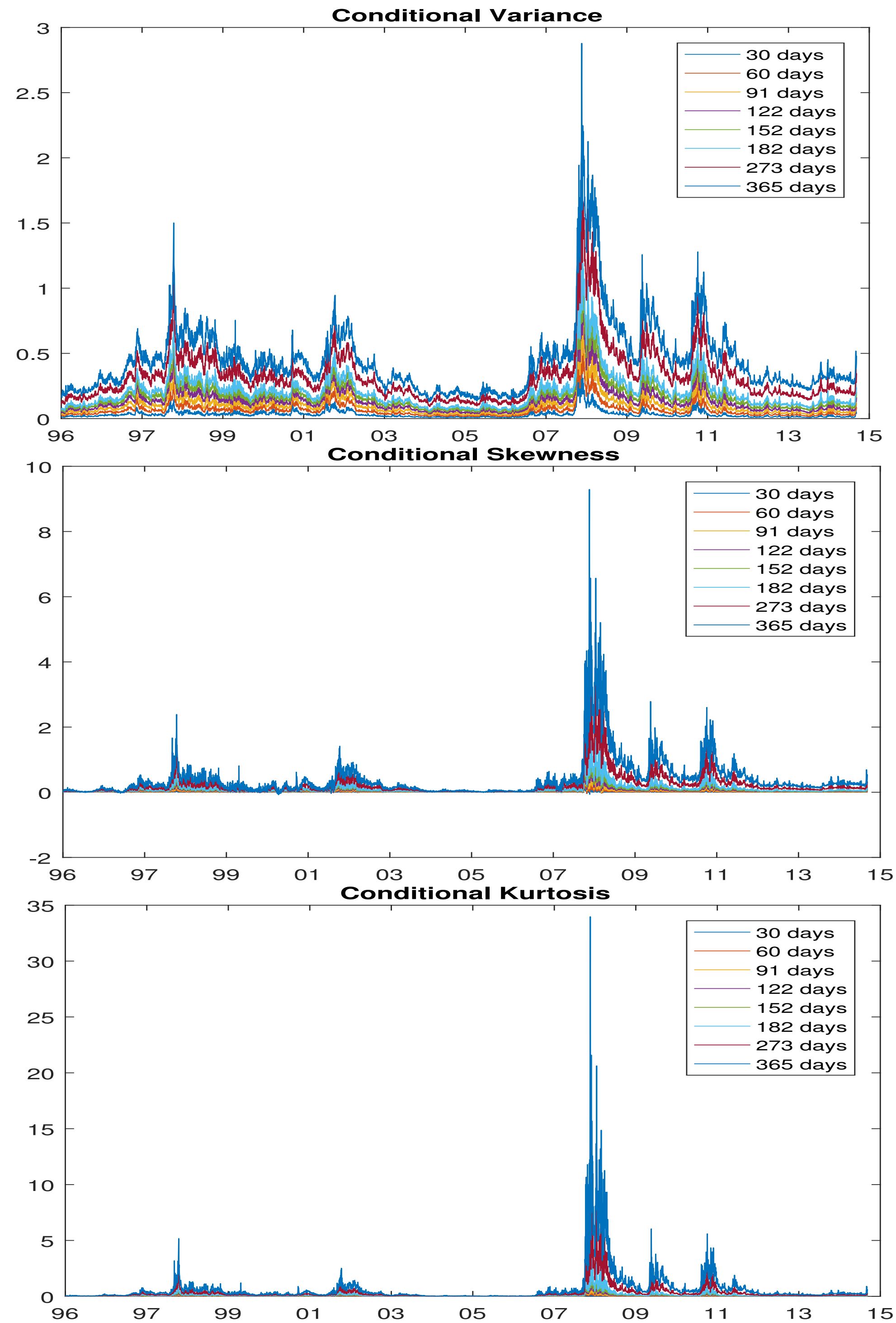

Fig. 1. **Conditional Physical Moments of the SDF under CRRA Preferences.** I plot daily variance, skewness, and kurtosis of the SDF. The moments are computed from the S&P 500 index option prices and are not annualized. The data runs from January 4, 1996, through August 31, 2015. The maturity of options used to compute the moments is labeled in days.

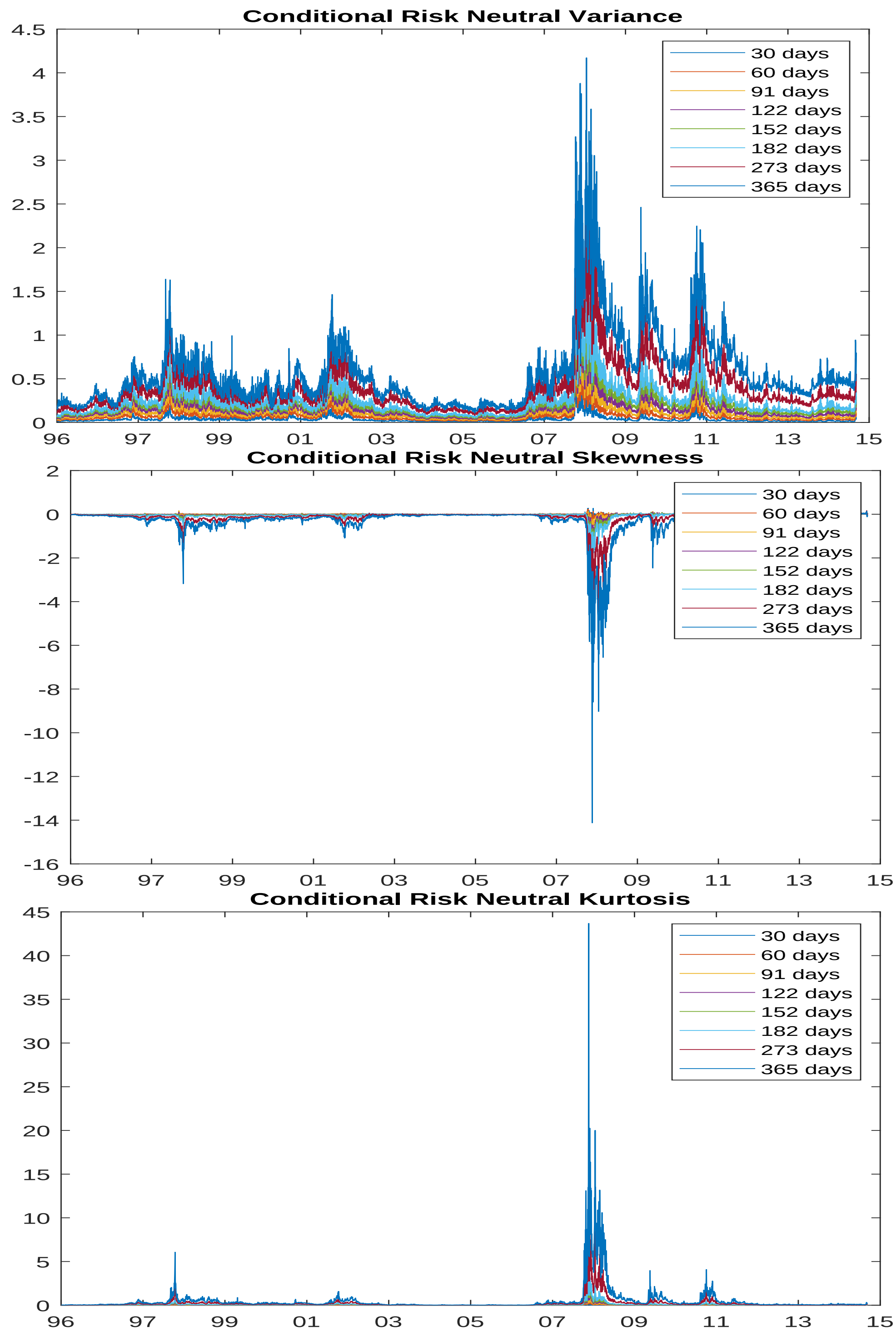

Fig. 2. **Conditional Risk-Neutral Moments of the SDF under CRRA Preferences.** I plot daily variance, skewness, and kurtosis of the SDF. The moments are computed from the S&P 500 index option prices and are not annualized. The data runs from January 4, 1996, through August 31, 2015, and is not annualized. The maturity of options used to compute the moments is labeled in days.

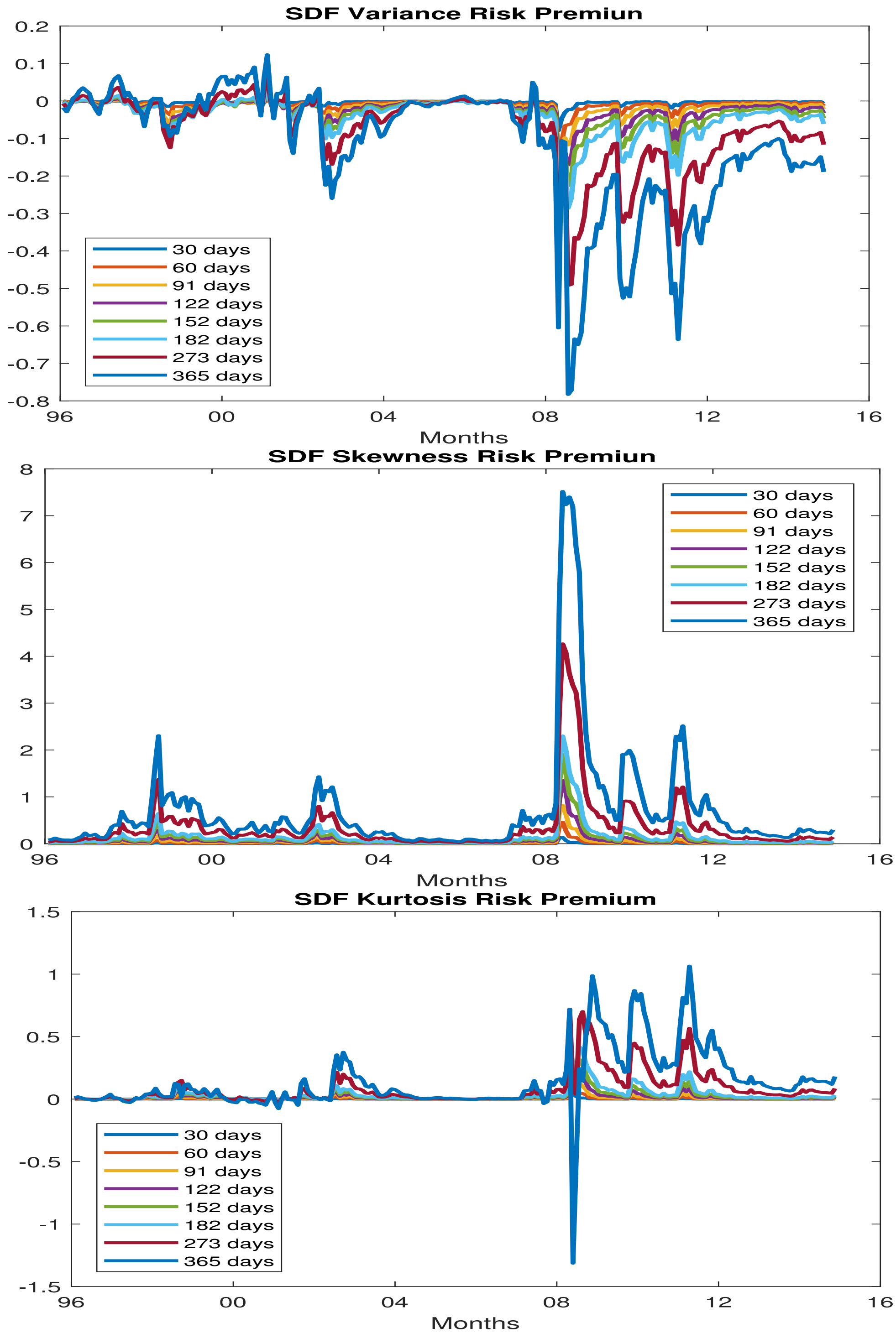

Fig. 3. **SDF Moments Risk Premium under CRRA preferences.** I plot SDF moments risk premium at monthly frequency. The data runs from January 4, 1996, through August 31, 2015, and is not annualized.

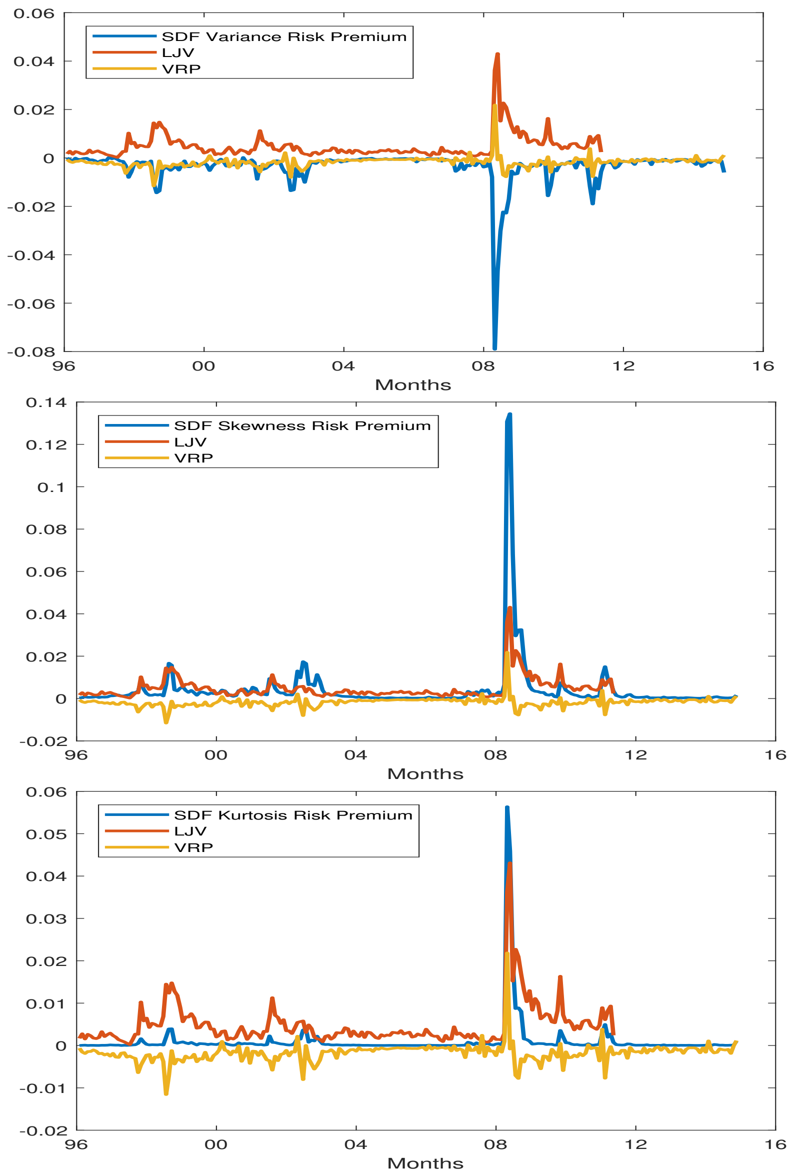

Fig. 4. **SDF Moments Risk Premium under CRRA preferences.** I plot SDF moments risk premium at monthly frequency. In this figure, the maturity of options used is 30 days to facilitate comparison. Monthly estimates of SDF moments risk premium are computed by averaging within a month all daily estimates of the SDF moments risk premium. The moments are computed from the S&P 500 index option prices and are not annualized. The data runs from January 4, 1996, through August 31, 2015, and are not annualized. The figure also plots the variance risk premium obtained form Hao Zhou's web-page (https://sites.google.com/site/haozhouspersonalhomepage/). To facilitate comparison with the SDF moments risk premium, the variance risk premium is negative of the variance risk premia defined in Hao Zhou's calculation. The latter is computed by taking the difference between the risk-neutral variance and the physical variance. I also plot in the same figure the Left Risk Neutral Jump Variation (LJV) of Bollerslev, Todorov, and Xu (2015). Monthly estimates of LJV data are available from January 1996 to December 2011.

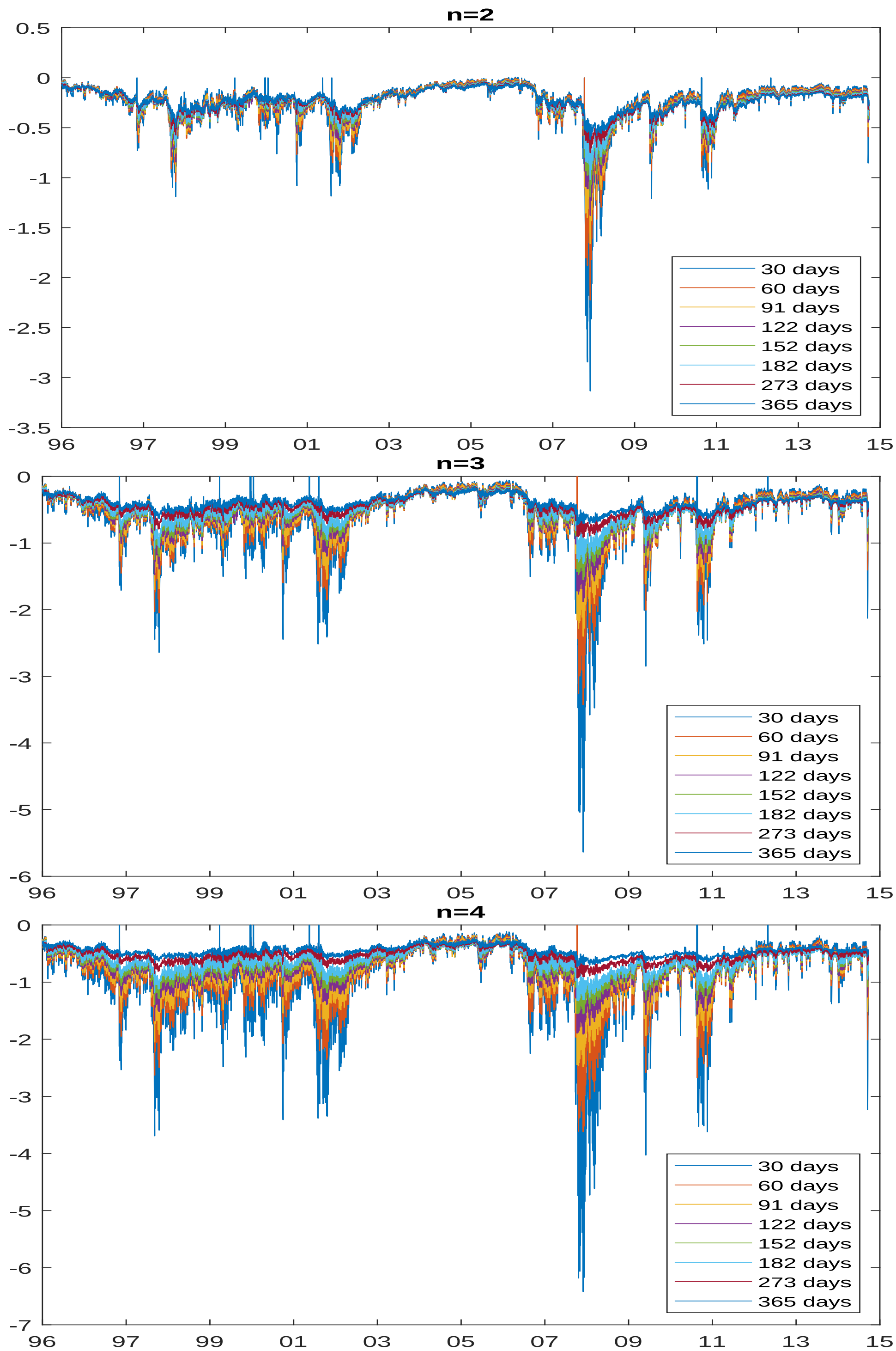

Fig. 5. **Conditional Expected Excess Returns on SDF-Based Moments under CRRA Preferences.** I plot conditional expected excess returns on SDF-based moments under CRRA preferences. The expected excess returns are annualized and cover the period from January 4, 1996, through August 31, 2015. The maturity of options used to compute the moments is labeled in days.

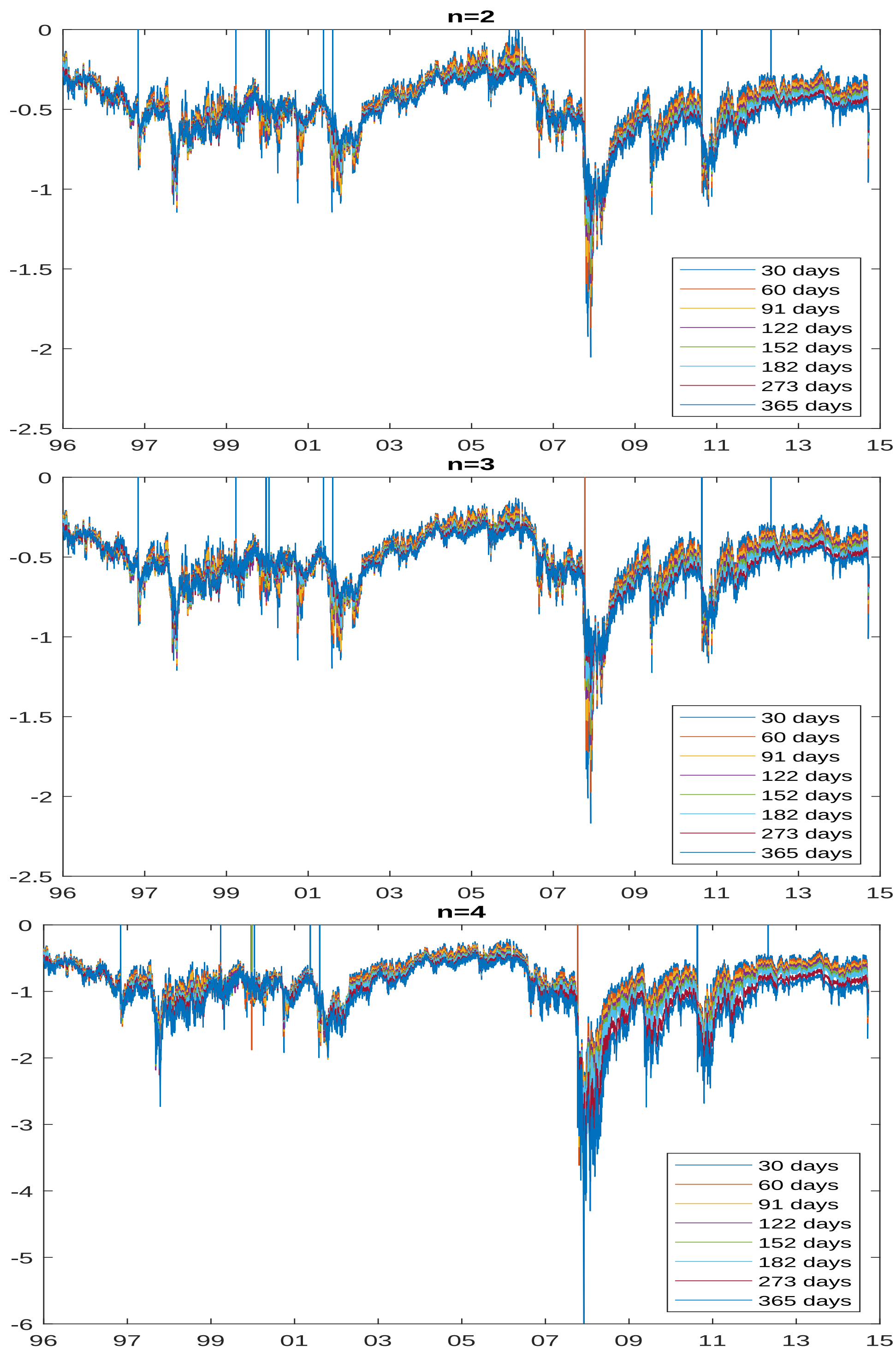

Fig. 6. **Sharpe Ratios of Returns on SDF-Based Moments** I plot conditional Sharpe ratio of returns on SDF-based moments under CRRA preferences. Conditional Sharpe ratios are annualized and cover the period from January 4, 1996, through August 31, 2015. The maturity of options used to compute the moments is labeled in days.

Real-Time Distribution of Stochastic Discount Factors

Internet Appendix: Not for Publication

Abstract

The Internet Appendix consists of three sections. In Section A, we present result of beta pricing models when the test assets comprise 25 Fama and French portfolios. Section B provides the proof of the spanning formula used in the paper. Section C presents the proof of the SDF when preferences depart from CRRA preferences and also closed-form expression of the conditional moments of the SDF.

A. Results for Fama and French 25 Portfolios Formed on Size and Book-to-Market

A.1. Results Obtained With the Pricing Model (40)

Table 25 reports the estimation results of the beta pricing model. The estimates of the price of SDF variance risk, $\lambda_{\mathcal{M}^{(2)}}$, are all positive and significant at 30-, 60-, and 91-day maturity. The price of SDF volatility risk decreases with the maturity, featuring a decreasing term structure of the price of the SDF variance. The Fama and MacBeth (1973) t-ratio, the Shanken (1992) t-ratio, and the Jagannathan and Wang (1998) t-ratio show that the estimates of the price of SDF skewness and SDF kurtosis are significant at 30-, 60-, and 91-day maturity. The price of risk of the Fama and French factors, SMB, HML, RMW, and CMA are not significant at all maturities. The adjusted R^2 ranges from 36.4% (at 30-day maturity) to 42.2% (at 365-day maturity). When I use only the Fama and French five factors, the adjusted R^2 is 19.7%. This further shows a significant improvement of the beta pricing model (40) over the Fama and French five factor model.

A.2. Results Obtained with the Pricing Model (41)

Table 26 reports the estimation results of the beta pricing model (41). The estimates of the price of the SDF variance risk premium, $\lambda_{\mathcal{MRP}^{(2)}}$ are negative and significant at 30-, 60-, and 91-day maturity when the Fama and MacBeth (1973) t-ratio, the Shanken (1992) t-ratio, and the Jagannathan and Wang (1998) t-ratio are used. They are not significant when the Kan, Robotti, and Shanken (2013) t-ratio is used.

In contrast, regardless of the t-ratio used to gauge the significance of the price of risks, the estimates of the price of the SDF third moment risk premium, $\lambda_{\mathcal{MRP}^{(3)}}$, are positive for all maturities and significant at 30-, 60-, and 91-day maturity. The estimates decrease from 0.152 (at 30-day horizon) to 0.005 (at 365-day) horizon. The price of the SDF fourth moment risk premium, $\lambda_{\mathcal{MRP}^{(4)}}$, is marginally significant at 60-day horizon, regardless of the t-ratios used.

Finally, the price of risk of the Fama and French factors, SMB, HML, RMW, and CMA is not significant at all maturities. The R^2 ranges from 33.8% to 39.8%. When I use only the Fama and French five factors, the adjusted R^2 is 19.7%. Thus, there is an improvement of the beta pricing model (41) over the Fama and French five factor model.

B. Appendix

Before I give a formal proof of various propositions, I show that for any function $g[x]$ that is continuous and has first- and second-order derivative that exist, the following identity holds

$$g[x] \quad = \quad g[x_0] + g^{'}[x_0](x - x_0) + \int_0^{x_0} g^{''}[K] \max(K - x, 0)\, dK + \int_{x_0}^{\infty} g^{''}[K] \max(x - K, 0)\, dK.$$

Note that the following identity holds:

$$\begin{aligned}
\int_0^{\infty} g^{''}[K] \max(x - K, 0)\, dK &= \left[g^{'}[K]x - \left(Kg^{'}[K] - g[K]\right)\right]_0^x \\
&= \left(g^{'}[x]x - \left(xg^{'}[x] - g[x]\right)\right) - \left(g^{'}[0]x - \left(0g^{'}[0] - g[0]\right)\right) \\
&= g[x] - xg^{'}[0] - g[0].
\end{aligned}$$

Thus,

$$g[x] = g[0] + xg^{'}[0] + \int_0^{\infty} g^{''}[K] \max(x - K, 0)\, dK. \tag{I-A1}$$

Observe that

$$\int_0^{\infty} g^{''}[K] \max(x - K, 0)\, dK = \int_0^{x_0} g^{''}[K] \max(x - K, 0)\, dK + \int_{x_0}^{\infty} g^{''}[K] \max(x - K, 0)\, dK. \tag{I-A2}$$

Since

$$(x - K) = \max(x - K, 0) - \max(K - x, 0),$$

I multiply this identity by $g^{''}[K]$ to obtain

$$g^{''}[K](x - K) = g^{''}[K] \max(x - K, 0) - g^{''}[K] \max(K - x, 0)$$

and

$$g^{''}[K]\max(x-K,0)=g^{''}[K](x-K)+g^{''}[K]\max(K-x,0). \tag{I-A3}$$

Replace (I-A3) in (I-A2), and get

$$\int_0^{\infty} g^{''}[K]\max(x-K,0)\,dK=\int_0^{x_0} g^{''}[K](x-K)\,dK+\int_0^{x_0} g^{''}[K]\max(K-x,0)\,dK+\int_{x_0}^{\infty} g^{''}[K]\max(x-K,0)\,dK. \tag{I-A4}$$

Now, replace the following expression in (I-A1):

$$g[x] = g[0]+xg^{'}[0]+\int_0^{x_0} g^{''}[K](x-K)\,dK+\int_0^{x_0} g^{''}[K]\max(K-x,0)\,dK+\int_{x_0}^{\infty} g^{''}[K]\max(x-K,0)\,dK.$$

This simplifies to

$$g[x] = g[0]+xg^{'}[0]+\left[g^{'}[K]x-\left(Kg^{'}[K]-g[K]\right)\right]_0^{x_0}+\int_0^{x_0} g^{''}[K]\max(K-x,0)\,dK+\int_{x_0}^{\infty} g^{''}[K]\max(x-K,0)\,dK$$

and

$$g[x] = g[x_0]+g^{'}[x_0](x-x_0)+\int_0^{x_0} g^{''}[K]\max(K-x,0)\,dK+\int_{x_0}^{\infty} g^{''}[K]\max(x-K,0)\,dK. \tag{I-A5}$$

This is similar to the spanning formula of Carr and Madan (2001) and Bakshi, Kapadia, and Madan (2003).

C. Appendix

Proof of the Moments of the Simple Return $R_{M,t\to T}$. I use the spanning formula (I-A5) to show that

$$\begin{aligned}(R_{M,t\to T}-R_{f,t\to T})^n &= \left(\frac{S_T}{S_t}-R_{f,t\to T}\right)^n \\ &= \left(\frac{S_t}{S_t}-R_{f,t\to T}\right)^n+n\frac{1}{S_t}\left(\frac{S_t}{S_t}-R_{f,t\to T}\right)^{n-1}(S_T-S_t) \\ &\quad +\frac{n(n-1)}{S_t^2}\left\{\begin{array}{l}\int_{S_t}^{\infty}\left(\frac{K}{S_t}-R_{f,t\to T}\right)^{n-2}(S_T-K)^+\,dK \\ +\int_0^{S_t}\left(\frac{K}{S_t}-R_{f,t\to T}\right)^{n-2}(K-S_T)^+\,dK\end{array}\right\}\end{aligned} \tag{I-B1}$$

Then, I take the conditional expectation of

$$
\begin{aligned}
& \mathbb{E}_t^*\left(R_{M,t\to T}-\mathbb{E}_t^*\left(R_{M,t\to T}\right)\right)^n \\
= \quad & (1-n)\left(1-R_{f,t\to T}\right)^n+\frac{n(n-1)R_{f,t\to T}}{S_t^2}\left\{\begin{array}{c} \int_{S_t}^{\infty}\left(\frac{K}{S_t}-R_{f,t\to T}\right)^{n-2}C_t[K]\,dK \\ +\int_0^{S_t}\left(\frac{K}{S_t}-R_{f,t\to T}\right)^{n-2}P_t[K]\,dK \end{array}\right\}.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\mathbb{VAR}_t^*\left(R_{M,t\to T}\right) &= \mathbb{E}_t^*\left(R_{M,t\to T}-\mathbb{E}_t^*\left(R_{M,t\to T}\right)\right)^2, \\
\mathbb{SKEW}_t^*\left(R_{M,t\to T}\right) &= \mathbb{E}_t^*\left(R_{M,t\to T}-\mathbb{E}_t^*\left(R_{M,t\to T}\right)\right)^3, \text{ and} \\
\mathbb{KURT}_t^*\left(R_{M,t\to T}\right) &= \mathbb{E}_t^*\left(R_{M,t\to T}-\mathbb{E}_t^*\left(R_{M,t\to T}\right)\right)^4.
\end{aligned}
$$

■ **Proof of Proposition 11.** Consider the following maximization problem:

$$
\max_{\substack{\omega_t \\ W_{t\to T}=W_t\left(R_{f,t}+\omega_t'\left(R_{t\to T}-R_{f,t}\right)\right)}} \mathbb{E}_t\left(\vartheta\left[W_{t\to T}\right]\right)=\max_{\substack{\omega_t \\ W_{t\to T}=W_t\left(R_{f,t}+\omega_t'\left(R_{t\to T}-R_{f,t}\right)\right)}} \mathbb{E}_t\left(\upsilon\left[u\left[W_{t\to T}\right]\right]\right).
$$

The FOCs are

$$
\mathbb{E}_t\left(\upsilon'\left[u\left[W_{t\to T}\right]\right]u'\left[W_{t\to T}\right]\left(R_{i,t\to T}-R_{f,t\to T}\right)\right)=0 \text{ for } i=1,\ldots,N.
$$

The FOCs imply a SDF of the form

$$
m_{t\to T}^{SDF}=\frac{1}{R_{f,t\to T}}\frac{\upsilon'\left[u\left[W_{t\to T}\right]\right]u'\left[W_{t\to T}\right]}{\mathbb{E}_t\left(\upsilon'\left[u\left[W_{t\to T}\right]\right]u'\left[W_{t\to T}\right]\right)}=a_t\upsilon'\left[u\left[W_{t\to T}\right]\right]u'\left[W_{t\to T}\right], \tag{I-B2}
$$

with

$$
a_t=\frac{1}{R_{f,t\to T}}\frac{1}{\mathbb{E}_t\left(\upsilon'\left[u\left[W_{t\to T}\right]\right]u'\left[W_{t\to T}\right]\right)}.
$$

To begin this proof, I omit the time subscript for simplicity. The Taylor expansion series of υ around $u[x]=u[x_0]$ is

$$
\upsilon[u[x]]=\upsilon[u[x_0]]+\sum_{k=1}^{\infty}\frac{1}{k!}\left(u[x]-u[x_0]\right)^k\left(\frac{\partial^k\upsilon[y]}{\partial^k y}\right)_{y=u[x_0]}
$$

and

$$\upsilon\left[u\left[x\right]\right] = \upsilon\left[u\left[x_0\right]\right] + \sum_{k=1}^{\infty} \frac{1}{k!}\left(u\left[x\right] - u\left[x_0\right]\right)^k \left(\frac{\partial^k \upsilon\left[y\right]}{\partial^k y}\right)_{y=u[x_0]}. \tag{I-B3}$$

I use (I-B3) to show that

$$\upsilon^{'}\left[u\left[W_{t\to T}\right]\right] = \sum_{k=1}^{\infty} \frac{k}{k!}\left(u\left[W_{t\to T}\right] - u\left[x_0\right]\right)^{k-1} \left(\frac{\partial^k \upsilon\left[y\right]}{\partial^k y}\right)_{y=u[x_0]}.$$

Hence, the SDF is

$$m^{SDF}_{t\to T} = a_t u^{'}\left[W_{t\to T}\right] \sum_{k=1}^{\infty} \frac{1}{(k-1)!}\left(u\left[W_{t\to T}\right] - u\left[x_0\right]\right)^{k-1} \left(\frac{\partial^k \upsilon\left[y\right]}{\partial^k y}\right)_{y=u[x_0]} = c_t u^{'}\left[W_{t\to T}\right] z_{M,t\to T}, \tag{I-B4}$$

with

$$\begin{aligned}
z_{M,t\to T} &= 1 + \sum_{k=2}^{\infty} (-1)^{k+1} \frac{\rho^{(k)}}{(k-1)!}\left(\frac{u\left[W_{t\to T}\right]}{u\left[x_0\right]} - 1\right)^{k-1}, \text{and} \\
\rho^{(k)} &= \left((-1)^{k+1} \frac{\frac{y^{k-1}\partial^k \upsilon[y]}{\partial^k y}}{\frac{\partial \upsilon[y]}{\partial y}}\right)_{y=u[x_0]}, \\
c_t &= a_t \left(\frac{\partial \upsilon\left[y\right]}{\partial y}\right)_{y=u[x_0]}.
\end{aligned}$$

From (I-B4), observe that

$$m^{SDF}_{t\to T}\left(u^{'}\left[W_{t\to T}\right]\right)^{-1} = c_t z_{M,t\to T}.$$

Now, I take the expected value of this equation and obtain

$$\mathbb{E}_t\left(m^{SDF}_{t\to T}\left(u^{'}\left[W_{t\to T}\right]\right)^{-1}\right) = c_t \mathbb{E}_t\left[z_{M,t\to T}\right].$$

Thus, I derive c_t,

$$\frac{\mathbb{E}_t\left(m^{SDF}_{t\to T}\left(u^{'}\left[W_{t\to T}\right]\right)^{-1}\right)}{\mathbb{E}_t\left[z_{M,t\to T}\right]} = c_t. \tag{I-B5}$$

Now, replace c_t in (I-B4) and get

$$m_{t\to T}^{SDF}=\left(u^{'}\left[W_{t\to T}\right]\right)\left(\mathbb{E}_t\left(m_{t\to T}^{SDF}\left(u^{'}\left[W_{t\to T}\right]\right)^{-1}\right)\right)\frac{z_{M,t\to T}}{\mathbb{E}_t\left(z_{M,t\to T}\right)}=\frac{1}{R_{f,t\to T}}u^{'}\left[W_{t\to T}\right]\frac{\mathbb{E}_t^*\left(\frac{1}{u^{'}\left[W_{t\to T}\right]}\right)}{\mathbb{E}_t\left(z_{M,t\to T}\right)}z_{M,t\to T}. \tag{I-B6}$$

Next, I determine $\mathbb{E}_t\left[z_{M,t\to T}\right]$. I first observe that $m_{t\to T}^{SDF}=c_t u^{'}\left[W_{t\to T}\right]z_{M,t\to T}$ and use it to show that

$$m_{t\to T}^{SDF}\left(u^{'}\left[W_{t\to T}\right]z_{M,t\to T}\right)^{-1}=c_t. \tag{I-B7}$$

I then apply the conditional expectation operator to c_t and show that

$$c_t=\mathbb{E}_t\left(m_{t\to T}^{SDF}\left(u^{'}\left[W_{t\to T}\right]z_{M,t\to T}\right)^{-1}\right). \tag{I-B8}$$

This reduces to

$$c_t=\frac{1}{R_{f,t\to T}}\mathbb{E}_t^*\left(\left(u^{'}\left[W_{t\to T}\right]\right)^{-1}z_{M,t\to T}^{-1}\right). \tag{I-B9}$$

Identities (I-B5) and (I-B9) are identical:

$$\frac{\mathbb{E}_t\left(m_{t\to T}^{SDF}\left(u^{'}\left[W_{t\to T}\right]\right)^{-1}\right)}{\mathbb{E}_t\left(z_{M,t\to T}\right)}=\frac{1}{R_{f,t\to T}}\mathbb{E}_t^*\left(\left(u^{'}\left[W_{t\to T}\right]\right)^{-1}z_{M,t\to T}^{-1}\right). \tag{I-B10}$$

From the equation (I-B10), I deduce

$$\mathbb{E}_t\left(z_{M,t\to T}\right)=\frac{\mathbb{E}_t^*\left(\frac{u^{'}\left[W_t\right]}{u^{'}\left[W_{t\to T}\right]}\right)}{\mathbb{E}_t^*\left(\frac{u^{'}\left[W_t\right]}{u^{'}\left[W_{t\to T}\right]}z_{M,t\to T}^{-1}\right)}.$$

Finally, (I-B6) can be expressed as

$$m_{t\to T}^{SDF}=\left(\frac{1}{R_{f,t\to T}}u^{'}\left[W_{t\to T}\right]\left(\mathbb{E}_t^*\left(\frac{1}{u^{'}\left[W_{t\to T}\right]}\right)\right)\right)\times\left(\frac{z_{M,t\to T}}{\mathbb{E}_t\left(z_{M,t\to T}\right)}\right), \tag{I-B11}$$

with

$$\mathbb{E}_t\left(z_{M,t\to T}\right)=\frac{\mathbb{E}_t^*\left(\frac{1}{u^{'}\left[W_{t\to T}\right]}\right)}{\mathbb{E}_t^*\left(\frac{1}{u^{'}\left[W_{t\to T}\right]}z_{M,t\to T}^{-1}\right)}$$

To summarize, the SDF has the form

$$m^{SDF}_{t\to T} = m^{T}_{t\to T} \times m^{P}_{t\to T}, \tag{I-B12}$$

with

$$\begin{aligned} m^{T}_{t\to T} &= \left(\frac{1}{R_{f,t\to T}} u^{'}\left[W_{t\to T}\right]\left(\mathbb{E}^{*}_{t}\left(\frac{1}{u^{'}\left[W_{t\to T}\right]}\right)\right)\right) \text{ and} \\ m^{P}_{t\to T} &= \frac{z_{M,t\to T}}{\mathbb{E}_{t}\left(z_{M,t\to T}\right)}, \end{aligned}$$

and

$$\begin{aligned} z_{M,t\to T} &= 1 + \sum_{k=2}^{\infty} (-1)^{k+1} \frac{\rho^{(k)}}{(k-1)!} \left(\frac{u\left[W_{t\to T}\right]}{u\left[x_0\right]} - 1\right)^{k-1} \\ \mathbb{E}_{t}\left[z_{M,t\to T}\right] &= \frac{\mathbb{E}^{*}_{t}\left[\frac{1}{u^{'}\left[W_{t\to T}\right]}\right]}{\mathbb{E}^{*}_{t}\left[\frac{1}{u^{'}\left[W_{t\to T}\right]} z^{-1}_{M,t\to T}\right]}, \end{aligned}$$

where

$$\rho^{(k)} = \left((-1)^{k+1} \frac{z^{k-1}\partial^{k}\upsilon\left[z\right]}{\partial^{k} z} / \frac{\partial\upsilon\left[z\right]}{\partial z}\right)_{z=u[x_0]}.$$

Setting $W_{t\to T} = S_T$ and $x_0 = S_t$ ends the proof. ∎

Proof of the Conditional Moments when $m^{SDF}_{t\to T} = m^{T}_{t\to T} \times m^{P}_{t\to T}$.

To proceed, I set $x = S_T$, $x_0 = S_t$, $z = z_{M,t\to T}$, and $k = 2$. I recall that

$$u\left[x\right] = \frac{x^{1-\alpha} - 1}{1-\alpha}.$$

Thus,

$$u^{'}\left[x\right] = x^{-\alpha},\; u^{''}\left[x\right] = -\alpha x^{-\alpha-1},\; u^{'''}\left[x\right] = \alpha\left(\alpha+1\right) x^{-\alpha-2}$$

and

$$z_{M,t\to T} = 1 - \rho^{(2)}\left(\frac{u\left[x\right]}{u\left[x_0\right]} - 1\right).$$

My first goal is to determine

$$\mathbb{E}_t^*\left[\frac{1}{zu'[x]}\right].$$

I first observe that

$$\frac{1}{zu'[x]} = F[x],$$

with

$$F[x] = \frac{1}{u'[x] - \rho^{(2)}\frac{u[x]u'[x]}{u[x_0]} + \rho^{(2)}u'[x]}.$$

Observe that

$$F[x_0] = \frac{1}{u'[x_0]}.$$

Observe that the first derivative of $F[x]$ is

$$F_x[x] = \frac{f[x]}{(g[x])^2},$$

with

$$\begin{aligned} f[x] &= -u''[x] + \rho^{(2)}\frac{\left(u'[x]\right)^2}{u[x_0]} + \rho^{(2)}\frac{u[x]u''[x]}{u[x_0]} - \rho^{(2)}u''[x] \text{ and} \\ g[x] &= u'[x] - \rho^{(2)}\frac{u[x]u'[x]}{u[x_0]} + \rho^{(2)}u'[x]. \end{aligned}$$

The second derivative of $F[.]$ with respect to x is

$$F_{xx}[x] = \frac{f'[x]g[x] - 2g'[x]f[x]}{(g[x])^3},$$

with

$$\begin{aligned} f'[x] &= -u'''[x] + 3\rho^{(2)}\frac{u'[x]u''[x]}{u[x_0]} + \rho^{(2)}\frac{u[x]u'''[x]}{u[x_0]} - \rho^{(2)}u'''[x] \text{ and} \\ g'[x] &= u''[x] - \rho^{(2)}\frac{\left(u'[x]\right)^2}{u[x_0]} - \rho^{(2)}\frac{u[x]u''[x]}{u[x_0]} + \rho^{(2)}u''[x]. \end{aligned}$$

Hence,

$$F_{xx}[K] = \frac{f'[K]\,g[K] - 2g'[K]\,f[K]}{(g[K])^3}.$$

Using the spanning formula from Appendix I-A, the following identity holds:

$$F[S_T] = F[S_t] + (S_T - S_t)\,F_S[S_t] + \int_{S_t}^{\infty} F_{SS}[K]\,(S_T - K)^+\,dK + \int_0^{S_t} F_{SS}[K]\,(K - S_T)^+\,dK.$$

Hence,

$$\mathbb{E}_t^*[F[S_T]] = F[S_t] + (R_{f,t\to T} - 1)\,S_t F_S[S_t] + R_{f,t\to T}\left(\int_{S_t}^{\infty} F_{SS}[K]\,C_t[K]\,dK + \int_0^{S_t} F_{SS}[K]\,P_t[K]\,dK\right).$$

I recall that

$$\mathbb{E}_t^*\left[\frac{1}{u'[x]}\right] = \mathbb{E}_t^*[S_T^{\alpha}] = S_t^{\alpha}\mathbb{E}_t^*\left[R_{M,t\to T}^{\alpha}\right] \text{ (since } R_{M,t\to T} = \frac{S_T}{S_t}\text{).}$$

Finally,

$$\begin{aligned}
\mathbb{E}_t[z] &= \frac{\mathbb{E}_t^*[S_T^{\alpha}]}{\mathbb{E}_t^*[F[S_T]]} \\
&= \frac{S_t^{\alpha}\mathbb{E}_t^*\left[R_{M,t\to T}^{\alpha}\right]}{\mathbb{E}_t^*[F[S_T]]} \\
&= \frac{S_t^{\alpha}\delta_t}{\mathbb{E}_t^*[F[S_T]]} \text{ (}\delta_t\text{ in defined in the proof of Proposition 8).}
\end{aligned}$$

<u>Conditional Physical Moments of the SDF</u>

Now, I compute the conditional moments of the SDF. To compute the physical moments of the SDF, I derive a closed-form expression of $\mathcal{N}^{*(n)}[T]$:

$$\mathcal{N}_t^{*(n)}[T] = \mathbb{E}_t^*\left[\left(m_{t\to T}^{SDF} - \mathbb{E}_t\left(m_{t\to T}^{SDF}\right)\right)^n\right] = \mathbb{E}_t^*\left[\left(m_{t\to T}m_{t\to T}^{P} - \frac{1}{R_{f,t\to T}}\right)^n\right].$$

Note that

$$m_{t\to T}^{SDF} = \frac{\delta_t}{R_{f,t\to T}} R_{M,t\to T}^{-\alpha}\frac{z_{M,t\to T}}{\mathbb{E}_t(z_{M,t\to T})}.$$

Hence,

$$\begin{aligned}\mathcal{N}_t^{*(n)}[T] &= \mathbb{E}_t^*\left[\left(\frac{\delta_t}{R_{f,t\to T}}R_{M,t\to T}^{-\alpha}\frac{z_{M,t\to T}}{\mathbb{E}_t\left(z_{M,t\to T}\right)}-\frac{1}{R_{f,t\to T}}\right)^n\right]\\ &= \frac{1}{R_{f,t\to T}^n}\mathbb{E}_t^*\left[\left(\widetilde{\delta}_t R_{M,t\to T}^{-\alpha}z_{M,t\to T}-1\right)^n\right]\\ &= \frac{1}{R_{f,t\to T}^n}\mathbb{E}_t^*\left[\left(\widetilde{\delta}_t H\left[S_T\right]-1\right)^n\right].\end{aligned}$$

Denote

$$\widetilde{\delta}_t=\frac{\delta_t}{\mathbb{E}_t\left(z_{M,t\to T}\right)}\text{ and }H\left[S_T\right]=\frac{u'\left[S_T\right]}{u'\left[S_t\right]}\left(1-\rho^{(2)}\left(\frac{u\left[S_T\right]}{u\left[S_t\right]}-1\right)\right).$$

It follows that

$$\begin{aligned}H\left[S_t\right] &= 1,\\ H_S\left[S_T\right] &= \frac{u''\left[S_T\right]}{u'\left[S_t\right]}\left(1-\rho^{(2)}\left(\frac{u\left[S_T\right]}{u\left[S_t\right]}-1\right)\right)-\rho^{(2)}\frac{\left(u'\left[S_T\right]\right)^2}{u'\left[S_t\right]u\left[S_t\right]},\text{ and}\end{aligned}$$

$$H_{SS}\left[S_T\right]=\frac{u'''\left[S_T\right]}{u'\left[S_t\right]}\left(1-\rho^{(2)}\left(\frac{u\left[S_T\right]}{u\left[S_t\right]}-1\right)\right)-\rho^{(2)}\frac{u''\left[S_T\right]}{u'\left[S_t\right]}\frac{u'\left[S_T\right]}{u\left[S_t\right]}-2\rho^{(2)}\frac{u'\left[S_T\right]u''\left[S_T\right]}{u'\left[S_t\right]u\left[S_t\right]}.$$

Now I use the spanning formula in Appendix I-A to show that

$$\begin{aligned}\left(\widetilde{\delta}_t H\left[S_T\right]-1\right)^n &= \left(\widetilde{\delta}_t-1\right)^n+n\left(S_T-S_t\right)\left(\widetilde{\delta}_t H\left[S_t\right]-1\right)^{n-1}\widetilde{\delta}_t H_S\left[S_t\right]\\ &\quad+\int_{S_t}^{\infty}G_{SS}[K]\left(S_T-K\right)^+dK+\int_0^{S_t}G_{SS}[K]\left(K-S_T\right)^+dK,\end{aligned}\tag{I-B13}$$

with

$$G_{SS}\left[S_T\right]=n\left(n-1\right)\left(\widetilde{\delta}_t H\left[S_T\right]-1\right)^{n-2}\left(\widetilde{\delta}_t\right)^2\left(H_S\left[S_T\right]\right)^2+n\left(\widetilde{\delta}_t H\left[S_T\right]-1\right)^{n-1}\widetilde{\delta}_t H_{SS}\left[S_T\right].$$

I take the expected value of (I-B13):

$$\begin{aligned}\mathbb{E}_t^*\left[\left(\widetilde{\delta}_t H\left[S_T\right]-1\right)^n\right] &= \left(\widetilde{\delta}_t-1\right)^n+n\left(R_{f,t\to T}-1\right)\left(\widetilde{\delta}_t-1\right)^{n-1}\widetilde{\delta}_t S_t H_S\left[S_t\right]\\ &\quad+R_{f,t\to T}\left(\int_{S_t}^{\infty}G_{SS}[K]C_t[K]dK+\int_0^{S_t}G_{SS}[K]P_t[K]dK\right).\end{aligned}$$

Conditional Risk-Neutral Moments of the SDF The conditional risk-neutral moment is

$$\begin{aligned}\mathcal{M}_t^{*(n)}[T] &= \mathbb{E}_t^*\left[\left(m_{t\to T}^{SDF}-\mathbb{E}_t^*\left(m_{t\to T}^{SDF}\right)\right)^n\right] \\ &= \left(\mathbb{E}_t^*\left(m_{t\to T}^{SDF}\right)\right)^n\mathbb{E}_t^*\left[\left(\frac{m_{t\to T}^{SDF}}{\mathbb{E}_t^*\left(m_{t\to T}^{SDF}\right)}-1\right)^n\right].\end{aligned}$$

Since

$$m_{t\to T}^{SDF}=\frac{\delta_t}{R_{f,t\to T}\mathbb{E}_t\left(z_{M,t\to T}\right)}R_{M,t\to T}^{-\alpha}z_{M,t\to T},$$

we have

$$\frac{m_{t\to T}^{SDF}}{\mathbb{E}_t^*\left(m_{t\to T}^{SDF}\right)}=\delta_t^* R_{M,t\to T}^{-\alpha}z_{M,t\to T},$$

where

$$\delta_t^*=\frac{\left(\frac{\delta_t}{\mathbb{E}_t\left(z_{M,t\to T}\right)}\right)}{R_{f,t\to T}\xi_t},$$

with

$$\xi_t=\mathbb{E}_t^*\left(m_{t\to T}^{SDF}\right)=R_{f,t\to T}\mathbb{E}_t\left(\left(m_{t\to T}^{SDF}\right)^2\right)=R_{f,t\to T}\left(\mathcal{M}_t^{(2)}[T]+\frac{1}{R_{f,t\to T}^2}\right).$$

Thus,

$$\begin{aligned}\mathcal{M}_t^{*(n)}[T] &= \xi_t^n\mathbb{E}_t^*\left[\left(\delta_t^* R_{M,t\to T}^{-\alpha}z_{M,t\to T}-1\right)^n\right] \\ &= \xi_t^n\mathbb{E}_t^*\left[\left(\delta_t^* H[S_T]-1\right)^n\right] \\ &= \xi_t^n\mathbb{E}_t^*\left(G[S_T]\right).\end{aligned}$$

Following the proof of the physical moments of the SDF, we define

$$G_{SS}[S_T]=n(n-1)\left(\delta_t^* H[S_T]-1\right)^{n-2}\left(\delta_t^*\right)^2\left(H_S[S_T]\right)^2+n\left(\delta_t^* H[S_T]-1\right)^{n-1}\delta_t^* H_{SS}[S_T].$$

and show that

$$\begin{aligned}\mathbb{E}_t^*\left[\left(\delta_t^* H[S_T]-1\right)^n\right] &= \left(\delta_t^*-1\right)^n+n\left(R_{f,t\to T}-1\right)\left(\delta_t^*-1\right)^{n-1}\delta_t^* S_t H_S[S_t] \\ &\quad +R_{f,t\to T}\left(\int_{S_t}^{\infty}G_{SS}[K]C_t[K]\,dK+\int_0^{S_t}G_{SS}[K]P_t[K]\,dK\right).\end{aligned}$$

Therefore,

$$\mathcal{M}_t^{*(n)}[T] = \xi_t^n \mathbb{E}_t^*\left[\left(\delta_t^* H[S_T] - 1\right)^n\right].$$

Risk-Neutral Moments of $\mathbb{E}_t^*\left(\left(m_{t\to T}^{SDF}\right)^{n-2}\right)$

I first observe that

$$m_{t\to T}^{SDF} = \frac{\delta_t}{R_{f,t\to T}} R_{M,t\to T}^{-\alpha} \frac{z_{M,t\to T}}{\mathbb{E}_t\left(z_{M,t\to T}\right)}.$$

Thus,

$$\left(m_{t\to T}^{SDF}\right)^{n-2} = \left(\frac{\frac{\delta_t}{R_{f,t\to T}}}{\mathbb{E}_t\left(z_{M,t\to T}\right)}\right)^{n-2} R_{M,t\to T}^{-\alpha(n-2)} z_{M,t\to T}^{(n-2)} = \xi^{n-2} R_{M,t\to T}^{-\alpha(n-2)} z_{M,t\to T}^{(n-2)},$$

with $\xi_t = \frac{\frac{\delta_t}{R_{f,t\to T}}}{\mathbb{E}_t(z_{M,t\to T})}$.

It follows that $\mathbb{E}_t^*\left(\left(m_{t\to T}^{SDF}\right)^{n-2}\right) = \xi_t^{n-2}\mathbb{E}_t^*\left(H[S_T]\right)$ with $H[S_T] = f[S_T]\,g[S_T]$, where

$$f[S_T] = \left(\frac{S_T}{S_t}\right)^{-\alpha(n-2)} \text{ and } g[S_T] = 1 - \rho^{(2)}\left(\frac{u[S_T]}{u[S_t]} - 1\right). \tag{I-B14}$$

I, thereafter, observe that

$$f[S_T] = \left(\frac{S_T}{S_t}\right)^{-\alpha(n-2)} \text{ and } H[S_T] = 1 - \rho^{(2)}\left(\frac{S_T^{1-\alpha} - 1}{S_t^{1-\alpha} - 1} - 1\right). \tag{I-B15}$$

Thus $H[S_t] = 1$ and

$$H_S[S_t] = f^{'}[S_T]\,g[S_T] + f[S_T]\,g^{'}[S_T] = \alpha(2-n)\frac{1}{S_t} - \rho^{(2)}\frac{u^{'}[S_t]}{u[S_t]},$$

and

$$H_{SS}[K] = f_{SS}[K]\,g[K] + 2 f_S[K]\,g_S[K] + f[K]\,g_{SS}[K],$$

where

$$f_S[S_T] = \alpha(2-n)\frac{1}{S_t}\left(\frac{S_T}{S_t}\right)^{\alpha(2-n)-1},\ f_{SS}[S_T] = \left(\alpha(2-n)\right)^2\frac{1}{S_t^2}\left(\frac{S_T}{S_t}\right)^{\alpha(2-n)-2}, \tag{I-B16}$$

$$
\begin{aligned}
g_S[S_T] &= -\rho^{(2)}\frac{u'[S_T]}{u[S_t]} = -\rho^{(2)}(1-\alpha)\frac{S_t^{-\alpha}}{S_t^{1-\alpha}-1}\left(\frac{S_T}{S_t}\right)^{-\alpha}, \text{ and} \\
g_{SS}[S_T] &= -\rho^{(2)}\frac{u''[S_T]}{u[S_t]} = -\rho^{(2)}(-\alpha)(1-\alpha)\frac{S_t^{-\alpha-1}}{S_t^{1-\alpha}-1}\left(\frac{S_T}{S_t}\right)^{-\alpha-1}.
\end{aligned}
$$

Using the spanning formula,

$$
H[S_T] = H[1] + (S_T - S_t)H_S[S_t] + \int_{S_t}^{\infty} H_S[S_t](S_T-K)^+\,dK + \int_0^{S_t} H_S[S_t](K-S_T)^+\,dK.
$$

Finally,

$$
\mathbb{E}_t^*\left(\left(m_{t\to T}^{SDF}\right)^{n-2}\right) = \mathbb{E}_t^*(H[S_T]) = H[1] + (R_{f,t\to T}-1)S_t H_S[S_t] + R_{f,t\to T}\left(\int_{S_t}^{\infty} H_S[S_t]C_t[K]\,dK + \int_0^{S_t} H_S[S_t]P_t[K]\,dK\right). \tag{I-B17}
$$

■

Table 25

Estimates of the Price of Risk This table presents the estimation results of the beta pricing model (40). The model is estimated using daily returns on the 25 Fama and French portfolios formed on size and book-to-market. The column Only FF5 presents results when only the five Fama French factors are used. The data are from January 1996 to August 2015. I report the Fama and MacBeth (1973) t-ratio under correctly specified models (t_{FM}), the Shanken (1992) t-ratio (t_S), the Jagannathan and Wang (1998) t-ratio under correctly specified models that account for the EIV problem (t_{JW}), and the Kan, Robotti, and Shanken (2013) misspecification robust t-ratios (t_{KRS}). The table also presents the sample cross-sectional R^2 of the beta pricing model (40). p($R^2 = 1$) is the p-value for the test of H0 : $R^2 = 1$, p($R^2 = 0$) is the p-value for the test of H0 : $R^2 = 0$, and p(W) is the p-value of Wald test under the null hypothesis that all prices of risk are equal to zero. se($\widehat{R}^2$) is the standard error of $\widehat{R}^2$ under the assumption that $0 < R^2 < 1$.

Maturity	"Only FF5"	30	60	91	122	152	182	273	365
λ_0	0.013	0.117	0.124	0.128	0.129	0.130	0.131	0.126	0.122
t_{FM}	5.653	5.211	5.544	5.715	5.890	6.046	6.132	6.016	5.812
t_S	5.635	4.893	5.225	5.390	5.551	5.680	5.744	5.552	5.317
t_{JW}	5.820	5.721	6.054	6.217	6.372	6.563	6.645	6.230	5.928
t_{KRS}	5.743	5.154	5.341	5.334	5.561	5.770	6.020	5.818	5.438
λ_{MKT}	-0.064	-0.051	-0.062	-0.068	-0.072	-0.073	-0.073	-0.073	-0.069
t_{FM}	-2.923	-2.894	-3.152	-3.281	-3.367	-3.453	-3.490	-3.366	-3.217
t_S	-2.917	-2.781	-3.036	-3.162	-3.245	-3.323	-3.353	-3.205	-3.046
t_{JW}	-3.330	-3.387	-3.672	-3.786	-3.839	-3.945	-3.963	-3.693	-3.492
t_{KRS}	-3.208	-3.141	-3.352	-3.371	-3.460	-3.576	-3.662	-3.461	-3.235
$\lambda_{\mathcal{M}^{(2)}}$		0.094	0.057	0.046	0.042	0.048	0.050	0.043	0.036
t_{FM}		3.078	2.726	2.389	2.185	2.023	1.595	0.765	0.518
t_S		2.893	2.571	2.254	2.061	1.902	1.495	0.706	0.474
t_{JW}		3.024	2.644	2.301	2.090	2.027	1.624	0.756	0.513
t_{KRS}		2.369	2.055	1.744	1.593	1.407	1.028	0.483	0.321
$\lambda_{\mathcal{M}^{(3)}}$		0.112	-0.044	-0.067	-0.069	-0.072	-0.076	-0.064	-0.042
t_{FM}		2.324	2.421	2.217	2.056	1.720	1.275	0.424	0.183
t_S		2.184	2.283	2.092	1.939	1.616	1.195	0.392	0.168
t_{JW}		2.363	2.437	2.176	1.968	1.703	1.276	0.409	0.178
t_{KRS}		1.504	1.632	1.409	1.294	1.022	0.724	0.252	0.111
$\lambda_{\mathcal{M}^{(4)}}$		-0.088	0.041	0.046	0.039	0.033	0.032	0.021	0.011
t_{FM}		2.106	2.369	2.377	2.306	2.005	1.641	0.887	0.623
t_S		1.979	2.234	2.243	2.175	1.885	1.538	0.819	0.570
t_{JW}		2.071	2.390	2.343	2.204	1.963	1.623	0.839	0.593
t_{KRS}		1.400	1.674	1.560	1.486	1.231	0.955	0.543	0.391

Table 25
Estimates of the Price of Risk, continued

Maturity (days)	Only FF5	30	60	91	122	152	182	273	365
λ_{SMB}	0.024	0.043	0.048	0.049	0.051	0.050	0.050	0.045	0.043
t_{FM}	0.709	0.798	0.798	0.795	0.804	0.795	0.793	0.779	0.784
t_S	0.709	0.796	0.796	0.793	0.802	0.793	0.791	0.777	0.781
t_{JW}	0.688	0.776	0.780	0.776	0.785	0.774	0.770	0.752	0.754
t_{KSR}	0.685	0.775	0.777	0.774	0.782	0.771	0.767	0.745	0.745
λ_{HML}	0.061	0.080	0.094	0.102	0.108	0.106	0.105	0.095	0.090
t_{FM}	1.125	1.173	1.138	1.128	1.102	1.093	1.087	1.064	1.063
t_S	1.125	1.171	1.136	1.126	1.100	1.091	1.085	1.061	1.059
t_{JW}	1.084	1.113	1.083	1.076	1.055	1.050	1.046	1.028	1.029
t_{KSR}	1.078	1.107	1.079	1.069	1.046	1.042	1.037	1.015	1.013
λ_{RMW}	0.013	0.022	0.023	0.021	0.020	0.011	0.003	-0.018	-0.020
t_{FM}	0.915	1.044	1.205	1.255	1.289	1.225	1.187	0.974	0.895
t_S	0.913	1.000	1.157	1.206	1.237	1.173	1.133	0.920	0.839
t_{JW}	0.723	0.826	0.952	1.004	1.047	1.025	1.023	0.906	0.864
t_{KSR}	0.710	0.803	0.914	0.953	0.984	0.944	0.924	0.808	0.763
λ_{CMA}	-0.130	-0.147	-0.186	-0.205	-0.219	-0.213	-0.212	-0.196	-0.182
t_{FM}	-0.287	-0.315	-0.526	-0.638	-0.717	-0.649	-0.629	-0.525	-0.441
t_S	-0.286	-0.299	-0.501	-0.608	-0.682	-0.616	-0.595	-0.490	-0.409
t_{JW}	-0.337	-0.370	-0.621	-0.750	-0.847	-0.780	-0.759	-0.634	-0.532
t_{KSR}	-0.299	-0.333	-0.554	-0.658	-0.729	-0.649	-0.611	-0.501	-0.419
R^2	0.197	0.364	0.354	0.362	0.365	0.370	0.381	0.408	0.422
p($R^2=1$)	0.001	0.003	0.002	0.002	0.002	0.001	0.001	0.003	0.004
p($R^2=0$)	0.075	0.125	0.107	0.105	0.097	0.105	0.114	0.085	0.075
p(W)	0.030	0.016	0.015	0.013	0.012	0.011	0.010	0.009	0.008
se($\widehat{R}^2$)	0.097	0.144	0.136	0.134	0.133	0.132	0.134	0.147	0.151

Table 26

Estimates of the Price of Risk This table presents the estimation results of the beta pricing model (41). The model is estimated using daily returns on the 25 Fama and French portfolios formed on size and book-to-market. The data are from January 1996 to August 2015. I report the Fama and MacBeth (1973) t-ratio under correctly specied models (t_{FM}), the Shanken (1992) t-ratio (t_S), the Jagannathan and Wang (1998) t-ratio under correctly specied models that account for the EIV problem (t_{JW}), and the Kan, Robotti, and Shanken (2013) misspecification-robust t-ratios (t_{KRS}). The table also presents the sample cross-sectional R^2 of the beta pricing model ((41)). p($R^2 = 1$) is the p-value for the test of H0 : $R^2 = 1$, p($R^2 = 0$) is the p-value for the test of H0 : $R^2 = 0$, and p(W) is the p-value of Wald test under the null hypothesis that all prices of risk are equal to zero. se($\widehat{R}^2$) is the standard error of $\widehat{R}^2$ under the assumption that $0 < R^2 < 1$.

Maturity	Only FF5	30	60	91	122	152	182	273	365
λ_0	0.013	0.111	0.116	0.120	0.121	0.123	0.136	0.133	0.128
t_{FM}	5.653	4.994	5.191	5.305	5.439	5.577	6.083	6.310	6.135
t_S	5.635	4.718	4.897	4.997	5.150	4.913	5.035	5.199	5.696
t_{JW}	5.820	5.243	5.448	5.613	5.707	5.559	5.155	5.685	6.554
t_{KRS}	5.743	4.771	4.994	5.033	5.205	5.039	4.967	5.147	6.330
λ_{MKT}	-0.064	-0.050	-0.059	-0.061	-0.066	-0.060	-0.070	-0.076	-0.062
t_{FM}	-2.923	-2.700	-2.871	-2.986	-3.064	-3.133	-3.563	-3.614	-3.404
t_S	-2.917	-2.605	-2.767	-2.875	-2.961	-2.889	-3.145	-3.194	-3.255
t_{JW}	-3.330	-3.069	-3.252	-3.370	-3.395	-3.409	-3.423	-3.527	-3.833
t_{KRS}	-3.208	-2.819	-3.019	-3.070	-3.144	-3.151	-3.315	-3.281	-3.719
$\lambda_{\mathcal{MRP}^{(2)}}$		-0.235	-0.070	-0.035	0.007	-0.121	-0.152	0.154	0.038
t_{FM}		-2.308	-2.307	-1.864	-1.461	-1.783	-1.849	1.262	0.138
t_S		-2.182	-2.178	-1.757	-1.384	-1.572	-1.532	1.040	0.128
t_{JW}		-2.330	-2.253	-1.812	-1.366	-1.723	-1.687	1.164	0.141
t_{KRS}		-1.592	-1.550	-1.111	-0.797	-0.786	-0.704	0.541	0.073
$\lambda_{\mathcal{MRP}^{(3)}}$		0.152	0.048	0.029	0.015	0.017	0.014	0.008	0.005
t_{FM}		2.322	2.730	2.773	2.396	2.711	2.805	0.486	1.186
t_S		2.195	2.577	2.614	2.270	2.390	2.324	0.401	1.102
t_{JW}		2.054	2.299	2.366	1.976	2.386	2.359	0.421	1.169
t_{KRS}		1.626	2.006	1.829	1.425	1.219	1.063	0.231	0.656
$\lambda_{\mathcal{MRP}^{(4)}}$		-0.238	-0.037	-0.018	0.014	-0.078	-0.094	0.073	0.014
t_{FM}		1.966	2.432	2.253	2.002	1.918	1.823	-0.494	0.242
t_S		1.858	2.296	2.123	1.896	1.691	1.510	-0.407	0.225
t_{JW}		1.864	2.336	2.199	1.883	1.880	1.674	-0.458	0.246
t_{KRS}		1.207	1.640	1.397	1.153	0.903	0.751	-0.245	0.152

Table 26
Estimates of the Price of Risk, continued

Maturity (days)	Only FF5	30	60	91	122	152	182	273	365
λ_{SMB}	0.024	0.044	0.049	0.051	0.053	0.040	0.029	0.074	0.050
t_{FM}	0.709	0.792	0.818	0.820	0.831	0.866	0.851	0.809	0.793
t_S	0.709	0.790	0.816	0.818	0.829	0.862	0.843	0.802	0.791
t_{JW}	0.688	0.771	0.801	0.802	0.813	0.844	0.831	0.769	0.769
t_{KRS}	0.685	0.771	0.797	0.797	0.807	0.835	0.827	0.763	0.758
λ_{HML}	0.061	0.074	0.089	0.099	0.102	0.126	0.160	0.045	0.078
t_{FM}	1.125	1.175	1.122	1.116	1.096	1.037	1.026	1.197	1.166
t_S	1.125	1.173	1.120	1.113	1.095	1.032	1.018	1.187	1.163
t_{JW}	1.084	1.112	1.062	1.057	1.040	0.979	0.970	1.138	1.112
t_{KRS}	1.078	1.101	1.059	1.053	1.037	0.973	0.961	1.127	1.100
λ_{RMW}	0.013	0.031	0.037	0.042	0.042	0.066	0.051	-0.055	0.013
t_{FM}	0.915	0.971	1.137	1.246	1.291	1.393	1.225	0.991	1.259
t_S	0.913	0.934	1.093	1.196	1.242	1.273	1.068	0.859	1.194
t_{JW}	0.723	0.754	0.874	0.960	1.009	1.015	0.806	0.891	1.051
t_{KRS}	0.710	0.735	0.835	0.903	0.924	0.939	0.775	0.756	0.929
λ_{CMA}	-0.130	-0.139	-0.178	-0.200	-0.214	-0.221	-0.244	-0.116	-0.180
t_{FM}	-0.287	-0.331	-0.569	-0.712	-0.787	-0.800	-0.862	0.022	-0.447
t_S	0.286	-0.315	-0.542	-0.677	-0.752	-0.719	-0.734	0.018	-0.420
t_{JW}	-0.337	-0.371	-0.644	-0.802	-0.902	-0.847	-0.813	0.023	-0.520
t_{KRS}	-0.299	-0.332	-0.571	-0.710	-0.770	-0.749	-0.725	0.016	-0.393
R^2	0.197	0.338	0.333	0.343	0.341	0.367	0.398	0.433	0.389
$p(R^2 = 1)$	0.001	0.004	0.004	0.004	0.004	0.056	0.226	0.085	0.003
$p(R^2 = 0)$	0.075	0.167	0.179	0.161	0.163	0.159	0.168	0.131	0.109
$p(W)$	0.030	0.026	0.026	0.023	0.019	0.017	0.013	0.010	0.012
$se(\widehat{R}^2)$	0.097	0.145	0.145	0.147	0.143	0.176	0.211	0.203	0.160

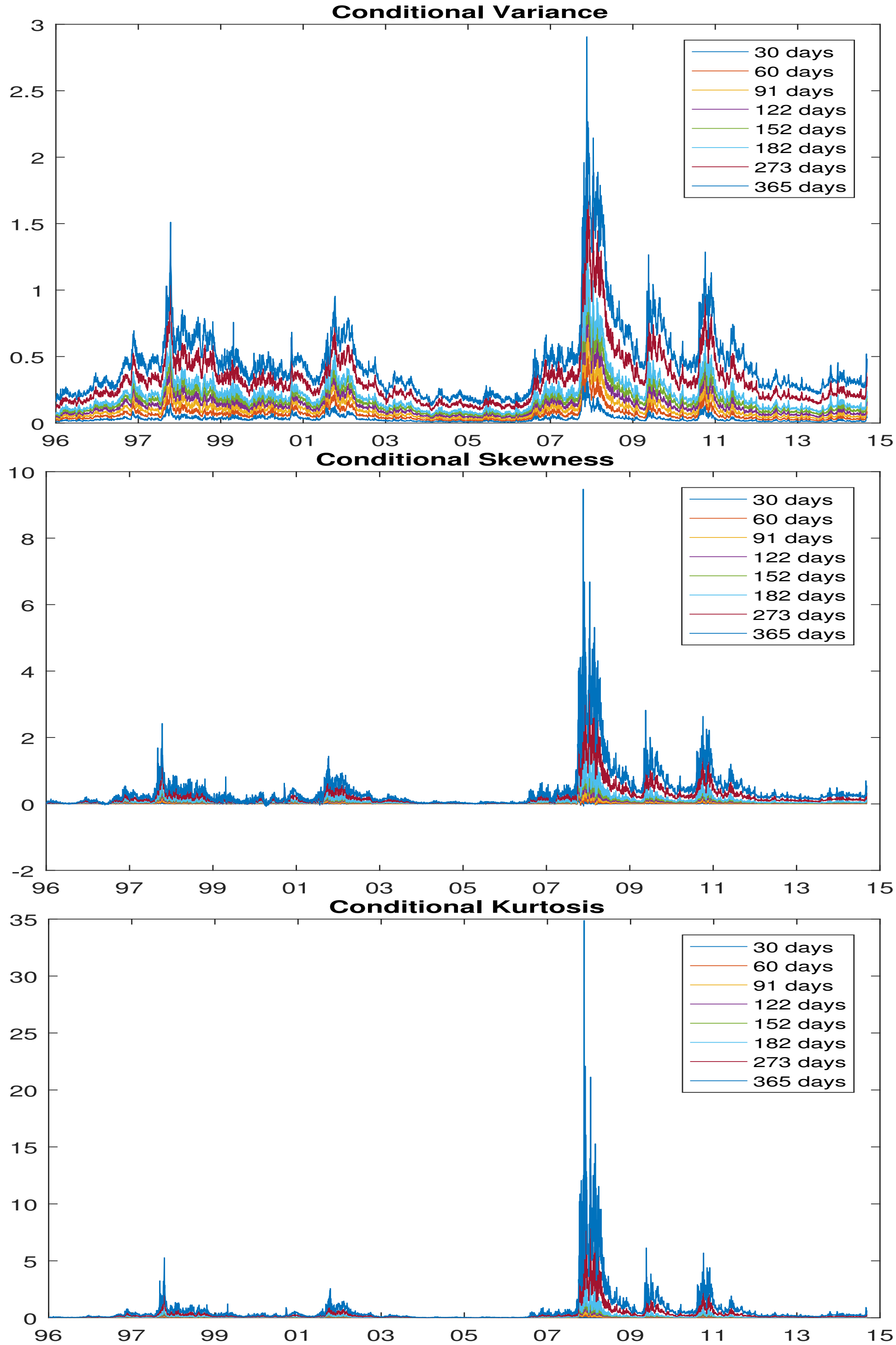

Fig. 7. **Conditional Moments of the SDF under Generalized Preferences** I plot daily variance, skewness, and kurtosis of the SDF. The moments are computed from the S&P 500 index option prices from January 4, 1996, through August 31, 2015, and are not annualized. I use preferences that depart from CRRA preferences with $\alpha = 2$, $\rho^{(2)} = 5$ and $\rho^{(k)} = 0$ for $k > 2$. The maturity of options used to compute the moments is labeled in days.

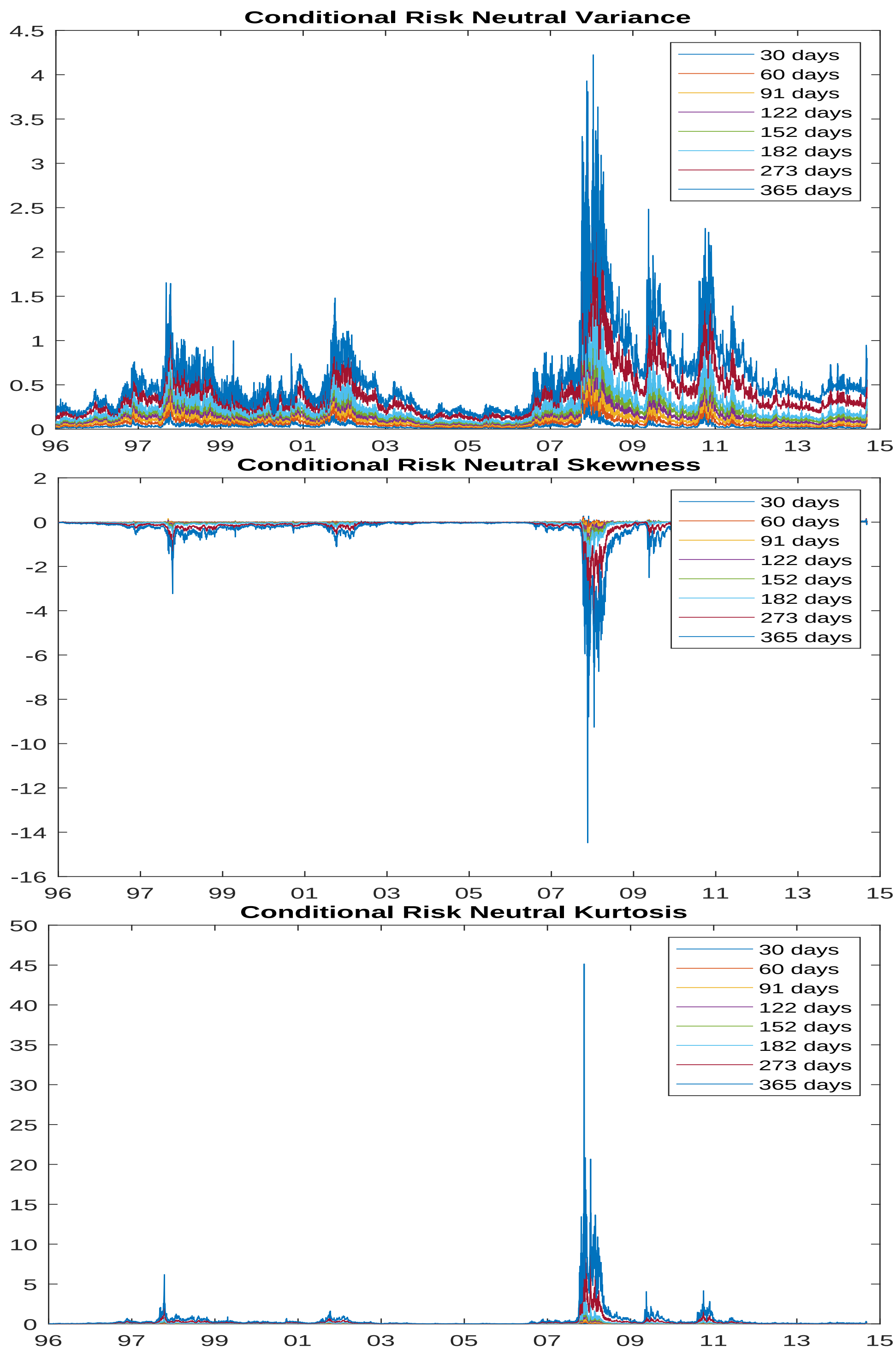

Fig. 8. **Conditional Risk-Neutral Moments of the SDF under Generalized Preferences.** I plot daily variance, skewness and kurtosis of the SDF. The moments are computed from the S&P 500 index option prices from January 4, 1996, through August 31, 2015, and are not annualized. I use preferences that depart from CRRA preferences with $\alpha = 2$, $\rho^{(2)} = 5$, and $\rho^{(k)} = 0$ for $k > 2$. The maturity of options used to compute the moments is labeled in days.

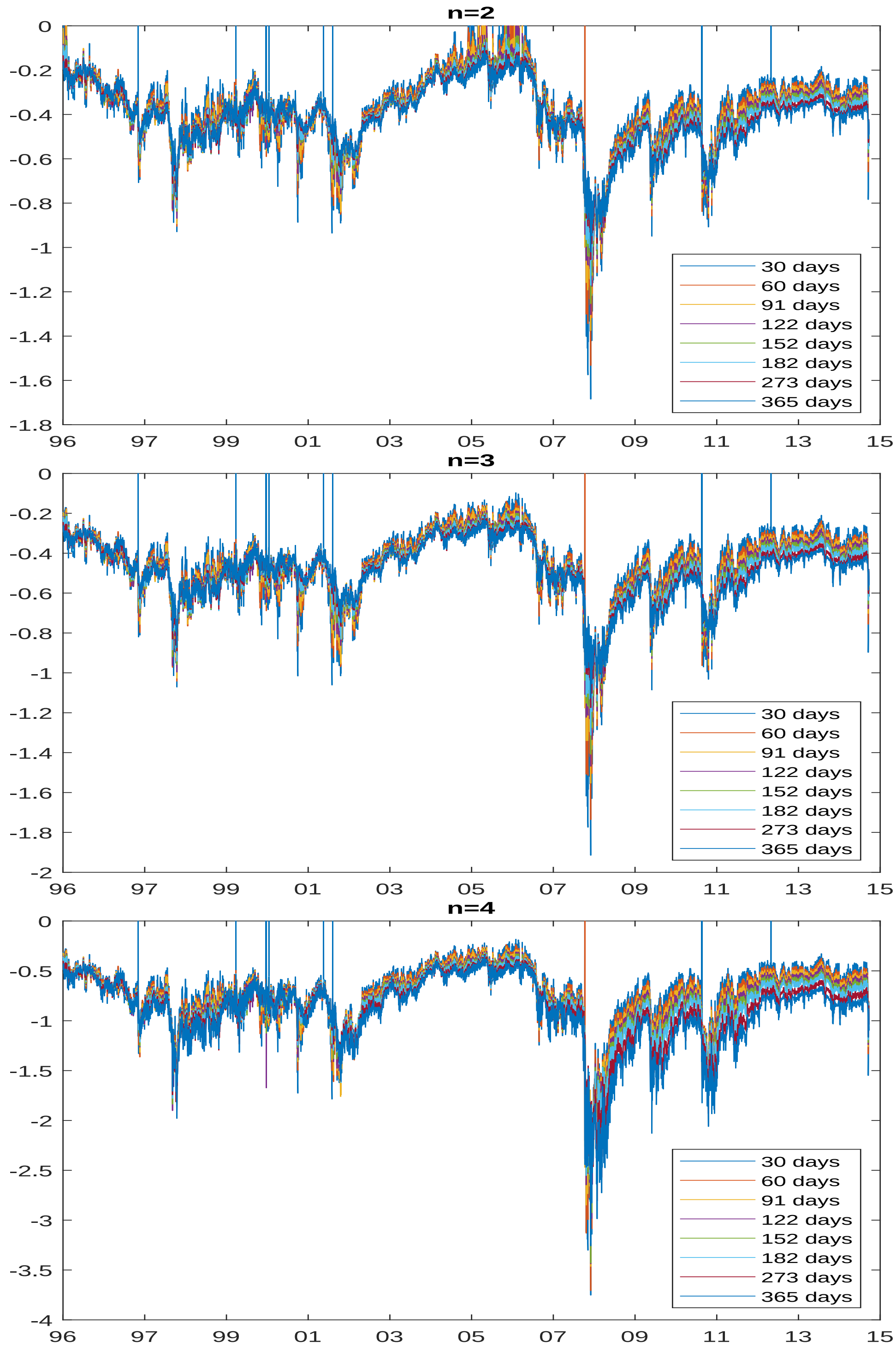

Fig. 9. **Sharpe Ratios on SDF Moments under Generalized Preferences.** I plot conditional Sharpe Ratios under preferences that depart from CRRA preferences. Sharpe Ratios are annualized and cover the period from January 4, 1996, through August 31, 2015. I use preferences that depart from CRRA preferences with $\alpha = 2$, $\rho^{(2)} = 5$, and $\rho^{(k)} = 0$ for $k > 2$. The maturity of options used to compute the moments is labeled in days.

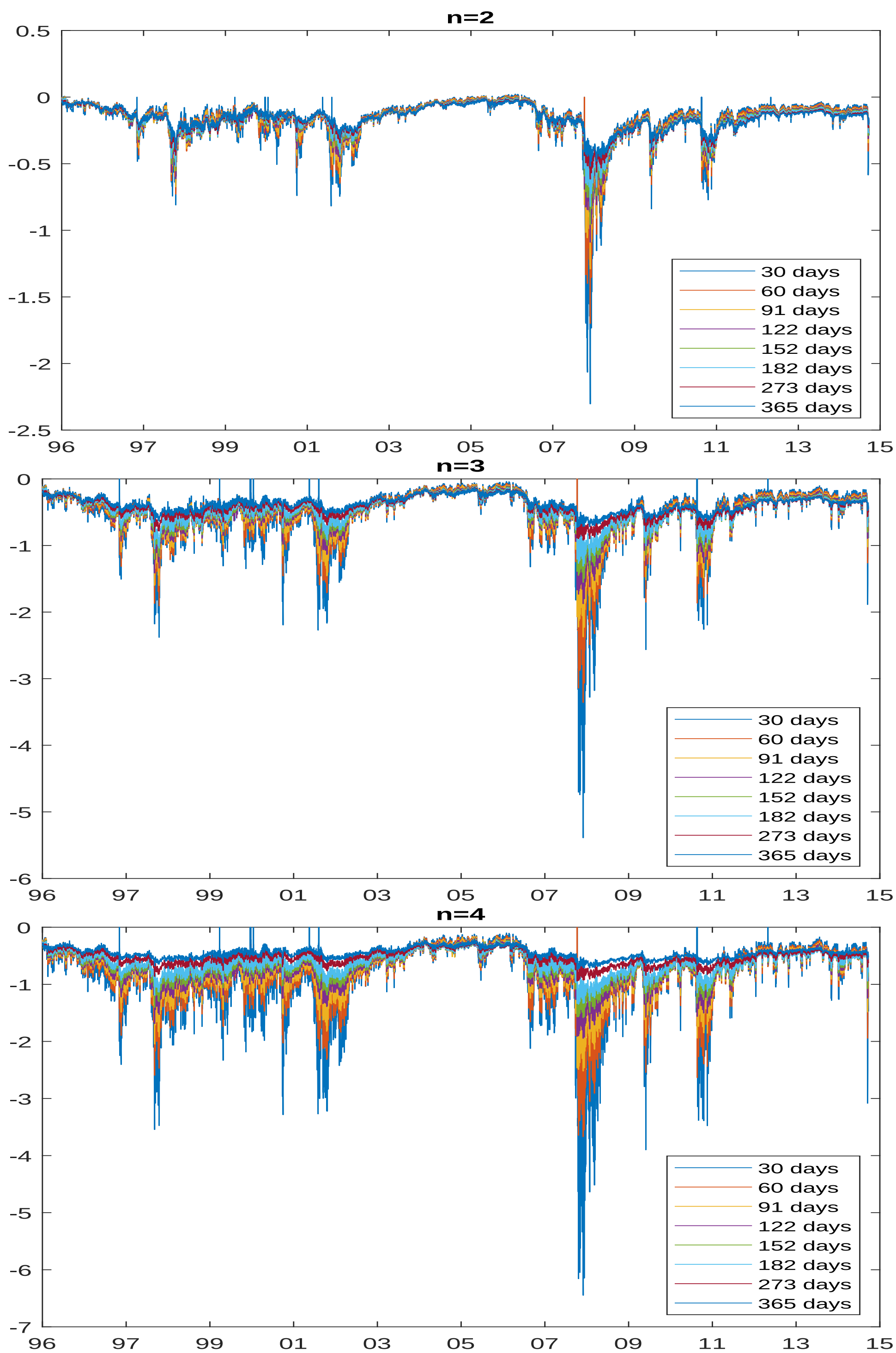

Fig. 10. **Expected Excess Returns on SDF Moments under Generalized Preferences.** I plot conditional expected excess returns on SDF-based moments under preferences that depart from CRRA preferences. I use preferences that depart from CRRA preferences with $\alpha = 2$, $\rho^{(2)} = 5$, and $\rho^{(k)} = 0$ for $k > 2$. The expected excess returns are annualized and cover the period from January 4, 1996, through August 31, 2015. The maturity of options used to compute the moments is labeled in days.