Index Investing and Price Discovery*

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Abstract

We study the effect of the rise of indexing on price discovery. We show that this effect critically depends on the causes of the rise of indexing and the cost structure of information acquisition. If the rise of indexing is due to increased cost of participating in the non-index market, then the price informativeness of the non-index *decreases*. In contrast, if it is due to improved market transparency and thus lower profitability in the non-index market, then the price informativeness of the non-index *increases*. In both cases, the price informativeness of the index tends to increase (resp. decrease) and thus market risk premium decreases (resp. increases) if acquiring information in one market increases (resp. decreases) information acquisition cost in another market. In addition, after an exogenous rise of indexing, price informativeness in both index and non-index markets declines. Moreover, social welfare may increase with indexing and active investors may "free ride" on discretionary indexers' information acquisition in the index market. Our analysis provides testable implications that can help identify the driving force of the rise of indexing.

JEL Classification Codes: G11, G12, G14, D82.

Keywords: Indexing, Information Acquisition, Risk Premium, Welfare.

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Abstract

We study the effect of the rise of indexing on price discovery. We show that this effect critically depends on the causes of the rise of indexing and the cost structure of information acquisition. If the rise of indexing is due to increased cost of participating in the non-index market, then the price informativeness of the non-index *decreases*. In contrast, if it is due to improved market transparency and thus lower profitability in the non-index market, then the price informativeness of the non-index *increases*. In both cases, the price informativeness of the index tends to increase (resp. decrease) and thus market risk premium decreases (resp. increases) if acquiring information in one market increases (resp. decreases) information acquisition cost in another market. In addition, after an exogenous rise of indexing, price informativeness in both index and non-index markets declines. Moreover, social welfare may increase with indexing and active investors may "free ride" on discretionary indexers' information acquisition in the index market. Our analysis provides testable implications that can help identify the driving force of the rise of indexing.

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1. Introduction

Index investing has grown substantially during the past few decades.[1] As a result of such growth, more investors enjoy lower investment fees and more index funds and ETFs save on stock picking costs. However, as such growth continues over the recent years, some major concerns arise in the industry, the media, and the academia. One of such major concerns is whether a significant rise of index investing will hurt price discovery and thus reduce market efficiency. In this paper, we develop an equilibrium model to study how price discovery changes after a rise of index investing. We show that qualitative and quantitative impact of index investing critically depends on what caused the growth of index investing. Our analysis highlights the importance of identifying the underlying driving force of the rise of index investing for the understanding of its impact and provides empirically testable implications that can help this identification.

In our model, in addition to a risk free asset, investors can also trade two risky assets, the index and the non-index, to maximize their expected utility at the end of period. Trading in the index is free for all investors, but some investors have to incur a participation cost (e.g., security learning cost) in order to trade the non-index. We consider four types of investors: 1. active investors ("A investors") who have zero cost for trading the non-index and thus always trade both risky assets; 2. discretionary investors ("D investors") who must incur a participation cost to trade the non-index. They may choose to be indexers who only trade the index ("DI investors") or active investors who trade both the index and the non-index ("DA investors"); 3. exogenous indexers ("I investors") who only trade the index; and 4. liquidity traders ("L investors") who must trade both risky assets to hedge their endowment risk. Only active traders and discretionary investors may acquire private information about the fundamental value of the risky assets by paying a cost. Depending on how high the participation cost is, in equilibrium, either all discretionary investors are indifferent between being indexing and staying active or all prefer indexing or all prefer staying active.

In actual financial markets, most hedge funds can be considered as "A investors" since they do not invest only in the index. Mutual funds, by contrast, have to basically stick to certain stocks or bonds, and are usually long-only. Therefore, we can consider them as "D investors" in our model. Some of them may choose to become active mutual funds ("DA investors") and the rest choose to be passive mutual funds ("DI investors"). Most index funds are considered as "I investors" who do not do stock selections or acquire private information. "L investors" can be considered as individual traders or hedgers who trade assets for hedging needs instead of fundamental

[1]During 2016, actively-managed funds experienced \$285 billion of outflows while passive funds attracted \$429 billion of inflows. The proliferation of ETFs is now approaching 2,000 funds and nearly \$3.0 trillion of asset under management.

value of assets.

Our model allows us to study different possible causes of the rise of index investing. To make our main points clearer, we focus on three main possible causes in our analysis: 1. increases in participation cost in the non-index market; 2. increases in transparency of the non-index market; and 3. increases in exogenous indexers.

We show that the equilibrium effects of the rise of index investing on price discovery critically depend on what causes the rise of indexing and how information acquisition by an investor in one market affects his cost of acquiring information in another market ("information acquisition externality" or "IAE" for short).

To see why the sign of IAE matters, consider the case when the IAE is positive (i.e., acquiring information in one market lowers the cost of acquiring information in another market).[2] Because discretionary active investors also acquire private information in the non-index market which lowers their information acquisition cost in the index market, discretionary index investors' equilibrium precision of private information about the index is lower than that of discretionary active investors. The opposite is true if the IAE is negative. Our model implies that, in contrast to the intuition that indexers are getting a free ride on the information acquisition of the active investors, active traders might sometimes free-ride on the discretionary indexers' information acquisition as indexing increases if the sign of the IAE is negative. Only when the IAE is positive, discretionary indexers choose to obtain less precise information than the active investors and thus indexers tend to free-ride active investors.

If the rise of index investing is due to increases in the participation cost for trading the non-index,[3] then price informativeness in the index market decreases and thus market risk premium increases if and only if the IAE is positive. The intuition is simple. When the IAE is positive, DI investors' equilibrium precision of private information about the market portfolio is lower than that of DA investors, because DA investors also acquire information in the non-index market which lowers their information acquisition cost in the index market. Therefore, as more discretionary investors choose to be DI investors, the aggregate precision of private information about the index decreases, and thus the price informativeness in the index market decreases, which drives up the market risk premium. The opposite is true if the IAE is negative. In contrast, even though A and DA investors both acquire more precise information due to the increase of marginal benefit of information as a result of the decrease in the competition, price informativeness in the non-index market *decreases* regardless of the sign of IAE. This is because as more discretionary active traders

[2]Acquiring information in one market may lower the cost of acquiring information in another market because of economics of scope or commonality in information acquisition across the two markets.

[3]A possible reason for an increase in the participation cost is the increase in the number of risk factors for an individual security and the increase in the variety of non-index securities which makes it more difficult to learn about a particular security and to choose which ones to invest in.

switch to index investment, the fraction of investors who acquire information about the fundamental value of non-index (e.g., A and DA investors) decreases.

In addition, social welfare can improve with the increase in the participation cost for trading the non-index. More index investment improves the welfare of active and discretionary traders[4] because there are less informed investors competing in the non-index market. A rise in index investing does not affect significantly the welfare of exogenous indexers since they don't trade the non-index to get the benefit from reduced competition and the effect of the changed price informativeness of the index market is small. On the other hand, a rise in indexing may hurt liquidity traders significantly, because there are less investors in the non-index market to share risk and thus the adverse price impact of liquidity trades becomes greater. Therefore, the social welfare may decrease if there are sufficient liquidity investors. However, if the uncertainty in the non-index market is low, the adverse price impact of liquidity trades increases less as indexing rises, therefore the social welfare may improve with indexing.

Another possible reason for the rise of index investing is that it becomes less profitable to trade in the non-index market as a result of the improvement in the transparency in the non-index market. When the profitability in the non-index market decreases, more discretionary investors choose to be indexers. Both exogenous active traders and discretionary active traders optimally acquire less precise private information due to reduced profitable opportunities, yet the price informativeness in the non-index market *increases* because of improved non-index market transparency.

When more discretionary active traders switch to index investment because of reduced profitability in the non-index market, the price informativeness in the index market increases if and only if the IAE is negative. This follows from similar intuitions as the previous case. When the IAE is negative, discretionary indexers acquire more precise private information about the index than that of discretionary active investors. As more discretionary investors choose to be indexers, the aggregate precision of private information about the index increases. In addition, the social welfare tends to increase when more indexing is due to increased transparency of the non-index market.

The third possible reason for the rise of index investment is increased exogenous indexers who don't trade in non-index and do not acquire private information about the index because of high information acquisition cost. This exogenous increase could be triggered by market efficiency education and cumulated evidence against persistent outperformance of active funds over index funds. Not surprisingly, in this case, the price informativeness of the index *decreases* and thus market risk premium *increases* regardless of the sign of the IAE. This is because the fraction of traders who acquire

[4]Note that in equilibrium a discretionary investor is indifferent between being an active trader and being an indexer. Therefore, the welfare of a discretionary active trader is the same as the welfare of a discretionary indexer.

information about the index decreases. In some sense, these exogenous indexers are always getting a free ride on the information acquisition of active traders and discretionary indexers.

With more discretionary investors switching to indexers, even though both exogenous active traders and discretionary active traders acquire more precise private information about the non-index, the price informativeness of the non-index tends to *decrease* because fewer investors acquire information in the market. In this case, the social welfare tends to be reduced while both exogenous active traders and discretionary traders are better off.

To summarize, our analysis implies that a rise of indexing due to increased exogenous indexers harms price discovery in both index and non-index markets and reduces social welfare. For the other two causes of the rise of indexing, the price informativeness of the index decreases (resp. increases) if and only if the IAE is positive (resp. negative). Regardless of the sign of IAE, the price informativeness of non-index market *decreases* if the rise of index investing is due to increases in the participation cost while it *increases* if the rise of index investing is due to low profitability of the non-index market.

Our paper highlights that it is important to understand the underlying mechanisms that drive the rise of index investing. These predictions can help researchers use empirically found relationship between the rise of index investment and price informativeness to identify what likely caused the increase in indexing and thus offer regulators some guidance on regulations adjustment if necessary.

The closest work to ours is Bond and Garcia (2017). Different from Bond and Garcia (2017), however, in our model, indexing is endogenous and some indexers can acquire private information about the assets they trade before and after they become indexers. In contrast to our above findings, Bond and Garcia (2017) find that as indexing increases, price informativeness improves, market risk premium declines, and social welfare decreases. The driving force behind the finding of Bond and Garcia (2017) on price informativeness and market risk premium is that all indexers in their model are essentially liquidity traders whose trades in a market only add noise and decrease the price informativeness. After they leave the non-index market, there is less noise trading in this market. As a result, the price informativeness in the non-index market increases, and thus the informed investors acquire less information in the non-index market. With the assumption of a negative IAE in their model, informed investors then acquire more information in the index market, which increases the price informativeness in the index market and lowers the market risk premium. In our model even in the case where we find the same qualitative results, the underlying driving force can be different from theirs. For example, consider the case where a rise of indexing is due to an increase in the participation cost in the non-index market and the IAE is negative (as in Bond and Garcia (2017)). As reported above, because A and DA investors acquire information in the non-index market which increases their information acquisition cost in the index market, they acquire *less* precise information in the index market, opposite to the finding of Bond and Garcia (2017). However, as Bond and Garcia (2017), we also find that as indexing increases, the price informativeness in the index market increases and market risk premium decreases. As explained before, the driving force for obtaining these same qualitative results in this case as theirs is that the endogenous indexers (DI investors) obtain more precise information than both exogenous active traders and DA traders in the index market, and as the population of the endogenous indexers increases, the mass of investors who acquire more precise information increases, and thus the price informativeness in the index market increases and the market risk premium decreases.

As for welfare implications of indexing, Bond and Garcia (2017) find that the welfare of indexers decrease with indexing. In contrast, we show that the welfare of indexers may be improved with indexing. The difference in findings comes from the endogenizing of indexing and the consideration of the effect of the causes of indexing. With endogenous indexing, discretionary indexers can share the benefit of less competition in the non-index market as indexing increases, because otherwise they would choose to be active. Some causes of the rise of indexing such as an improvement in transparency of the non-index market tends to improve the welfare of some investors. Overall, our analysis complements Bond and Garcia (2017) by providing additional channels associated with a rise of index investing.

Our model extends the canonical Grossman and Stiglitz (1980) framework to a multi-asset setting to study the impact of indexing. As in Admati (1985), but different from Diamond and Verrecchia (1981), Ganguli and Yang (2009), and Bond and Garcia (2017), the liquidity trades are entirely exogenous in our model. However, this assumption is unlikely critical for our results because it is equivalent to a limiting case of endogenous liquidity trades as risk aversion approaches infinity. Van Nieuwerburgh and Veldkamp (2009) also study information acquisition in multi-asset markets. In their study, they use information acquisition capacity constraints to model the tension between acquiring information in different markets. In contrast, we explicitly model the cost of information acquisition that is a convex function of precisions. Qualitatively, the case with negative IAE in our model is similar to their setting in that acquiring more information in one market increases the cost of acquiring information in another market and thus tends to lower information acquisition in the other market. The case with positive IAE in our model captures another possibility where acquiring more information in one market helps lower the cost of information acquisition in another market. Qualitatively, this is similar to the case where the experience of acquiring information in one market helps information acquisition efficiency in another market. Baruch and Zhang (2017) also study the impact of indexing. They find that indexing does not affect the validity of the CAPM risk-return relation and non-index portfolios suffer in terms of both Sharpe ratio and conditional payoff uncertainty. In contrast, we focus on how indexing affects price informativeness and market risk premium through its impact on information acquisition. Ganguli and Yang (2009) considers how multiple sources of information can lead to information acquisition complementarity and multiple equilibria. The information acquisition complementarity is qualitatively similar to a positive information acquisition externality in our model, but information acquisition complementarity endogenously arise in equilibrium, while information acquisition externality as we defined is determined by the exogenous information acquisition cost structure. Benchmarking to an index is qualitatively similar to indexing. Breugem and Buss (2017) show that benchmarking to an index reduces price informativeness and increases return volatility. In contrast to our model, a change in benchmarking is exogenous in Breugem and Buss (2017). Consequently, their results are also qualitatively different from ours in many cases.

The remainder of the paper proceeds as follows. In Section 2, we present the model. In Section 3 we derive the equilibrium and provide some comparative statics on information precision and price informativeness. In Section 4, we conduct numerical analysis to illustrate the impact of indexing that rises from different causes. We conclude in Section 5. All proofs are provided in the Appendix.

2. The Model

The Asset Market We consider a one-period model where a continuum of investors can trade at time 0 one risk free and two risky assets to maximize their expected utility at time 1. The first risky asset m is the market portfolio ("index") and the second risky asset s is called a non-index portfolio. The risk-free asset and the non-index portfolio have a net supply of zero, and the market portfolio has a net supply of 1 share. We assume that the time 1 payoff V_m of the market portfolio is distributed as $N(\mu_{vm}, \tau_{vm}^{-1})$, the time 1 payoff V_s of the non-index portfolio is distributed as $N(0, \tau_{vs}^{-1})$, and for simplicity of exposition, we assume that V_m is independent of V_s, where μ_{vm}, τ_{vm}, and τ_{vs} are constants.[5] Some investors need to incur a participation cost in terms of utility loss (e.g., from time and attention consumption) to trade the non-index portfolio, but there is no such cost for trading the market portfolio.

Types of Investors A mass λ_A of investors are exogenous active investors ("A" investors) who have zero cost of trading the non-index portfolio and thus always trade in both risky assets. They also acquire private information about both assets. These investors can represent funds that have relatively low cost in picking stocks.

[5]It can be easily shown that this is equivalent to a setting where investors can trade two stocks S_1 and S_2 with the same expected payoff and a net supply of one share each, because trading S_1 and S_2 is equivalent to trading the market portfolio $\frac{S_1+S_2}{2}$ and the spread portfolio $\frac{S_1-S_2}{2}$ ("nonindex"), and the net supply of the market portfolio is 1 and the net supply of the non-index portfolio is zero.

A mass λ_D of investors are discretionary traders ("D" investors) who can choose to incur a fixed participation cost k to participate in the non-index market. In equilibrium, an endogenously solved mass η^* of these discretionary traders choose to become indexers ("DI" investors) and a mass of $\lambda_{DA} \equiv \lambda_D - \eta^*$ become active investors ("DA" investors) who trade both risky assets. "D" investors can represent funds who have relatively high cost of picking stocks and can switch to passive investing from active management.

A mass λ_I investors are exogenous indexers ("I" investors) who represent most index funds that do not trade the non-index portfolio and do not acquire any private information. A change in the mass λ_I of exogenous indexers can be used to examine the impact of an exogenous change of indexing due to a large shock to some investors' participation cost and information acquisition cost.

A mass λ_L of investors are liquidity traders ("L" investors) whose trading in both markets are exogenous. For $j \in \{m, s\}$, each liquidity trader i has a random endowment $e_{ji} = Z_j + u_{ji}$ shares of a nontraded asset j (e.g., two streams of labor income), with each share of the nontraded asset j paying the same amount at time 1 as V_j, where Z_j and u_{ji} are independently distributed as $N(0, \tau_{zj}^{-1})$ and $N(0, \tau_{uj}^{-1})$ respectively, and τ_{zj} and τ_{uj} are all constants.[6] Z_j (resp. u_{ji}) can be viewed as an aggregate (resp. idiosyncratic) shock in the endowment. Liquidity traders have mean-variance preferences over the final wealth and infinite risk aversion.[7] As a result, they must perfectly hedge the endowment risk by selling the same number of shares in markets m and s as their endowment of the nontraded assets at the market prices and have no incentive to acquire any information. As in Grossman and Stiglitz (1980), the presence of liquidity traders is necessary for the existence of an equilibrium in our model. The liquidity traders are similar to noise traders commonly assumed in the literature. The main difference is that we can measure liquidity traders' utility by the expected terminal wealth which depends on market prices. Given this setup, we can shed some light on how indexing affects liquidity traders' welfare. Because the total mass of investors is 1, we have $\lambda_A + \lambda_D + \lambda_I + \lambda_L = 1$.

Every investor has an initial endowment of 1 share of the market portfolio, but has no risk-free asset or the non-index portfolio. All non-liquidity-traders have constant absolute risk averse (CARA) preferences with a risk aversion coefficient of $\gamma > 0$.

In actual financial markets, hedge funds can be considered as "A investors" who do not passively invest only in index funds. Mutual funds can be considered as "D investors" in our model. Some of them may choose to become active mutual funds ("DA investors") and others choose to be passive mutual funds ("DI investors").

[6]It is sufficient to assume that the payoff of the nontraded asset j is perfectly correlated with V_j so that hedging motive is present. This is equivalent to assuming that liquidity traders are noise traders who have exogenous trading demand.

[7]With finite risk aversion, the derivation is more complicated, but the qualitative results are the same.

Most index fund investors are considered as "I investors" who do not select stocks and do not care about acquiring private information about assets. "L investors" can be considered as individual traders or hedgers who trade assets for hedging or rebalancing needs instead of fundamental value of assets.

Information Acquisition For $t \in \{A, D\}$, $j \in \{m, s\}$), each investor i of types A and D can observe independent private signals Y_{tji} at time 0 about the payoff of the risky asset j, where

$$Y_{tji} = V_j + \varepsilon_{tji}, \quad j \in \{m, s\},$$

and all ε_{tji} are independently distributed as $N(0, \tau_{tj}^{-1})$. The cost of acquiring private information with precisions τ_{tm} and τ_{ts} is $C_t(\tau_{tm}, \tau_{ts})$ for $t \in \{A, D\}$. We only consider symmetric equilibria where investors of the same type make the same trading and information acquisition decision. As a result, the precision choices are the same across investors of the same type, and thus we omit the i index in the precision variables.

Investors' Problems Let P_m and P_s be the time 0 equilibrium prices of the market portfolio and the non-index portfolio respectively, I_{ti} be time 0 information set of investor i of type t, and Θ_{tji} be the number of shares of the j portfolio bought by investor i of type t at time 0, for $j \in \{m, s\}$ and $t \in \{A, D, I, L\}$. At time 0, for $t \in \{A, D\}$ investor i of type t chooses $(\Theta_{tmi}, \Theta_{tsi})$ to solve

$$\max E[-e^{-\gamma(\tilde{W}_{ti} - k1_{\{t=DA\}})}) | I_{ti}] \tag{1}$$

subject to the budget constraint

$$\tilde{W}_{ti} = V_m + \Theta_{tmi}(V_m - P_m) + \Theta_{tsi} 1_{\{t \neq DI\}}(V_s - P_s) - C_t(\tau_{tm}, \tau_{ts}). \tag{2}$$

At time 0, investor i of type $I-$traders who only invest in the index chooses Θ_{Imi} to solve

$$\max E[-e^{-\gamma \tilde{W}_{Ii}} | I_{Ii}] \tag{3}$$

subject to the budget constraint

$$\tilde{W}_{Ii} = V_m + \Theta_{Imi}(V_m - P_m). \tag{4}$$

For liquidity traders, $\Theta_{Lmi} = -(1 + Z_m + u_{mi})$ and $\Theta_{Lsi} = -(Z_s + u_{si})$.

Market-Clearing Condition The time 0 equilibrium is $\{P_j, \Theta_{tji}, j \in \{m, s\}, t \in \{A, D, I, L\}\}$ such that Θ_{tji} solves the above problems for investor i of type $t \in$

$\{A, D, I, L\}$ and the following market clearing condition is satisfied:

$$\sum_{t\in\{A,D,I,L\}} \int_i \Theta_{tji} di = 0, \quad j \in \{m, s\}. \tag{5}$$

Choice of Indexing and Precision Just before time 0, type D investors choose whether to become indexers. In equilibrium, either all D investors strictly prefer indexing (i.e., $\eta^* = \lambda_D$) or all D investors strictly prefer to be active (i.e., $\eta^* = 0$) or each D investor is indifferent between indexing and being active. Traders A and D then choose the optimal precisions of their private information given the indexing decision and information acquisition choices of other investors.

3. The Equilibrium

We first solve the equilibrium at time 0 given investors' information and participation choice. We conjecture and later verify that

$$P_m = a_m + b_m V_m - d_m Z_m, \quad P_s = b_s V_s - d_s Z_s, \tag{6}$$

where a_m, b_m, d_m, b_s, and d_s are constants to be determined. For $j \in \{m, s\}$, let

$$\tau_j := \lambda_A \tau_{Aj} + (\lambda_D - \eta^*)\tau_{DAj} + \eta^* \tau_{DIj} 1_{j=m}, \tag{7}$$

and

$$\rho_j = \frac{1}{Var[V_j|P_j]} = \tau_{vj} + \frac{\tau_j^2}{\gamma^2 \lambda_L^2} \tau_{zj} \tag{8}$$

denote the total precision of private information and the price informativeness in market j. For given values of τ_{vj} and τ_{zj}, price informativeness in market j increases with the total precision τ_j of private information in market j.

We focus on linear symmetric equilibrium where investors of the same type choose the same trading strategy and the same information precision.

Theorem 1 *Given the signal precisions τ_{tj}, $t \in \{A, DI, DA\}$, $j \in \{m, s\}$, there is* *a unique linear symmetric equilibrium, and the equilibrium price coefficients are*

$$a_m = \frac{(1-\lambda_L)\mu_{vm}\tau_{vm} - \gamma}{\tau_m + (1-\lambda_L)\rho_m}, \tag{9}$$

$$b_m = 1 - \frac{(1-\lambda_L)\tau_{vm}}{\tau_m + (1-\lambda_L)\rho_m}, \tag{10}$$

$$b_s = 1 - \frac{(\lambda_A + \lambda_{DA})\tau_{vs}}{\tau_s + (\lambda_A + \lambda_{DA})\rho_s}, \tag{11}$$

$$d_m = \frac{\gamma\lambda_L b_m}{\tau_m}, \quad d_s = \frac{\gamma\lambda_L b_s}{\tau_s}. \tag{12}$$

Theorem 1 implies that the risk premium of the market portfolio m is equal to[8]

$$E[V_m - P_m] = \frac{\gamma}{\tau_m + (1-\lambda_L)\rho_m}. \tag{13}$$

For given values of τ_{vm} and τ_{zm}, equation (13) implies that market risk premium and price informativeness ρ_m^* move in the opposition direction. This is because as price informativeness increases, the aggregate uncertainty in the market reduces and vise versa.

Just before time 0, investors optimally choose the precisions of their private signals. We consider symmetric Nash equilibrium where investors of the same type choose the same precision and in equilibrium, each investor's information precision is optimal given other investors' choice.

Let C_{tm}, C_{ts}, C_{tmm}, and C_{tss} denote respectively the first and the second derivative of C_t with respect to τ_{tm} and τ_{ts}, and C_{tms} denote the cross derivative, for $t \in \{A, D\}$. To ensure the existence and the uniqueness of equilibrium, we make the following assumption:

Assumption 1 *$C_{tj} \geq 0$ for $j \in \{m, s\}$ with equality only at $\tau_{tm} = \tau_{ts} = 0$. $C_{tmm} > 0$, $C_{tss} > 0$, and $C_{tmm}C_{tss} - C_{tms}^2 \geq 0$ for $t \in \{A, D\}$.*

The above assumption ensures the convexity of the cost function. The information acquisition externality (IAE) for type t ($t = A, D$) investors can be measured by

$$\varphi_t \equiv -C_{tms} = -\frac{\partial^2 C_t(\tau_{tm}, \tau_{ts})}{\partial\tau_{tm}\partial\tau_{ts}}. \tag{14}$$

A negative φ_t means that acquiring more information in one market increases the cost of information acquisition in another market. In this sense, there is a negative

[8]The risk premium of the non-index portfolio is zero because the aggregate supply of the non-index portfolio is zero and thus there is no aggregate risk.

IAE across the two markets. The negative IAE can represent qualitatively the case where an investor has a fixed total information acquisition capacity, which implies that acquiring more information in one market lowers the capacity of acquiring information in another market, and as a result, information acquisition in the other market is reduced. A positive φ_t means that information acquisition in one market reduces the cost of information acquisition in another market. In this sense, there is a positive IAE across the two markets. The positive IAE can represent qualitatively the case where the experience of acquiring information in one market helps make information acquisition in another market more efficient and thus lowers the cost of information acquisition in the other market.[9]

Given the optimal choice of precisions of signals and trading strategies, D investors optimally choose whether to incur the participation cost k to trade the non-index risky asset.

Theorem 2 *Under Assumption 1, there exists a unique linear symmetric equilibrium where the equilibrium information precisions solve the following five equations:*

$$2\gamma C_{tj}(\tau_{tm}, \tau_{ts}) = \frac{1}{\tau_{tj} + \rho_j}, \quad t \in \{A, DA\}, j \in \{m, s\}, \tag{15}$$

and

$$2\gamma C_{Dm}(\tau_{DIm}, 0) = \frac{1}{\tau_{DIm} + \rho_m}, \tag{16}$$

and the equilibrium mass η^* of DA investors is such that either all D investors strictly prefer to be DA investors or all D investors strictly prefer to be DI investors or all D investors are indifferent between being DA or DI investors.

Equation (15) is the first order conditions of the A and DA investors for the choice of precisions in the index and the non-index markets. Equation (16) is the first order condition of the DI investors for the choice of precision in the index market.

First, we examine how changes in the participation cost k, in non-index market transparency τ_{zs}, and in the exogenous indexing λ_I affect endogenous fraction of indexers η^*.

Proposition 1 *1. As the participation cost k increases, endogenous indexing η^* increases.*

[9] A positive IAE is also qualitatively consistent with the situation where acquiring information in one market helps reveal information about another market, and thus paying a lower cost in the other market can give an investor the same precision information in total, for example, due to correlations among the different assets. The information acquisition complementarity as termed by the existing literature is qualitatively similar to a positive IAE in our model, but information acquisition complementarity endogenously arises in equilibrium, while IAE as we defined is determined by the exogenous information acquisition cost structure.

2. *As the non-index market transparency τ_{zs} increases, endogenous indexing η^* increases.*

3. *As the exogenous indexing λ_I increases, keeping $\lambda_D + \lambda_I$ constant, endogenous indexing η^* decreases if $\varphi_D = 0$.*

As expected, when it becomes more costly to trade in the non-index market, more discretionary investors optimally choose to be indexers. When the non-index market becomes more transparent, trading in the non-index becomes less profitable, and thus more discretionary traders choose to be indexers. When exogenous indexing increases, the equilibrium endogenous indexers decreases if there is no information acquisition externality.[10]

One of the main questions we want to answer is how the rise of index investing affects price informativeness of both index and non-index markets. The following proposition implies that a rise in indexing due to either an increase of participation cost or an increase in exogenous indexing tends to *decrease* the price informativeness of the non-index. We will provide more detailed analysis and intuitions about this result in next section.

In addition, if there is no information acquisition externality, then the rise of endogenous indexing does not have any impact on the price informativeness of the index. In next section, we will show that how the rise of endogenous indexing affects the price informativeness of the index as well as the market risk premium critically depends on the sign of information acquisition externality φ_D while the rise of indexing due to increases in the exogenous indexing always decreases the price informativeness of both the index and the non-index.

Proposition 2 *Suppose there is no information acquisition externality. Then*

1. *holding λ_A, λ_D, γ, λ_L, τ_{zm}, and τ_{zs} constant, as the equilibrium mass of discretionary indexers increases, the price informativeness ρ_m in the index market does not change, i.e., $\frac{\partial \rho_m}{\partial \eta^*} = 0$.*

2. *holding λ_A, λ_D, γ, λ_L, τ_{zm}, and τ_{zs} constant, as the equilibrium mass of discretionary indexers increases, the price informativeness ρ_s in the non-index market decreases.*

One common concern on the rise of index investing is that indexers may free ride on active traders for information acquisition. The following proposition shows that while this is true in some cases, discretionary indexers may sometimes choose to

[10]It can be shown that as the index market transparency τ_{zm} increases, endogenous indexing η^* increases if and only if $\varphi_D > 0$.

acquire more precise information about the index than active investors and thus let others "free ride" on them.[11]

Proposition 3

$$Sign(\tau_{DAm} - \tau_{DIm}) = Sign\left(\varphi_D\right), \tag{17}$$

where

$$Sign(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0. \end{cases}$$

Proposition 3 shows that if the information acquisition externality is negative, then discretionary indexers acquire more precise information than discretionary active investors. In this case, discretionary active investors in some sense "free ride" on discretionary indexers in the index market.

The intuition is simple. When the information acquisition externality is negative (acquiring information in one market increases the cost of acquiring information in another market). Because discretionary active investors also acquire private information in the non-index market which increases their information acquisition cost in the index market, discretionary index investors' equilibrium precision of private information about the index is therefore higher than that of discretionary active investors.

4. Effects of the Rise of Index Investing

In this section we conduct numerical analysis of the equilibrium effects of the rise of indexing on price informativeness, market risk premium, and welfare. We focus on three possible causes of the rise of index investing:

1. Increases in the participation cost k for discretionary investors;
2. Increases in the market transparency τ_{zs} in the non-index market;
3. Increases in exogenous indexers.

We will show that the relationship between indexing and market risk premium, welfare, and price informativeness depends critically on the causes of the rise of indexing. In the subsequent analysis, we use the following default parameter values: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.5$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_I = 0.1$, $\lambda_L = 0.1$, $k = 0.01$, and $C_t(\tau_{tm}, \tau_{ts}) = c_{tm}\tau_{tm}^2 + c_{ts}\tau_{ts}^2 + c_{tms}\tau_{tm}\tau_{ts}$ with $c_{tm} = c_{ts} = 0.01$,

[11] Note that exogenous indexers in our model are always getting a free ride on other traders. Active investors do pay to acquire private information even though the precision can be lower than that of discretionary indexers in the index market.

for $t = A, DI, DA$. To illustrate the impact of the sign of information acquisition externality, we plot three cases in all the figures below: positive IAE (i.e., $\varphi_D > 0$ with parameter value $c_{tms} = -0.005$); negative IAE (i.e., $\varphi_D < 0$ with parameter value $c_{tms} = 0.005$); and zero IAE (i.e., $\varphi_D = 0$).

Note that we do not attempt to calibrate our model to match empirical data. Instead, these default parameter values are chosen such that with minimum variations in some parameter values, we can illustrate all the key qualitative results.

In addition, to help understand our key results, we focus on the case where $0 < \eta^* < \lambda_D$, i.e., there is an interior equilibrium where the utilities of DI and DA investors must equal.[12] For simplicity of exposition and to make our intuitions more clear, in this numerical section we use the same information cost parameter values for both A and D investors and thus the only difference between A and D investors lies in the participation cost. As a result, the optimal information precisions chosen by A investors and DA investors are always the same in both markets.[13]

4.1 Effects of Changes in Participation Cost

The participation cost k for trading the non-index portfolio may change over time.[14] The first possible reason for the rise of indexing is that it becomes more costly to trade in the non-index market. Indeed, Figure 1 shows that as the participation cost k increases, the equilibrium endogenous mass of indexers increases until all discretionary investors become indexers.

Figure 1 also implies that, as participation cost k increases, the speed of the increase in η^* decreases. This is because trading in the non-index portfolio becomes more profitable as the number investors who trade in the non-index market decreases and thus the competition among traders becomes less intensive in the non-index market. In addition, the equilibrium mass of endogenous indexers is smaller with positive IAE (i.e., $\varphi_D > 0$) than that with negative IAE (i.e., $\varphi_D < 0$) due to the addition benefit of lowering information acquisition cost in the index market from participating in the non-index market.

As the endogenous indexing increases due to a higher participation cost in non-index market, competition in the non-index market decreases, and the marginal benefit of acquiring more precise information in this market increases. As a result, the optimal precisions for both A and DA investors increase, as shown in the left panel of Figure 2.[15] However, as illustrated in the right panel of Figure 2, the price infor-

[12]The impact of k in the corner equilibrium case where $\eta^* = 0$ or $\eta^* = \lambda_D$ is obvious.

[13]We also conduct similar analysis when A and D investors have different information cost parameter values. The main qualitative results are the same.

[14]For example, as wealth level increases, the marginal benefit of leisure and thus the marginal cost of time spent in investing may increase.

[15]Note that the precision with zero IAE (green line) is lower than that with negative IAE. This is

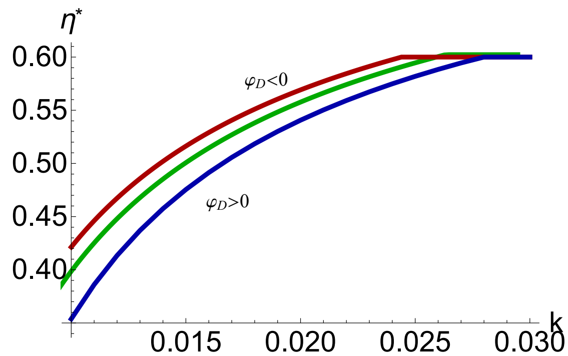

Figure 1: The equilibrium mass of endogenous indexers η^* against participation cost k. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.5$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_I = 0.1$, $\lambda_L = 0.1$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ ($\varphi_D > 0$) for the bottom curve, $c_{tms} = 0$ ($\varphi_D = 0$) for the middle curve, and $c_{tms} = 0.005$ ($\varphi_D < 0$) for the top curve, for $t = A, DI, DA$.

mativeness in the non-index market still decreases because fewer investors trade and acquire private information in the market.

We now examine how the rise of index investing caused by increases in the participation cost affects price informativeness of the index and market risk premium. As illustrated in Figure 3, how market risk premium (MRP) and price informativeness in the index market (ρ_m^*) change with the rise of indexing critically depends on the sign of IAE φ_D. In addition, as implied by equation (13), market risk premium changes in the opposite direction to that of price informativeness ρ_m^*. More specifically, we have the following results:

1. If there is no information acquisition externality (i.e., $\varphi_D = 0$), then neither MRP nor ρ_m^* changes;

2. If there is positive information acquisition externality (i.e., $\varphi_D > 0$), then MRP increases but ρ_m^* decreases;

3. If there is negative information acquisition externality (i.e., $\varphi_D < 0$), then MRP decreases but ρ_m^* increases.

When there is no information acquisition externality (i.e., $\varphi_D = 0$), trading in the non-index market does not affect the information acquisition cost in the index market, and thus the optimal precisions chosen by DI and DA investors are the same

because with negative IAE, there are less DA investors in the nonindex market than with zero IAE, and thus the marginal benefit of acquiring more information is greater than with zero IAE, which makes it optimal to acquire more precise information than with zero IAE.

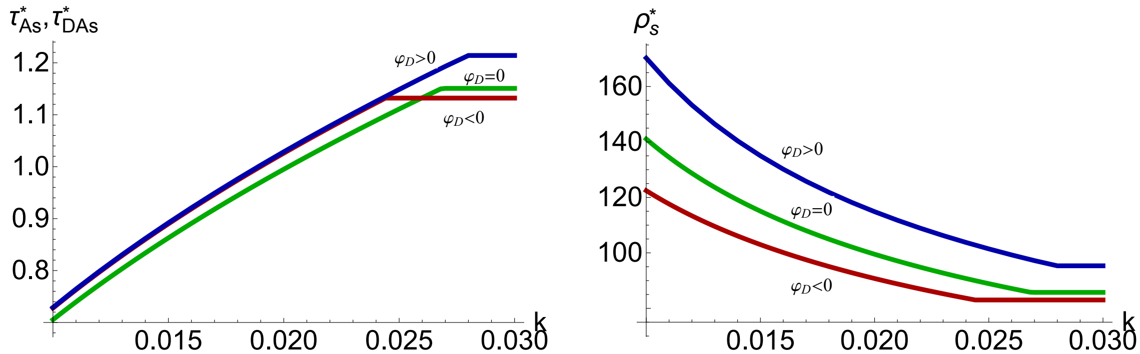

Figure 2: The price informativeness and equilibrium precisions in Market s against participation cost k. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.5$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_I = 0.1$, $\lambda_L = 0.1$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ for the blue line ($\varphi_D > 0$), $c_{tms} = 0$ for the green line ($\varphi_D = 0$), and $c_{tms} = 0.005$ for the red line ($\varphi_D < 0$), for $t = A, DI, DA$.

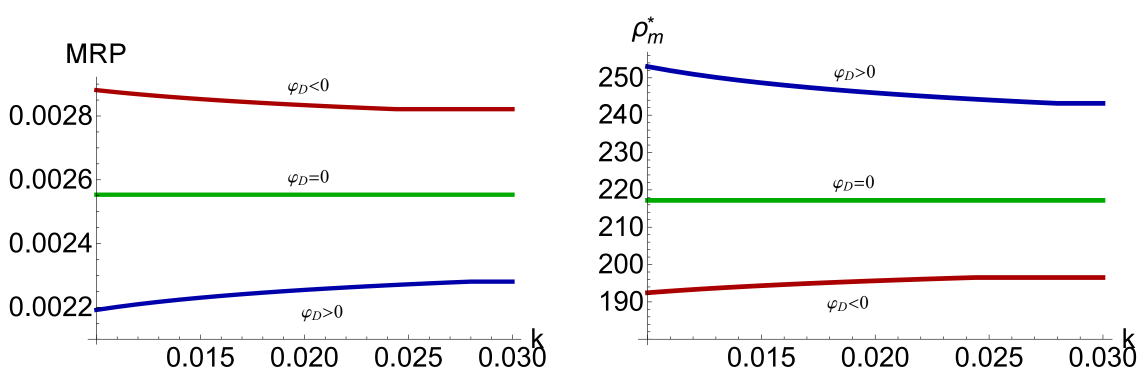

Figure 3: The market risk premium (MRP) and price informativeness of the index against participation cost k. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.5$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_I = 0.1$, $\lambda_L = 0.1$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ for the curves with $\varphi_D > 0$, $c_{tms} = 0$ for the curves with $\varphi_D = 0$, and $c_{tms} = 0.005$ for the curves with $\varphi_D < 0$, for $t = A, DI, DA$.

in the index market, as shown in the middle lines in Figure 4. Therefore, in this case, the total precision of private information

$$\tau_m := \lambda_A \tau_{Am} + (\lambda_D - \eta^*)\tau_{DAm} + \eta^* \tau_{DIm} = \lambda_A \tau_{Am} + \lambda_D \tau_{DAm}, \tag{18}$$

does not depend on η^*. As a result, for the case with $\varphi_D = 0$, both the price informativeness of the index and market risk premium do not depend on the participation k, as illustrated by the flat lines in Figure 3.

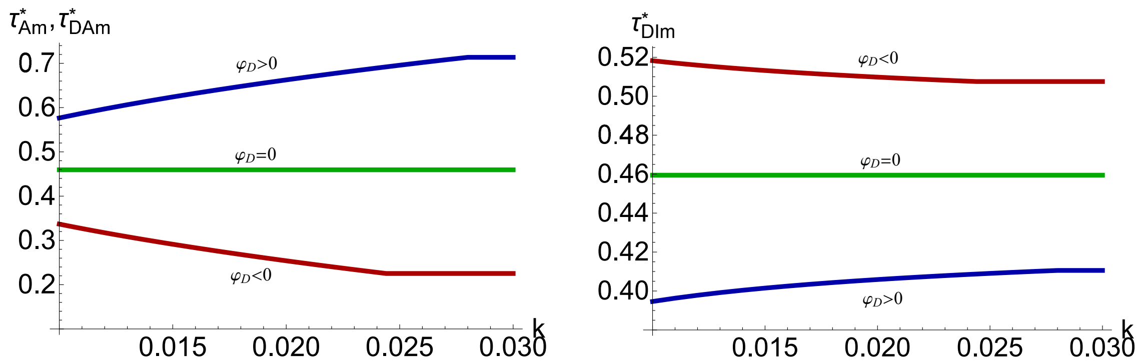

Figure 4: The equilibrium precisions of A and DA investors against participation cost k. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.5$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_I = 0.1$, $\lambda_L = 0.1$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ for the blue line ($\varphi_D > 0$), $c_{tms} = 0$ for the green line ($\varphi_D = 0$), and $c_{tms} = 0.005$ for the red line ($\varphi_D < 0$), for $t = A, DI, DA$.

When there is positive information acquisition externality (i.e., $\varphi_D > 0$), because both A investors and DA investors also acquire information in the non-index market, they have a lower cost of information acquisition in the index market, and thus obtain more precise information about the index than DI investors, as shown in the blue lines ($\varphi_D > 0$) in Figure 4. As their precisions of the private information about the non-index increase due to reduced competition, as shown in Figure 2, both A and DA investors increase their optimal precisions of the signal about the index. To reduce the information disadvantage against A and DA investors, DI investors also increase their optimal precision about the index.

Surprisingly, even though all informed participants in the index market (i.e., A, DA, and DI investors) increase their information precisions and the price informativeness ρ_m moves in the same direction as the total precision of the private information τ_m, the price informativeness of the index still decreases, as shown in the right panel of Figure 3. This is because as participation cost increases, the mass of endogenous indexer (η^*) increases and they optimally acquire less precise information than endogenous active traders in the case when $\varphi_D > 0$. Therefore, the aggregate precision of the private information in the index market τ_m decreases and so does the price informativeness ρ_m. When there is negative information acquisition externality (i.e., $\varphi_D < 0$), the opposite is true.

By contrast, Bond and Garcia (2017) find that market risk premium decreases and price informativeness in both markets increases with indexing. Their result is mainly driven by the assumption that indexers cannot have information-motivated trades before and after switching from active investors and the IAE is negative. Because indexers in their model only trade for liquidity reasons in the non-index market before becoming indexers, when they leave the non-index market, the price informativeness of the non-index increases. As a result, active investors can reduce information acquisition about the non-index, and thus lowering the cost of information acquisition about the index. Consequently, active investors increase their information acquisition in the index market, which leads to greater price informativeness in this market and a lower market risk premium.

Note that even in the case with negative IAE ($\varphi_D < 0$) as in their model, our results on information acquisition and price informativeness in the non-index market are the opposite to theirs: the active investors acquire less precise information about the index and more precise information about the non-index, and price informativeness in the non-index market decreases. In addition, as explained above, the mechanism that drives the result that market risk premium decreases and price informativeness in index market increases for the negative IAE case is different from that in their model. In our model, it is because the population weight on the less informed investors (i.e., DI investors) increases, while in their model, it is the increase of the precision of the informed investors. This shows that the key features that drive the difference between our results and theirs are endogenizing index investing and allowing discretionary indexers to acquire private information about the index.

One concern about the rapid growth of index investment is that indexers may free-ride on others for information acquisition, and thus reduce market information revelation. While this is always true for exogenous indexers and this is also true for discretionary indexers if the IAE is positive ($\varphi_D > 0$), Proposition 3 shows that active investors may sometimes free ride on discretionary indexers for information acquisition if there is negative IAE ($\varphi_D < 0$). Indeed, Figure 5 shows that when there is positive IAE, discretionary indexers acquire less precise information than both active traders and discretionary active traders. But when there is negative IAE, the opposite is true. In the case with negative IAE, as indexing increases due to higher participation cost, not only the information advantage of DI investors over A and DA investors increases, the fraction of investors with more precise information (i.e., the DI indexers) also increases, which further increases the free-riding extent of A and DA investors.

We next turn to the analysis of welfare. As shown in Figure 6, as indexing increases due to increased participation cost in the non-index market, the welfare of

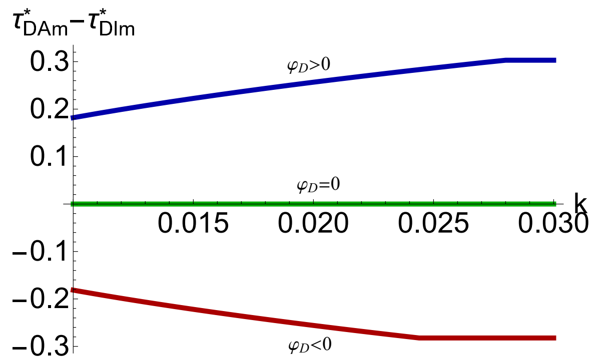

Figure 5: The free-ride measure $\tau^*_{DA} - \tau^*_{DI}$ against participation cost k. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.5$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_I = 0.1$, $\lambda_L = 0.1$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ for the blue line ($\varphi_D > 0$), $c_{tms} = 0$ for the green line ($\varphi_D = 0$), and $c_{tms} = 0.005$ for the red line ($\varphi_D < 0$), for $t = A, DI, DA$.

active investors increases, but that of liquidity traders decreases. The welfare of exogenous active investors increases because of less intensive competition in the non-index market. The welfare of liquidity investors decreases with more indexing because there are fewer traders in the non-index market who can share the risk with them.

Different from exogenous active traders, Figure 7 shows that, for discretionary investors, if there is no IAE ($\varphi_D = 0$), then their welfare does not change as long as some discretionary investors choose to be indexers.[16] This is because when there is no IAE, the precision choices of DA and DI investors in the index market are the same and thus as some DA investors become DI investors due to increased participation cost, neither the price informativeness nor individual investor's precision changes. As a result, the welfare of DI investors do not change. Because in an interior equilibrium the welfare of DA investors must be the same as that of DI investors, the welfare of DA investors does not change either.[17]

When there is negative IAE ($\varphi_D < 0$), DI investors acquire more precise information than DA investors in the index market. As more DA investors switch to be DI investors, price informativeness increases and profitability of the index decreases. As a result, the welfare of DI investors decreases, and so does that of DA investors. The

[16]When participation cost k is low enough, all D investors invest in both markets. Then if k increases by a small amount, it is still optimal for all D investors to invest in both markets and to adopt the same precision and trading strategy, and thus their utility decreases because they have to incur a higher cost.

[17]Indexing must increase enough so that the increase in the benefit from trading the non-index must exactly offset the increase in the participation cost. Otherwise, either some DA investors would become indexers (if benefit is less than cost) or some DI investors would prefer to trade in the non-index market.

opposite is true when there is positive IAE ($\varphi_D > 0$). In contrast to other investors, for exogenous indexers, their welfare does not change much with the rise in indexing because they can only trade in the index market and cannot acquire information, as shown in Figure 7.

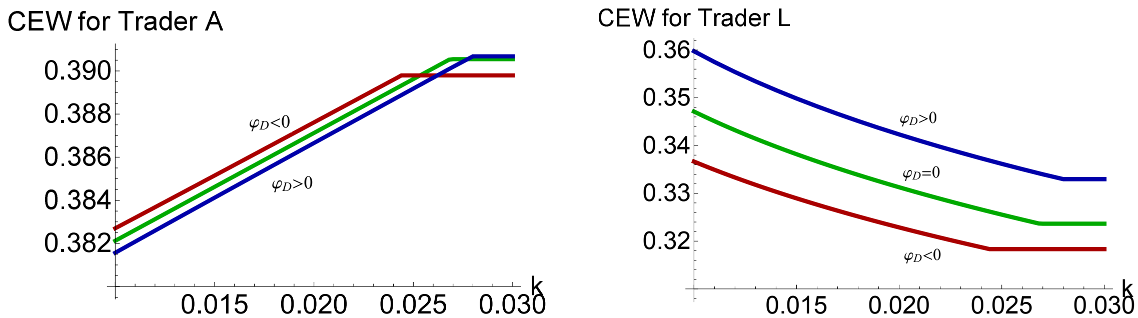

Figure 6: The certainty equivalent wealth of A and L investors against participation cost k. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.5$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_I = 0.1$, $\lambda_L = 0.1$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ for the blue line ($\varphi_D > 0$), $c_{tms} = 0$ for the green line ($\varphi_D = 0$), and $c_{tms} = 0.005$ for the red line ($\varphi_D < 0$), for $t = A, DI, DA$.

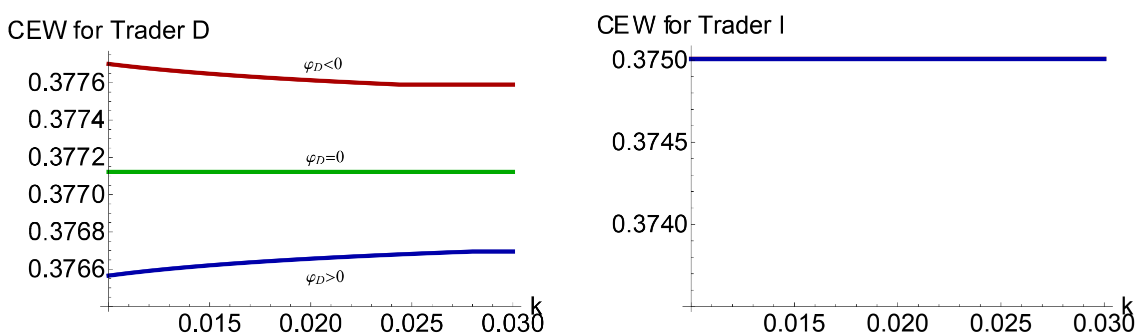

Figure 7: The certainty equivalent wealth of D and I investors against participation cost k. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.5$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_I = 0.1$, $\lambda_L = 0.1$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ for the blue line ($\varphi_D > 0$), $c_{tms} = 0$ for the green line ($\varphi_D = 0$), and $c_{tms} = 0.005$ for the red line ($\varphi_D < 0$), for $t = A, DI, DA$.

The left panel of Figure 8 shows that while some investors gain from the rise of indexing, overall the social welfare tends to decrease. However, the top curve in right panel of Figure 8 shows that social welfare might increase with an increase in indexing when the prior precision about the payoff of V_m is significantly higher. Because of the less uncertainty about the payoff, the adverse selection problem for the liquidity

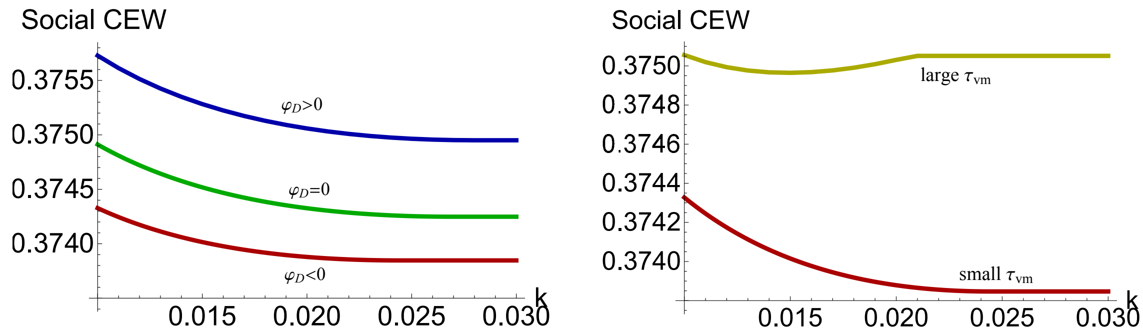

Figure 8: The social certainty equivalent wealth against participation cost k. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.5$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_I = 0.1$, $\lambda_L = 0.1$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ for the blue line ($\varphi_D > 0$), $c_{tms} = 0$ for the green line ($\varphi_D = 0$), $c_{tms} = 0.005$ for the red and yellow lines ($\varphi_D < 0$), and $\tau_{vm} = 20$ for the yellow line, for $t = A, DI, DA$.

traders becomes smaller. As a result, the welfare of liquidity traders decreases less as indexing increases, which may lead to a slight increase in the social welfare.

To summarize, if the rise of indexing is due to increased cost of participating in the non-index market, then the price informativeness of the non-index *decreases*. The price informativeness of the index tends to increase (resp. decrease) and thus market risk premium decreases (resp. increases) if acquiring information in one market increases (resp. decreases) information acquisition cost in another market. Overall the social welfare tends to be decreased while exogenous active traders are better off with more indexing.

4.2 Effects of Changes in Transparency of the Non-Index Market

We now look at another possible reason for the rise of index investment. By fixing the participation cost in the non-index market, if trading in the non-index market becomes less profitable, then we should expect that more discretionary traders optimally choose to be indexers. Intuitively, if the non-index market becomes more transparent (e.g., fewer noise traders), then it becomes less profitable for traders to acquire private information and trade in the non-index market.

A measure of the transparency of the non-index market in our model is the precision of the liquidity trades τ_{zs}. For example, as τ_{zs} increases, price in the non-index market becomes more informative as can be seen from equation (8).

Consistent with our intuition, as illustrated in Figure 9, the equilibrium mass of indexers indeed increases regardless of the sign of IAE as the non-index market

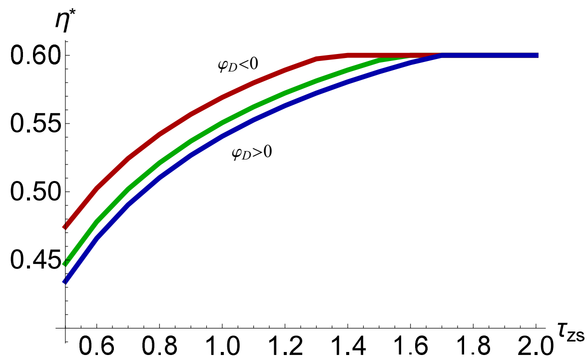

Figure 9: The equilibrium mass of endogenous indexers η^* against non-index market transparency τ_{zs}. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.5$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_I = 0.1$, $\lambda_L = 0.1$, $k = 0.02$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ for the blue line ($\varphi_D > 0$), $c_{tms} = 0$ for the green line ($\varphi_D = 0$), and $c_{tms} = 0.005$ for the red line ($\varphi_D < 0$), for $t = A, DI, DA$.

transparency τ_{zs} improves because the profitability from trading in the non-index market decreases.

In contrast to the case when the rise of index investment is due to increased participation cost, the right panel of Figure 10 shows that when the rise of index investment is due to increased transparency of the non-index market, price informativeness in the non-index market increases regardless of the sign of IAE. Even though active investors acquire less precise information about the non-index, the price informativeness in the non-index market still increases due to the increase in transparency τ_{zs}.

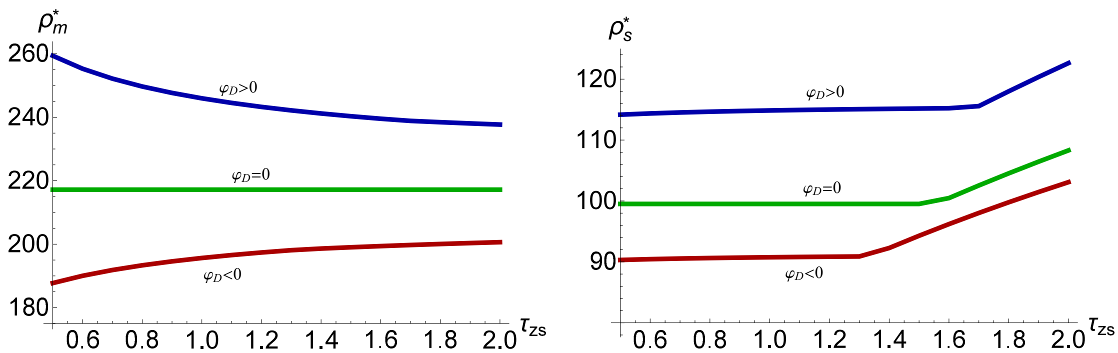

Figure 10: The price informativeness against the non-index market transparency τ_{zs}. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.5$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_I = 0.1$, $\lambda_L = 0.1$, $k = 0.02$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ for the blue line ($\varphi_D > 0$), $c_{tms} = 0$ for the green line ($\varphi_D = 0$), and $c_{tms} = 0.005$ for the red line ($\varphi_D < 0$), for $t = A, DI, DA$.

However, similar to the case when the rise of index investing is caused by increases in the participation cost, the left panel of Figure 10 illustrates that how market risk premium and price informativeness in the index market (ρ_m^*) change with the rise of indexing due to the increased transparency of non-index market also critically depends on the sign of IAE (i.e., the sign of φ_D). More specifically, if there is positive information acquisition externality (i.e., $\varphi_D > 0$), then market risk premium increases but ρ_m^* decreases. The opposite is true if there is a negative information acquisition externality (i.e., $\varphi_D < 0$). The intuition is as follows. When there is a negative information acquisition externality (i.e., $\varphi_D < 0$), with an increase in the transparency of the non-index market, both exogenous active traders and discretionary active traders optimally acquire less precise information in the non-index market, which lowers the cost of acquiring information and increases the benefit of trading in the index. Therefore, both active traders and discretionary traders acquire more precise information about the index which leads to a higher price informativeness of the index.

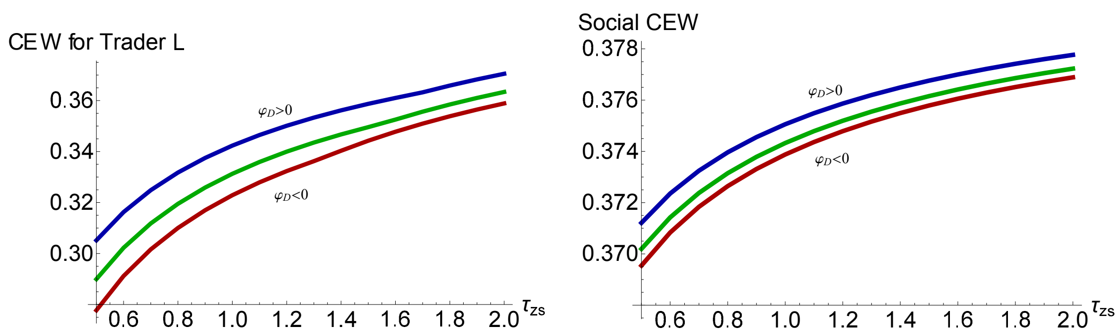

Figure 11: The certainty equivalent wealth of L and all investors against the non-index market transparency τ_{zs}. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.5$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_I = 0.1$, $\lambda_L = 0.1$, $k = 0.02$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ for the blue line ($\varphi_D > 0$), $c_{tms} = 0$ for the green line ($\varphi_D = 0$), and $c_{tms} = 0.005$ for the red line ($\varphi_D < 0$), for $t = A, DI, DA$.

Different from the case when the rise of indexing is due to increased participation cost in the non-index market, Figure 11 shows that both liquidity traders' welfare and the social welfare increase with increased transparency of the non-index market. The welfare of exogenous active traders and discretionary investors may also increase if there is positive IAE, as illustrated in Figure 12. Intuitively, the combination of positive IAE and less competition in the non-index market may overcome the low profitability in the non-index market by the improvement in the non-index market transparency.

To summarize, if more active traders switch to indexers because of lower profitability due to improved transparency of the non-index market, then the price informativeness of the non-index *increases*. The price informativeness of the index tends

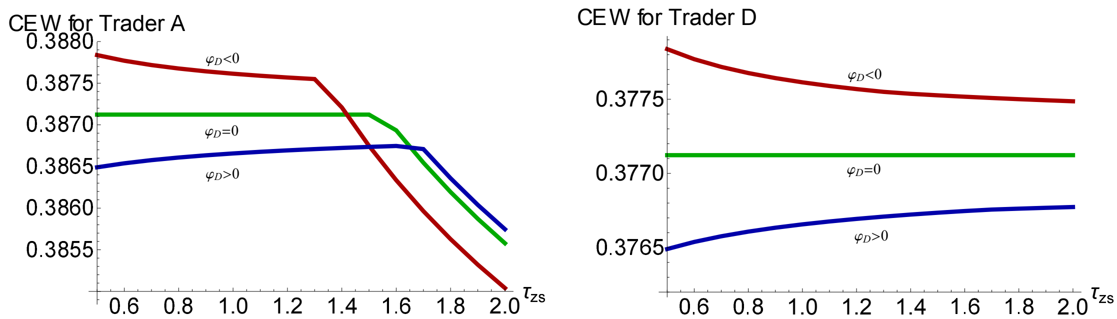

Figure 12: The certainty equivalent wealth of A and D investors against the non-index market transparency τ_{zs}. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.5$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_I = 0.1$, $\lambda_L = 0.1$, $k = 0.02$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ for the blue line ($\varphi_D > 0$), $c_{tms} = 0$ for the green line ($\varphi_D = 0$), and $c_{tms} = 0.005$ for the red line ($\varphi_D < 0$), for $t = A, DI, DA$.

to increase (resp. decrease) and thus market risk premium decreases (resp. increases) if acquiring information in one market increases (resp. decreases) information acquisition cost in another market. In addition, the social welfare tends to increase when more indexing is due to increased transparency of the non-index market.

4.3 Effects of Changes in Exogenous Indexers

In this subsection, we examine the third possible reason of the rise of indexing—increases in exogenous indexers. In practice, many passive index funds barely acquire private information about either index or non-index. Instead, they might spend great effort in retaining their existing clients and attracting more fund inflow. It is possible that some discretionary investors might simply switch to exogenous indexers and stop acquiring any private information.[18] As this happens, the mass of exogenous indexers λ_I increases.

Does the total fraction of both endogenous and exogenous indexers increase? The left panel of Figure 13 shows that when the exogenous indexers λ_I increases, the equilibrium mass of endogenous indexers η^* decreases, which suggests the presence of a “crowding out” effect on indexing. In addition, the crowding out effect can over- or under-offset the increase in exogenous indexers λ_I. This crowding-out effect arises because as some discretionary investors become indexers, the competition in the non-index market decreases, and thus more of other discretionary investors choose to remain in the non-index market. When there is positive IAE, because of the extra

[18]For example, some discretionary investors may experience a large shock to their participation cost and information acquisition cost so that it is optimal for them to become indexers and do not acquire any private information.

information acquisition benefit, the crowding-out effect becomes stronger and can overcome the increase of the mass of exogenous indexers λ_I. As a result, the total mass of indexers may decrease as the mass of exogenous indexers λ_I increases, as confirmed in the right panel of Figure 13. When there is negative IAE, the total mass of indexers increases as the exogenous indexers increases. When almost all discretionary investors have large shocks and become indexers, there are not enough remaining discretionary investors to offset the increase, and therefore the total mass of indexers increases irrespective of the sign of IAE.

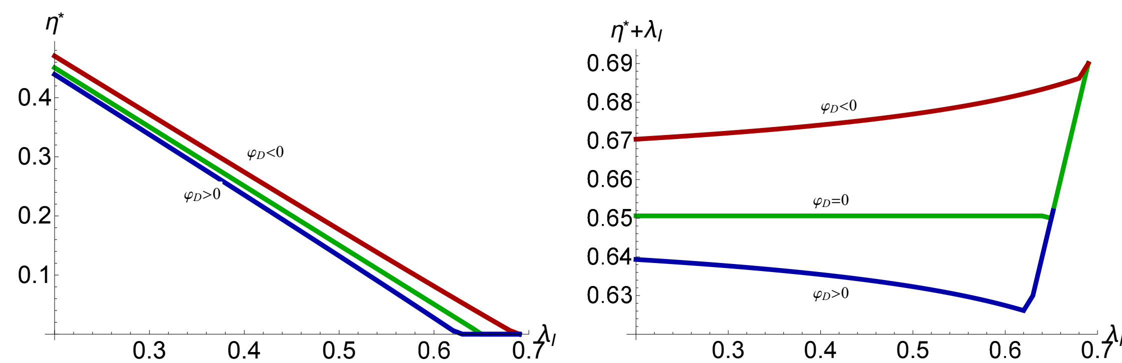

Figure 13: The equilibrium fraction of endogenous indexers η^* and total fraction of indexers $\eta^* + \lambda_I$ against the fraction of exogenous indexers. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.5$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_L = 0.1$, $k = 0.02$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ for the blue line ($\varphi_D > 0$), $c_{tms} = 0$ for the green line ($\varphi_D = 0$), and $c_{tms} = 0.005$ for the red line ($\varphi_D < 0$), for $t = A, DI, DA$.

As we explained, although the total mass of indexers may decrease, the mass of endogenous indexers η^* decreases irrespective of the sign of IAE. As a result, the price informativeness of the index decreases, which leads to greater market premium, as shown in Figure 14. In contrast, the price informativeness of the non-index can be non-monotonic if there is positive IAE. As shown in Figure 15, the precisions of all informed traders in the non-index market increase. The non-monotonicity of the price informativeness of the non-index then follows from the non-monotonicity of the total mass of traders who acquire information in the non-index market, as shown in the blue line (i.e., $\varphi_D > 0$) in Figure 13.

If the total fraction of indexers rises due to increases in exogenous indexers (e.g., the case when $\varphi_D < 0$ as in Figure 13), then the price informativeness of the non-index also decreases, as shown in Figure 15. In addition, as clearly illustrated in the left panel of Figure 15, when there is a large increase in the exogenous indexers, the price informativeness of the non-index also decreases irrespective of the sign of IAE.

Figures 16 and 17 show that, similar to the case when indexing rises because of

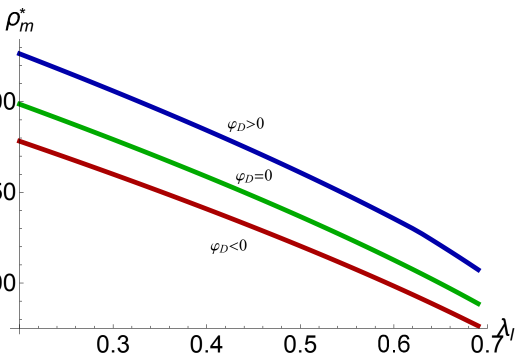

Figure 14: The market risk premium and price informativeness of the index against the fraction of exogenous indexers. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.5$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_L = 0.1$, $k = 0.02$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ for the blue line ($\varphi_D > 0$), $c_{tms} = 0$ for the green line ($\varphi_D = 0$), and $c_{tms} = 0.005$ for the red line ($\varphi_D < 0$), for $t = A, DI, DA$.

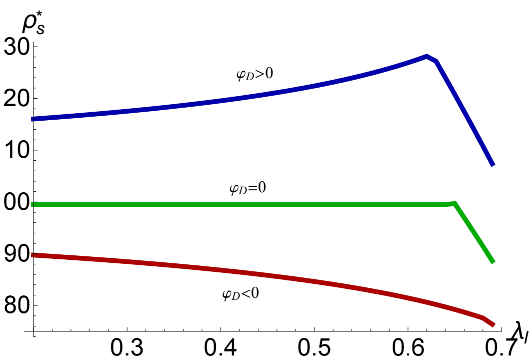

Figure 15: The price informativeness and equilibrium precisions in the non-index market against the fraction of exogenous indexers. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.5$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_L = 0.1$, $k = 0.02$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ for the blue line ($\varphi_D > 0$), $c_{tms} = 0$ for the green line ($\varphi_D = 0$), and $c_{tms} = 0.005$ for the red line ($\varphi_D < 0$), for $t = A, DI, DA$.

increased participation cost in the non-index market, the welfare of exogenous active traders and discretionary investors increases due to decreases in the competition, while the welfare of liquidity investors decreases because of reduced risk-sharing in the non-index market. In general, we can see from the right panel of Figure 17, an increase in the exogenous indexers tends to decrease the social welfare.[19]

To summarize, when the rise of total fraction of indexers is due to increases in

[19]The welfare of the I investors stays almost a constant as in the case of participation cost increase (not shown here to save space).

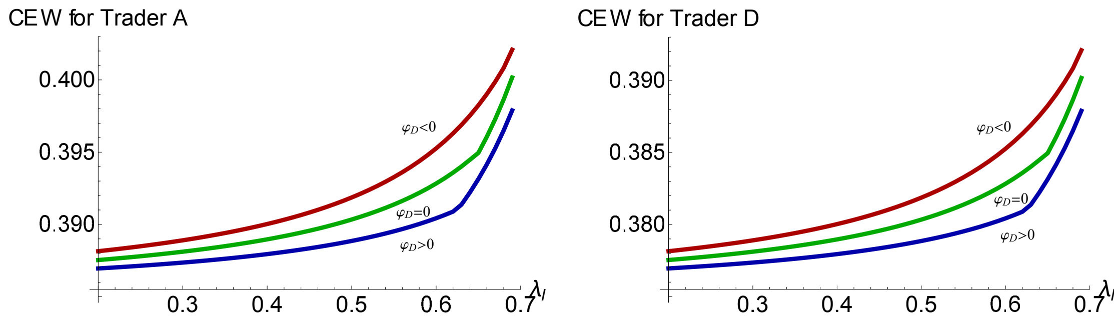

Figure 16: The certainty equivalent wealth of A and D investors against the fraction of exogenous indexers. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.5$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_L = 0.1$, $k = 0.02$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ for the blue line ($\varphi_D > 0$), $c_{tms} = 0$ for the green line ($\varphi_D = 0$), and $c_{tms} = 0.005$ for the red line ($\varphi_D < 0$), for $t = A, DI, DA$.

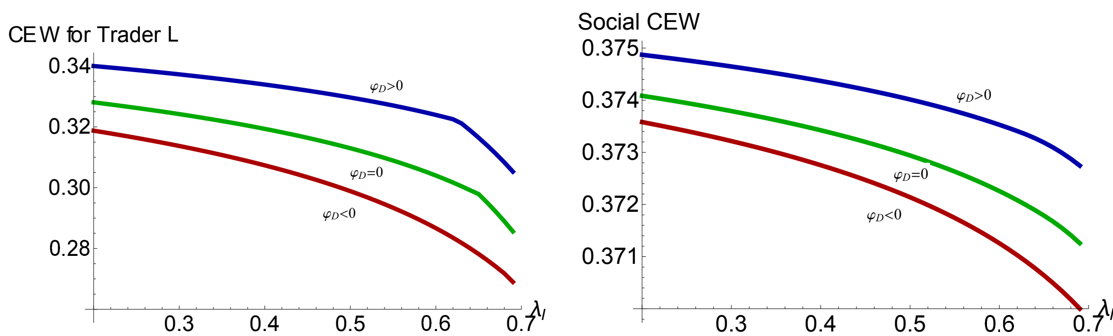

Figure 17: The certainty equivalent wealth of L investors and all traders against the fraction of exogenous indexers. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.5$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_L = 0.1$, $k = 0.02$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ for the blue line ($\varphi_D > 0$), $c_{tms} = 0$ for the green line ($\varphi_D = 0$), and $c_{tms} = 0.005$ for the red line ($\varphi_D < 0$), for $t = A, DI, DA$.

exogenous indexers, the price informativeness in both index and non-index markets as well as the social welfare tend to be reduced while both exogenous active traders and discretionary traders are better off.[20]

[20]This implies that, even though it is possible that some discretionary investors might switch to exogenous indexers, it is unlikely that most of them would switch since they are better off being discretionary and acquiring information. In some sense, if the market becomes less efficient as more investors shift to index funds, it will increase the likelihood that some investors will shift to active investing or acquire private information to take advantage of the inefficiency.

5. Conclusion

We study the equilibrium effects of the rise of indexing on price discovery. We show that these effects critically depend on the causes of the rise of indexing and the cost structure of information acquisition.

If the rise of indexing is due to increased cost of participating in the non-index market, then the price informativeness of the non-index *decreases* and the social welfare tends to decrease. In contrast, if more active traders switch to indexers due to lower profitability of the non-index, then the price informativeness of the non-index *increases* and the social welfare tends to increase. In both cases, the price informativeness of the index tends to increase (resp. decrease) and thus market risk premium decreases (resp. increases) if acquiring information in one market increases (resp. decreases) information acquisition cost in another market. In addition, a rise of exogenous indexers reduces price informativeness in both index and non-index markets and makes both exogenous active traders and discretionary traders better off.

In addition to exogenous indexers, we also consider discretionary indexers in the model. For these discretionary indexers, indexing decisions are endogenous. They can acquire information before and after they become indexers. As a result, active investors may sometimes free ride on discretionary indexers for information acquisition.

Our analysis highlights that the rise of indexing might have opposite effects if the mechanisms that drive the rise of indexing are different. These predictions can help researchers use empirically found relationship between the rise of indexing and price informativeness to identify what likely caused the increase in indexing and thus offer regulators some guidance on regulations adjustment if necessary.

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Appendix A

In this Appendix, we provide proofs of analytical results.

Proof of Theorem 1:

For $t = A, DA, DI, I$ and $j = m, s$, the optimal number of shares of security j bought by investor i of type t is

$$\Theta_{tji} = \frac{E[V_j|I_i] - P_j}{\gamma Var[V_j|I_i]} - s_j, \tag{A-1}$$

where $s_m = 1$ and $s_s = 0$. The information set of investor i of type t is $I_i = (Y_{tmi}, Y_{tsi}, P_m, P_s)$, where the precision of Y_{tji} ($j = m, s$) is zero for $t = I, L$, and the precision of Y_{tsi} is zero $t = DI$. Because Y_{tmi} and Y_{tsi} are independent, and V_m and V_s are independent, the conditional expectation of V_j only depends on (Y_{tji}, P_j). Direct computation yields that for $t = A, D, I$ and $j = m, s$,

$$E[V_j|I_i] = \frac{d_j^2(Y_{tji}\tau_{tji} + \mu_{vj}\tau_{vj}) + b_j(-a_j + P_j)\tau_{zj}}{d_j^2(\tau_{tji} + \tau_{vj}) + b_j^2\tau_{zj}} \tag{A-2}$$

and

$$Var[V_j|I_i] = \left(\tau_{tji} + \tau_{vj} + \frac{b_j^2}{d_j^2}\tau_{zj}\right)^{-1}, \tag{A-3}$$

where $a_s = 0$ and $\mu_{vs} = 0$. Using the market clearing conditions

$$\sum_{t\in\{A,DI,DA,I\}} \int_i \Theta_{tji} di - \lambda_L Z_j = 0, \tag{A-4}$$

where the integration is over all investors of the same type, Setting the coefficients of V_j and Z_j and the constant term to be zero, we get the results in Theorem 1.

Proof of Theorem 2:

Define $r_j = \frac{\tau_j}{\gamma\lambda_L}$ for $j = m, s$. First, it is straightforward to show that type A and type DA investors choose the precisions (τ_{tmi}, τ_{tsi}) to maximize

$$-\gamma C_t(\tau_{tmi}, \tau_{tsi}) + \frac{1}{2}\log(\tau_{tmi} + \tau_{vm} + r_m^2\tau_{zm}) + \frac{1}{2}\log(\tau_{tsi} + \tau_{vs} + r_s^2\tau_{zs}), \quad t \in \{A, DA\}, \tag{A-5}$$

while type DI investors choose the precision τ_{tmi} to maximize

$$-\gamma C_t(\tau_{tmi}, 0) + \frac{1}{2}\log(\tau_{tmi} + \tau_{vm} + r_m^2 \tau_{zm}), \quad t = DI. \tag{A-6}$$

Under Assumption 1, it can be easily verified that the objective functions are all globally strictly concave in the choice precision variables, and therefore given r_m and r_s, there are unique solutions. Since investors of the same type choose the same precisions, we omit the index i. Define the optimal precision functions as

$$\tau_{tj}^* = f_j^t(r_m, r_s), j \in \{m, s\}, t \in \{A, DA, DI\}, \tag{A-7}$$

with $f_s^{DI}(r_m, r_s) = 0$. Taking derivatives with respect to r_m in the first order conditions (15)-(16), Assumption 1 then implies that for $t = A, DA$,

$$\frac{\partial f_m^t(r_m, r_s)}{\partial r_m} < 0, \quad Sign\left(\frac{\partial f_m^t(r_m, r_s)}{\partial r_s}\right) = -Sign\left(\varphi_D\right) \tag{A-8}$$

and

$$\frac{\partial f_m^{DI}(r_m, r_s)}{\partial r_m} < 0, \quad Sign\left(\frac{\partial f_m^{DI}(r_m, r_s)}{\partial r_s}\right) = 0 \tag{A-9}$$

By a similar argument, we have for $t = A, DA$,

$$\frac{\partial f_s^t(r_m, r_s)}{\partial r_s} < 0, \quad Sign\left(\frac{\partial f_s^t(r_m, r_s)}{\partial r_m}\right) = -Sign\left(\varphi_D\right). \tag{A-10}$$

Note that in equilibrium

$$r_m = \frac{\lambda_A \tau_{Am} + (\lambda_D - \eta^*)\tau_{DAm} + \eta^* \tau_{DIm}}{\gamma \lambda_L} \tag{A-11}$$

and

$$r_s = \frac{\lambda_A \tau_{As} + (\lambda_D - \eta^*)\tau_{DAs}}{\gamma \lambda_L}. \tag{A-12}$$

Therefore, we must show that there is a unique solution (r_m^*, r_s^*) to the equations

$$f_m(r_m, r_s) \equiv \lambda_A f_m^A(r_m, r_s) + (\lambda_D - \eta^*) f_m^{DA}(r_m, r_s) + \eta^* f_m^{DI}(r_m, r_s) - \gamma \lambda_L r_m = 0 \tag{A-13}$$

and

$$f_s(r_m, r_s) \equiv \lambda_A f_s^A(r_m, r_s) + (\lambda_D - \eta^*) f_s^{DA}(r_m, r_s) - \gamma \lambda_L r_s = 0. \tag{A-14}$$

It is clear that for any given r_s, $f_m(0, r_s) > 0$ and $f_m(\infty, r_s) < 0$ because as optimal precisions $f^A_{im}(0, r_s) \geq 0$, $f^{DA}_{im}(0, r_s) \geq 0$, $f^{DI}_{im}(0, r_s) > 0$, and $f^t_m(\infty, r_s) = 0$ for $t \in \{A, DA, DI\}$ as implied by the first order conditions. In addition, we have $\frac{\partial f_m(r_m, r_s)}{\partial r_m} > 0$ by (A-8) for any given r_s. Therefore, for any given r_s, there is a unique positive solution $r_m = g(r_s)$ such that equation (A-13) holds. In addition, by implicit function theorem,

$$g'(r_s) = -\frac{\frac{\partial f_m(r_m, r_s)}{\partial r_s}}{\frac{\partial f_m(r_m, r_s)}{\partial r_m}}. \tag{A-15}$$

Plugging $r_m = g(r_s)$ into the second equation (A-14), we have $f_s(g(r_s), r_s) = 0$. We have $f_s(g(0), 0) > 0$ because the precisions $f^A_{is}(r_m, r_s)$ and $f^{DA}_{is}(r_m, r_s)$ are all positive for any r_m and $f_s(g(\infty), \infty) < 0$ because $f^A_{is}(r_m, \infty) = 0$ and $f^{DA}_{is}(r_m, \infty) = 0$ for any r_m. In addition, using the first order conditions and Assumption 1, through straightforward but tedious calculation of Jacobian matrix J of (f_m, f_s), one can show that

$$\frac{df_s(g(r_s), r_s)}{dr_s} = \frac{\partial f_s(r_m, r_s)}{\partial r_m} g'(r_s) + \frac{\partial f_s(r_m, r_s)}{\partial r_s} = \frac{|J|}{\frac{\partial f_m(r_m, r_s)}{\partial r_m}} < 0 \tag{A-16}$$

for all $r_s > 0$, because $|J|$ can be verified to be strictly positive. By continuity and monotonicity, there must exist a unique solution $r^*_s > 0$ to $f_s(g(r_s), r_s) = 0$ which implies that there exist unique $r^*_m = g(r^*_s) > 0$ and $r^*_s > 0$ that solve $f_m(r_m, r_s) = 0$ and $f_s(r_m, r_s) = 0$. Therefore, there exists a unique equilibrium for a given η^*. To show the existence and uniqueness of η^*, note that when the participation cost $k = 0$, it is always better to invest in both of the risky assets because of the diversification effect, so the fraction of DI among discretionary investors is zero, while when $k = \infty$, it is always better to always invest only in the market portfolio, and so the fraction of DI is one. The negative of the log of the ratio of the utility of DA investors to that of DI investors, which has the same sign of the difference in utilities, is equal to

$$\begin{aligned} h(\eta) \quad = \quad & -\gamma(C_D(\tau_{DAm}, \tau_{DIm}) + k) - \frac{1}{2}\log\left(\frac{\tau_{vm} + \tau_{DIm} + \tau_m^2\tau_{zm}}{\tau_{vm} + \tau_{DAm} + \tau_m^2\tau_{zm}}\right) \\ & + \frac{1}{2}\log\left(\frac{(\tau_{vs} + \tau_{DAs} + \tau_s^2\tau_{zs})\left(\tau_{vs}\tau_{zs} + \left(\frac{\gamma\lambda_L}{\lambda_A + \lambda_D - \eta^*} + \tau_s\tau_{zs}\right)^2\right)}{\tau_{zs}\left(\frac{\gamma\lambda_L}{\lambda_A + \lambda_D - \eta^*} + \tau_{vs} + \tau_s^2\tau_{zs}\right)^2}\right). \end{aligned} \tag{A-17}$$

Note that τ_m, τ_s, τ_{DAm}, τ_{DAs}, and τ_{DIm} are all functions of η and η^* solves $h(\eta) = 0$. Using the first order conditions, Propositions 3 and 2, it can be shown that $h'(\eta) > 0$. Therefore, there exists a unique fraction of DI η^* such that the utility of DA is equal to that of DI, and thus there is a unique equilibrium.

Proof of Proposition 1:

Part 1. Suppose given $k = k_0$ the equilibrium endogenous indexing is η_0^*. When k is increased to k_1, the utility of DA becomes smaller than that of DI because DI investors do not pay the participation cost. Therefore, some DA investors must switch to be DI investors and thus the new equilibrium endogenous indexing η_1^* must be greater than η_0^*.

Part 2. Taking derivative in $h(\eta^*) = 0$ with respect to k, using the Envelop Theorem for the endogenous precisions (which are all functions of r_m, r_s, and η^*), we have

$$\frac{\partial h}{\partial \eta^*}\frac{\partial \eta^*}{\partial k} - \gamma = 0, \tag{A-18}$$

which implies that

$$\frac{\partial h}{\partial \eta^*} > 0, \tag{A-19}$$

because $\frac{\partial \eta^*}{\partial k} > 0$ by Part 1.

Taking derivative in $h(\eta^*) = 0$ with respect to τ_{zm}, we have

$$\frac{\partial h}{\partial \tau_{zm}} + \frac{\partial h}{\partial \eta^*}\frac{\partial \eta^*}{\partial \tau_{zm}} = 0. \tag{A-20}$$

It can be shown by direct computation that

$$Sign\left(\frac{\partial h}{\partial \tau_{zm}}\right) = Sign(\varphi_D). \tag{A-21}$$

Therefore, we have

$$Sign\left(\frac{\partial \eta^*}{\partial \tau_{zm}}\right) = -Sign(\varphi_D). \tag{A-22}$$

Part 3. Taking derivative in $h(\eta^*) = 0$ with respect to τ_{zs}, we have

$$\frac{\partial h}{\partial \tau_{zs}} + \frac{\partial h}{\partial \eta^*}\frac{\partial \eta^*}{\partial \tau_{zs}} = 0. \tag{A-23}$$

It can be shown by direct computation that

$$\frac{\partial h}{\partial \tau_{zs}} < 0. \tag{A-24}$$

Therefore, we have

$$\frac{\partial \eta^*}{\partial \tau_{zs}} > 0. \tag{A-25}$$

Part 4. Taking derivative in $h(\eta^*) = 0$ with respect to λ_I, we have

$$\frac{\partial h}{\partial \lambda_I} + \frac{\partial h}{\partial r_m}\frac{\partial r_m}{\partial \lambda_I} + \frac{\partial h}{\partial r_s}\frac{\partial r_s}{\partial \lambda_I} + \frac{\partial h}{\partial \eta^*}\frac{\partial \eta^*}{\partial \lambda_I} = 0. \tag{A-26}$$

It can be shown by direct computation that

$$\frac{\partial h}{\partial \lambda_I} > 0 \tag{A-27}$$

and if $\varphi_D = 0$, then

$$\frac{\partial h}{\partial r_m} = 0, \quad \frac{\partial h}{\partial r_s} < 0, \quad \frac{\partial r_s}{\partial \lambda_I} < 0. \tag{A-28}$$

Therefore, we have

$$\frac{\partial \eta^*}{\partial \lambda_I} < 0. \tag{A-29}$$

Proof of Proposition 2:

Taking derivative with respect to r_m in (A-13), we have

$$\lambda_A \frac{\partial \tau_A^*}{\partial r_m} + (\lambda_D - \eta^*)\frac{\partial \tau_{DA}^*}{\partial r_m} + \eta^* \frac{\partial \tau_{DI}^*}{\partial r_m} - (\tau_{DA}^* - \tau_{DI}^*)\frac{\partial \eta^*}{\partial r_m} - \gamma \lambda_L = 0. \tag{A-30}$$

The last term and the first three terms are all negative. This implies that the fourth term must be positive. As shown above $\tau_{DA}^* - \tau_{DI}^*$ has the same sign as information acquisition externality φ_D, therefore $\frac{\partial \eta^*}{\partial r_m}$ must have the opposite sign of φ_D. the claim then follows because r_m moves in the same direction as the price informativeness ρ_m holding λ_L, τ_{vm} and γ constant. Similarly, one can show the second part of the proposition.

Proof of Proposition 3:

Because τ_{DAm} solves (note that the information acquisition cost function C is the same for DA and DI investors):

$$2\gamma C_{Dm}(\tau_{DAm}, \tau_{DAs}) = \frac{1}{\tau_{DAm} + \rho_m}. \tag{A-31}$$

If $\varphi_D > 0$, then we have

$$2\gamma C_{Dm}(\tau_{DAm}, 0) > 2\gamma C_{Dm}(\tau_{DAm}, \tau_{DAs}) = \frac{1}{\tau_{DAm} + \rho_m} \tag{A-32}$$

which implies that $\tau_{DAm} > \tau_{DIm}$ because τ_{DIm} solves

$$2\gamma C_{Dm}(\tau_{DIm}, 0) = \frac{1}{\tau_{DIm} + \rho_m}. \tag{A-33}$$

and $C_{Dm}(\tau_{DAm}, 0)$ increases in τ_{DAm}. The case where $\varphi_D \leq 0$ can be shown similarly.