

Option Pricing Anomalies in Times of Financial Distress

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Option Pricing Anomalies in Times of Financial Distress

Abstract

Option prices for calls and puts should generally reflect homogeneous market expectations about the future return behavior of the underlying asset. Additionally, such option prices should preclude the possibility of arbitrage, thus conforming to the requirements of put-call parity. We explore whether these requirements of finance theory hold during times of financial distress by investigating S&P 500 option prices during the stock market crash of 2008-2009. The task requires estimation of options' implied risk-neutral distributions utilizing the unique capabilities of Bayesian-based empirical methods. We demonstrate that put and call prices during the time period of our study reflect heterogeneous market expectations and also violate put-call parity. However, such violations do not necessarily result in arbitrage trading opportunities.

JEL Classification: G13, C15, C11

Key Words: S&P 500 options, put-call parity, skewness, kurtosis, implied risk-neutral distribution, Markov chain Monte Carlo (MCMC)

1. Introduction

Market index put and call option prices reflect market expectations about the distribution of future returns. In efficient markets these expectations should be homogeneous across option types and the underlying options prices should preclude the possibility of arbitrage. We examine future return expectations elicited from S&P 500 index call and put option prices during the financial crisis of 2008-2009 and find that they are mutually incompatible, thus implying a segmented market, and that they also violate the no arbitrage requirements of put-call parity. In doing so, we make three contributions to separate areas of the option-pricing literature.

First, we use Bayesian-based empirical methods in a new application as originally implemented by Sfiridis (2010) to effectively estimate the four-moment implied risk-neutral density used to model option prices. The critical starting point is the estimation of the historical or data-based likelihood function for S&P 500 daily returns using Bayesian-based Markov chain Monte Carlo (MCMC) methods. These offer unique and valuable capabilities that provide a large number of independent draws from the required marginal posterior distributions of each of the four moment parameters. Since Bayesian estimates of each higher moment of scale, skewness and kurtosis defining overall return volatility are independent, overall volatility is effectively partitioned among these three higher moments. Once the first moment of location, i.e., the mode, is found relative to the risk-free interest rate as in Brennan (1979), finding the sought-after *forward-looking* or implied risk-neutral specification extends the previous MCMC exercise by finding the unique implied risk-neutral density having specific higher moment values from the MCMC output that minimize the difference between option model and market prices.

Second, we show that the implied risk-neutral densities recovered from prices of matched put and call option series during the financial crisis are mutually incompatible, resulting in

implied volatility or IV plots for the matched option series that are quite different. Such inconsistencies are also revealed as differences in the implied higher moments of each density. These differences result in substantial pricing errors when applying the risk-neutral density from one option type to the other and indicate that during the period of our study, one of extreme financial distress, the market was segmented. Investor sentiment in both option sub-markets, though uniform about the general direction of expected market performance, differed as to the degree of such performance.

Third, we add to previous research on put-call parity by showing that the occurrence of market segmentation during the time period of our study leads to violations of put-call parity. Such violations reflect a segmented market where one option type is mispriced relative to its matched counterpart. Determination of a new implied risk-neutral density that eliminates put-call parity violations and thus gives new equilibrium option prices that would occur absent a financial crisis leads to the conclusion that, though mispricing did occur, arbitrage trading possibilities did not result.

The rest of the paper is organized as follows: Section 2 reviews prior relevant research. Section 3 gives data sources and presents the methodology used to determine the implied risk-neutral density and to partition implied volatility into its component parts. Section 4 presents empirical results that compare call and put market prices from the financial crisis of 2008-2009 relative to each other, their Black-Scholes equivalents and their derived counterparts for a non-distressed time period. Section 5 summarizes and concludes.

2. Literature Review

We begin by identifying two cases from the literature in which related risk-neutral densities are in reality quite different. Ait-Sahalia, Wang and Yared (2001) state that the

moments estimated from past S&P 500 returns are demonstrably different from those that additionally incorporate current option prices. Second, and more important, we consider the differences in implied volatility between puts and calls observed by Cremers and Weinbaum (2010). Cremers and Weinbaum find that implied risk-neutral volatilities for matched, contemporaneous puts and calls are not equivalent, and the gap between the two has predictive power.

There has been a plethora of recent research that shows that options are non-redundant securities containing useful information about expected future return performance of the underlying asset (Dennis and Mayhew, 2002; Bollen and Whaley, 2004; Buraschi and Jiltsov, 2006; Bing Han, 2008; Atilgan, 2009; Garleanu, Pedersen and Poteshman, 2009; Cremers and Weinbaum, 2010; Friesen, Zhang and Zorn, 2011). Researchers have generally considered two approaches; first, the occurrence of varying implied volatility or IV spreads for matched puts and calls as violations of put-call parity and, second, the degree of skewness of the implied risk-neutral density as indicated by the slope of the IV function that serves as a predictive mechanism. Such IV behavior results from option prices influenced by the trading activity of informed investors as information is received about upcoming information events. For example, this is termed “net buying pressure” by Bollen and Whaley (2004) or “demand pressure” by Garleanu, Pedersen and Poteshman (2009). Bing Han (2008) finds that more negative risk-neutral skewness for the S&P 500 index is related to more bearish market sentiment.

Bakshi, Kapadia and Madan (2003) draw attention to differences in risk-neutral skewness and kurtosis in situations where it may not intuitively be expected. They document differences in the risk-neutral distribution of index and individual stock returns. A crucial insight is that the data-based *historical* and *forward-looking* or implied risk-neutral distributions for a particular

asset can be quite different. Also, by comparing two risk-adjusted densities rather than a risk-adjusted density from option prices to an unadjusted density from index returns, assumptions about investor preferences are avoided, i.e., investor utility functions as required by Brennan (1979) to go to a discrete time framework are no longer necessary.

Much of the research in recovering information from options involves investigation of the difference between market and model prices as a function of the variables of interest. Jackwerth and Rubinstein (1996) consider various minimization criteria to determine the implied risk-neutral specification from historically observed option prices. They consider four methods to recover the risk-neutral distribution and find that with any option maturity date having at least eight strike prices, all four methods are equivalent. In this paper, we implement an approach similar to the maximum entropy function in Stutzer (1996) that adjusts a risk-neutral density from historical data to minimize the absolute price difference between model-derived and actual market prices.

3. Methodology

3.1. Data

We obtain daily S&P 500 index returns data for the historical risk-neutral distribution from CRSP and S&P 500 index option price data to then estimate the forward-looking risk-neutral distribution from the OptionMetrics database from IvyDB. We use a year of daily S&P 500 index price data prior to the observation dates in January, 2008 and 2009 to estimate the historical density and all pairs of put and call prices on the observation dates to estimate the implied density. LIBOR quotes for three, six, and twelve months from Bloomberg are used as a proxy for the risk-free rate at the respective time horizons. Pairs of liquid put and call options with positive trade volume on the observation dates with the midpoint of closing bid and ask

quotes are used as the market price for each contract. Those pairs maturing in March, June, and December are retained to examine changes in the implied risk-neutral densities through time.

3.2 The return model and Bayesian MCMC estimation

To determine the underlying moments of the risk-neutral distribution, we first start with the four-moment likelihood function introduced by Fernandez and Steel (1998), henceforth FS, as

$$h(r_i | \mu, \sigma^2, \lambda, \gamma) = \left(\frac{2}{\pi}\right)^{1/2} \left(\frac{1}{\gamma + \frac{1}{\gamma}}\right) \frac{\lambda_i^{1/2}}{\sigma} \exp \left[-\frac{\lambda_i (r_i - \mu)^2}{2\sigma^2} \left\{ \frac{1}{\gamma^2} I_{[0, \infty)}(r_i - \mu) + \gamma^2 I_{(-\infty, 0)}(r_i - \mu) \right\} \right] \quad (1)$$

which extends the normal likelihood by augmenting the location and scale parameters of μ and σ or the mean and variance for the normal case by λ and γ to introduce kurtosis and skewness respectively.

A normal distribution of returns is insufficient, since it fails to account for “fat tails” or excess kurtosis observed in index returns. We introduce kurtosis by using data augmentation from Tanner and Wong (1987) to replicate the *Student-t* distribution by a mixture of normals, giving

$$h(r_i | \mu, \sigma^2, \lambda, v, \gamma) = \left(\frac{2}{\pi}\right)^{1/2} \left(\frac{1}{\gamma + \frac{1}{\gamma}}\right) \int_0^\infty \frac{\lambda_i^{1/2}}{\sigma} \exp \left[-\frac{\lambda_i (r_i - \mu)^2}{2\sigma^2} \left\{ \frac{1}{\gamma^2} I_{[0, \infty)}(r_i - \mu) + \gamma^2 I_{(-\infty, 0)}(r_i - \mu) \right\} \right] \times f_G \left(\lambda_i \left| \frac{v}{2}, \frac{v}{2} \right. \right) d\lambda_i \quad (2)$$

where f_G is a gamma density specified by degrees of freedom v .¹ Thus, the kurtosis parameter λ is conditional on degrees of freedom and allows replication of a *Student-t* distribution from a mixture of normals. This accommodates various levels of leptokurtosis or “fat tails” with $DF < 30$. The γ skewness parameter controls symmetry, allocating probability mass to each side of the distribution’s mode. Values of $\gamma > 1.0$ produce positive skewness or skewing to the right and $\gamma < 1.0$ produces

¹ The symbol v is used to denote degrees of freedom to remain consistent with the notation from Fernandez and Steel (1998). However, in the remaining sections of the paper, the abbreviation “DF” will also be used.

negative skewness or skewing to the left. Values of γ and λ equal to unity provide the familiar normal likelihood.²

We obtain the joint posterior distribution of the model parameters following Bayes' Rule or

$$g(\mu, \sigma^2, \gamma, \lambda, \nu | \mathbf{r}) \propto h(\mathbf{r} | \mu, \sigma^2, \gamma, \lambda) \pi(\mu) \pi(\sigma^2) \pi(\gamma) \pi(\lambda | \nu) \pi(\nu) \quad (3)$$

where the posterior is proportional to the likelihood function times the parameters' prior distributions. We use this relation to formulate the full conditional distribution for each model parameter, defined as that distribution for the parameter conditional on the other model parameters and the data.³ After repeated iterative re-sampling from the full conditional distribution using MCMC methods, such as Gibbs sampling or the more general Metropolis-Hastings (M-H) algorithm, MCMC output as a long Markov chain comprised of candidate draws for the model parameter converges to the required marginal distribution defined as the parameter's distribution conditional only on the data. Thus, MCMC methods become a means of accomplishing sampling-based multidimensional integration.

For our analysis the Markov chains are all run for 600,000 iterations with the last 50,000 iterates retained as draws from the required marginal distributions. Of these, every fifth one is retained to minimize any potential serial correlation, providing Markov chains of 10,000 iterates each for the subsequent analysis. Such MCMC methods now provide a large sample of independent estimates of each model parameter, cross-sectionally independent across the parameter vector and serially independent within each parameter's Markov chain.

3.3 Option Pricing and the Expected Risk-Neutral Distribution of the S&P 500 Index

² A $\lambda = 1.0$ giving zero kurtosis requires degrees of freedom approximately equal to 30.

³ See the appendix in Sfiridis (2010) for a detailed description of the derivation of the full conditional distributions.

To estimate the market's latent beliefs about future market performance by incorporating option prices into the risk-neutral distribution, estimation of equation (2) must first be accomplished using MCMC methods.

Once the MCMC data-based marginal posterior distributions of the higher moments are acquired, the first moment or mode is then determined by equating the mean and the risk-free rate and then finding the mode as the maximum likelihood estimate of the distribution. This "location shift" is necessary in order to specify the *historical* risk-neutral density.

The observed prices of call (c) and put (p) options in a discrete setting with an underlying that follows any risk-neutral density $g(r_i)$ are

$$c = e^{-r_f(T-t)} \max\left[0, \sum_{i=1}^N (S_0 e^{r_i(T-t)} - k) g(r_i) \Delta r_i\right] \quad (4)$$

$$p = e^{-r_f(T-t)} \max\left[0, \sum_{i=1}^N (k - S_0 e^{r_i(T-t)}) g(r_i) \Delta r_i\right] \quad (5)$$

for calls and puts respectively. Equations (4) and (5) imply that observed put and call option prices provide restrictions on the unknowns that determine $g(r_i)$, the implied risk-neutral density that the market expects to obtain at time T .

Jackwerth and Rubinstein (1996) use option market prices to determine an implied risk-neutral distribution that produces near model and market price equivalency. Subramanian (2004) applies the same insight to the estimation of discontinuous distributions with jump processes representing the likelihood of acquisition in a merger. In both cases, their objective function is the cumulative absolute difference between option prices observed in the market, and those determined as a function of pricing model parameters. This cumulative absolute deviation (CAD) is defined as

$$\text{CAD} = \min \sum_{i=1}^{N_s} |P_{\text{mod}_i} - P_{\text{mkt}_i}| \quad (6)$$

where N_s designates the number of strike prices for an option price series of given maturity, P_{mod} is the model price per equations (4) and (5) for a specific strike price i and P_{mkt} is the actual market price of the option at that strike price. Following previous work, we employ the CAD objective function in our estimation, with P_{mod} from equations (4) and (5) as a function of parameters that determine $g(r_i)$.

The candidates to determine the new model prices that minimize the CAD come from independent combinations of the large number of draws from each parameter's marginal distribution comprising its Markov chain. Minimizing the CAD in equation (6) involves resampling from all iterates comprising the parameters' Markov chains until the unique optimal combination is found. The requirement that the risk-free interest rate is equal to the mean makes a new location shift necessary for each newly proposed risk-neutral specification. The theoretical optimum value of the CAD is zero, but pricing errors or deviations between implied model and market prices may reflect non-systematic effects or market microstructure noise as discussed by Figlewski (2009) that cannot be captured by the parametric risk-neutral specification.

3.3 Put-Call Parity and the Equilibrium Implied Risk-Neutral Distribution

A question that arises is what would option prices be if the time period of our study was not one of extreme financial distress? We assume that the observed decoupling between the prices of puts and calls during a financial crisis caused by heterogeneous market expectations results in violations of put-call parity that would not occur absent a financial crisis. A single *equilibrium* implied risk-neutral distribution is estimated based on eliminating violations of put-

call parity that restores homogeneous market expectations and consistent matched call and put option prices.

Stoll (1968) defined a fundamental relationship between matched puts and calls called *put-call parity* defined as

$$c + K \cdot e^{-r(T-t)} = S + p \quad (7)$$

where c and p are call and put prices respectively, S is the price of the underlying asset, K is the strike price, r is the risk-free interest rate and $(T-t)$ is the time to expiration. Quite simply, it states that portfolio A consisting of a European call option plus the present value of cash equal to the option strike price must equal portfolio B consisting of the underlying asset's spot price plus the corresponding European put having the same strike price and expiration date. At expiration it can be shown that both portfolios will be equivalent and therefore their present values must also be equivalent to preclude arbitrage opportunities that would materialize if there was divergence between the put and call values. Arbitrageurs would sell the overvalued portfolio and buy the undervalued one in order to make riskless profitable trades until restoration of the put-call parity relationship in equation (7).

Put-call parity validity depends on option mispricing relative to an equilibrium price for one option type that is lower than its market price and at the same time the equilibrium price for the other matched option type that is higher than its market price. The arbitrage trading opportunity materializes not because of contemporaneous option mispricing of one option type relative to the other but because of option mispricing relative to the equilibrium price, that market price that will be realized after arbitrage trading has restored market efficiency. However, under the conditions of a financial crisis where asset return volatility caused by investor

uncertainty most probably is at elevated levels, the above discussed pricing conditions to realize arbitrage might not be present.

Using the same methodology given previously to determine the CAD in equation (6), equation (7) in a different form

$$c + K \cdot e^{-r(T-t)} - S - p = 0 \quad (8)$$

becomes the second objective function used in our analysis. If portfolio A is expensive relative to portfolio B, then put-call parity or PCP will be greater than zero. If portfolio B is expensive relative to portfolio A, then PCP will be less than zero. The new unique *equilibrium* implied risk-neutral specification that restores put-call parity equal to zero must be determined from the model parameters' Markov chains as done previously for the CAD criterion. The resulting call and put option prices represent equilibrium prices that occur in the absence of financial distress and allow comparison of such prices with their financially distressed counterparts.

4 Empirical Results

4.1 Implementation

We first generate the historical risk-neutral density using daily return data for the S&P 500 index for a year prior to an observation date. We can now generate model-specified option prices as a function of the Markov chain values of the moment parameters describing the historical risk-neutral density in equation (2) for pricing and equations (4) and (5) for pairs of calls and puts respectively. Finally, the specific parameter values of the higher moments of the forward-looking or implied risk-neutral density for a specific option series are found by sequentially going through the Markov chain output to find that combination that minimizes the CAD criterion given in equation (6).

Our investigation focuses on differences between the implied risk-neutral densities of S&P 500 call and put option series for each of three maturities on two observation dates in January of 2008 and 2009. Risk-neutral densities are estimated for options maturing in March, June, and December of each year. Table 1 presents independent estimates of the volatility, skewness, and kurtosis parameters for the 2008 and 2009 observation dates. Table 2 shows the CADs from equation (6) as an indicator of the extent to which the six option contracts can be accurately priced. Tables 3-6 show the mispricing that occurs for puts and calls of similar maturity and the extent of put-call parity violations resulting from such mispricing.

4.2 Discussion

Figures 1 and 2 show the implied volatility or IV plots for put and call options on the S&P 500 index in 2008 and 2009 respectively. We can clearly see that for both time periods the IV's are different across the two option types. For each maturity the call IV plot lies below its respective put IV plot, strongly indicating relative mispricing with calls being cheap relative to puts and conversely puts being expensive relative to calls. This type of IV divergence between matched puts and calls is a bearish signal for future market performance. In both figures, except for the Mar08 calls, call prices are so low relative to B/S prices for deep in-the-money calls that IV's approach zero, thus giving an IV plot shape that resembles a "frown" rather than the familiar "smile" or "smirk", an unusual anomaly for equity options.⁴ The corresponding put IV's retain the familiar "smile" or "smirk", thus highlighting extreme relative option mispricing at low strike prices.

Figure 1 generally has IV plots that lie above the B/S standard deviation, revealing an expected relationship in 2008 that both calls and puts were relatively more expensive than their

⁴ The IV "frown" for equity call options is discussed in Sfiridis (2010).

B/S counterparts. However, the IV plots in figure 2 lie in large part below the B/S standard deviation, revealing 2009 option prices that were cheap relative to B/S prices. Such market underpricing relative to B/S indicates an increase in bearish market sentiment for 2009 compared to 2008.

Table 1 shows the first four moments of the implied risk-neutral densities at the start of 2008 (Panel A) and 2009 (Panel B). The historical risk-neutral density in Panel A, derived from the S&P returns from 2007, decomposes the Black-Scholes volatility of 16% into an 11% standard deviation with negative skewness and excess kurtosis. The implied distributions for the 2008 options reveal standard deviations a bit less than the B/S specification, but more than the Bayesian estimate of 11%, high levels of kurtosis and, most importantly, much higher levels of negative skewness than seen for the historical period of 2007. This very strong bearish indicator for 2008 relative to 2007 is signaled by both the put and call implied risk-neutral densities. Of the two option types the calls for March and June are sending a stronger bearish signal as indicated by the low γ parameter values, i.e., increased negative skewness, compared to those for puts. However, puts for March and June show higher levels of *symmetric* uncertainty about future market direction as revealed by increased kurtosis. These observations are not true for the Dec08 options. All the matched put and call options for each maturity have mutually inconsistent higher moments. Panel B of table 2 for the 2009 options shows very similar results to panel A for all three option maturities with one important difference. All of the Bayesian estimates for the implied second moment as measured by standard deviation are much less than the 2008 B/S estimate of 41%. This difference is a direct consequence of relatively low 2009 option market prices. Once again all the matched put and call options for each maturity in panel B have mutually inconsistent higher moments.

Table 2 describes the quality of fit of the risk-neutral distributions to observed option prices in 2008 and 2009. As expected, we see that the model prices using the implied or *forward-looking* risk-neutral distribution match market prices much better than the two using only historical return data. Since the differences between the observed market price and the model-derived one are seemingly random in direction, the remaining positive CAD for the *forward-looking* version may simply represent pricing noise. For both 2008 and 2009, maturity affects the quality of fit. On the observation dates in January, the December contracts are the least liquid and therefore do not move as cohesively as the near-term contracts in the earlier months. The CAD by maturity shows that the long-term December contracts are harder to model accurately than the near-term March and June ones.

Figures 1 and 2 give evidence of relative call and put option mispricing. Further evidence that such mispricing occurs is seen in table 1. However, the extent of such price differences is still an open question. We now explore this issue and whether put-call parity is violated during the time period of the study. We examine the near-term March contracts because of their desirable features of high liquidity, large span of strike prices, and good quality of fit as measured by CAD.

Table 3, panel A, shows what call and put option prices would be for both of the Mar08 option series using the implied risk-neutral specification for Mar08 calls previously presented in table 1, panel A. As expected the call model prices using equation (4) closely approximate the call market prices in columns 3 and 2 respectively. However, put model prices from equation (5) shown in column 5 reveal them to be significantly cheaper than their market price counterparts. Thus, puts are significantly more expensive relative to their matched call counterparts during times of financial distress. Additionally columns 6 and 7 show violations of put-call parity for

both market and model option prices respectively. All values are not equal to zero. Table 4, panel A, repeats the exercise for the March09 option series using the risk-neutral specification from table 1, panel B. All results are similar, though the magnitude of the difference in put model and market prices is not as great.

Panel B in tables 3 and 4 repeat the above exercise for puts. For example, in table 3 for the Mar08 option series the implied risk-neutral specification for puts given in table 1, panel A, is used. Now the put model and market prices shown in columns 5 and 4 respectively are very nearly equivalent. However, call market prices are consistently less than the corresponding model prices as seen in columns 2 and 3 respectively. Thus, we can conclude that calls relative to their corresponding puts are cheap in times of financial distress. Once again the nonzero values shown in columns 6 and 7 show violations of put-call parity for both market and model option prices respectively. The conclusion from the results in tables 1, 3 and 4 is that the put and call option markets were segmented during the extreme financial distress time period of our study. The relative mispricing that occurred, cheap calls relative to corresponding puts and expensive puts relative to corresponding calls, is reasonable in the presence of bearish investor sentiment and supports previous conclusions from the IV plots presented in figures 1 and 2.

What would option prices be if the time period of our study was not one of financial distress? How do such prices compare with actual market prices seen under extreme financial distress? Utilizing equation (8), column 4 of panel A in tables 5 and 6 show the *equilibrium* implied risk-neutral distributions for the Mar08 and Mar09 S&P 500 index options when put-call parity is not violated along with the actual implied distributions for calls and puts in columns 2 and 3 respectively. Panel B in tables 5 and 6 give market call and put prices in columns 2 and 4 and their *equilibrium* price counterparts in columns 3 and 5.

For the Mar08 options in table 5 an *equilibrium* risk-neutral distribution is presented that is different than its market-based counterparts. This is seen most clearly by noting the difference in kurtosis between the *equilibrium* distribution and the other two kurtosis values. Panel B of table 5 shows that the *equilibrium* call prices in column 3 are systematically and significantly less than the corresponding call market prices. Equilibrium put prices in column 5 are also systematically and significantly less than the corresponding put market prices. For the Mar09 options in panel A of table 6 the *equilibrium* risk-neutral distribution does not vary significantly from the actual market-based specifications for calls and puts. It does exhibit less negative skewness and less kurtosis. Panel B of table 6 shows that the *equilibrium* call prices in column 3 are systematically less than the corresponding call market prices, except for the deep in-the-money 600 call option. *Equilibrium* put prices in column 5 are also systematically less than the corresponding put market prices. Thus, we conclude that both calls and puts have become more expensive during this period of financial distress relative to their corresponding non-distressed counterparts. Increased return volatility resulting from an increase in market uncertainty during such financial distress is a reasonable explanation for such price behavior.

Were arbitrage trading opportunities available during the time period of our study because of violation of put-call parity? If so, how could such arbitrage opportunities exist when finance theory states that they are quickly eliminated by trading activity and, thus, not readily seen? Though the put-call parity relation was violated, we assert that arbitrage opportunities were not available during the time period of our study. To implement an arbitrage trade, the cheap portfolio must be bought and the expensive portfolio must be sold. However, our matched put and call portfolios were uniformly more expensive than their *equilibrium* counterparts. A cheap

portfolio did not exist, thus precluding the possibility of arbitrage trading to restore *equilibrium* option prices.

5. Summary and Conclusions

We demonstrate that the market for call and put options was segmented during the 2008-2009 financial crisis, i.e., the market's expectation that priced puts mispriced calls and vice versa. Specifically, the call-based risk-neutral densities underprice the puts, and the put-based ones overprice the calls. This effect is stronger in 2008 than in 2009. Its implication is that though our analysis revealed overall bearish market sentiment during the financial crisis, the extent of such bearishness was different for the S&P 500 put and call option markets. We further show that, as expected, this difference in expectations leads to put-call parity violations, but that these violations did not present immediate arbitrage opportunities.

We implement our analysis by independently estimating the three higher moments of the implied risk-neutral distribution of index returns based on historical return data and the additional incorporation of observed option market prices. Our approach effectively decomposes overall implied volatility into independent and unbiased moment estimates. Such capability should prove extremely valuable in pursuing further research in financial derivatives.

References

- Ait-Sahalia, Yacine, Yubo Wang and Francis Yared (2001), “Do option markets correctly price the probabilities of movement of the underlying asset?”, *Journal of Econometrics* 102; pages 67-110.
- Atilgan, Yigit, “Deviations from Put-Call Parity and Earnings Announcement Returns” (November 23, 2009). Available at SSRN; <http://ssrn.com/abstract=1512046>.
- Bakshi, Gurdip, Nikunj Kapadia and Dilip Madan (Spring 2003), “Stock Return Characteristics, Skew Laws, and the Differential Pricing of Individual Equity Options”, *The Review of Financial Studies*, vol. 16, no. 1; pages 101-143.
- Brennan, M.J. (March 1979),” The Pricing of Contingent Claims in Discrete Time Models”, *Journal of Finance*, vol. 34, no. 1; pages 53-68.
- Buraschi, Andrea and Alexei Jiltsov (December, 2006), “Model Uncertainty and Option Markets with Heterogeneous Beliefs”, *Journal of Finance*, vol. 61, no. 6; pages 2841-2897.
- Cremers, Martijn and David Weinbaum (April, 2010), “Deviations from Put-Call Parity and Stock Return Predictability”, *Journal of Financial and Quantitative Analysis*, Vol. 45, No. 2; pages 335-367.
- Dennis, P. and S. Mayhew (2002),”Risk-neutral Skewness: Evidence from Stock Options”, *Journal of Financial and Quantitative Analysis*, 37; pages 471-493.
- Fernandez, Carmen and Mark F.J. Steel (1998), “On bayesian modeling of fat tails and skewness”, *Journal of the American Statistical Association* 93; pages 359-371.
- Figlewski, Stephen (2009), “Estimating the Implied Risk Neutral Density for the U.S. Market Portfolio”, working paper, New York University Stern School of Business.
- Friesen, Geoffrey C., Zorn, Thomas S. and Zhang, Yi, “Heterogeneous Beliefs and Risk Neutral Skewness” (September 19, 2011). *Journal of Financial and Quantitative Analysis*, Forthcoming. Available at SSRN:<http://ssrn.com/abstract=1930365>.
- Garleanu, Nicolae, L.H. Pedersen and A.M. Poteshman, 2009. “Demand-based Option Pricing”, *Review of Financial Studies*, Vol. 22, No. 10; pages 4259-4299.

- Han, Bing (January, 2008), “Investor Sentiment and Option Prices”, *The Review of Financial Studies*, vol. 21, no. 1; pages 387-414.
- Jackwerth, Jens Carsten and Mark Rubinstein (December 1996), “Recovering Probability Distributions from Option Prices”, *The Journal of Finance*, vol. 51, no. 5; pages 1611-1631.
- Jackwerth, Jens Carsten (Winter 1999), “Option-Implied Risk-Neutral Distributions and Implied Binomial Trees: A Literature Review”, *Journal of Derivatives*, vol.7, no. 2; pages 66-82.
- Tanner, Martin A. and Wing Hung Wong (1987), “The calculation of posterior distributions by data augmentation”, *Journal of the American Statistical Association*, vol. 82; pages 528-540.
- Sfiridis, James M., (2010) “The Predictive Power of S&P 500 Option Prices and the Stock Market Crash of 2008-2009”. Available at SSRN: <http://ssrn.com/abstract=1624247>.
- Stoll, H. (1968), “The relationship between put and call option prices”, *The Journal of Finance* , vol. 24; pages 801-824.
- Stutzer, Michael (December 1996), “A Simple Nonparametric Approach to Derivative Security Valuation”, *The Journal of Finance*, vol. 51, no. 5; pages 1633-1652.
- Subramanian, A. (2004), “Option pricing on stocks in mergers and acquisitions”, *The Journal of Finance*, vol. 59; pages 795–829.

Table 1. Implied Risk Neutral Specifications

<p><i>Panel A.</i> Options maturing in 2008. Observation date is January 2, 2008. Physical historical S&P 500 pdf specifications for 2007 are standard deviation (σ) = 0.16 (σ_{MCMC} = 0.11), skewness = -0.42 (γ = 0.84), kurtosis = 1.32 (DF = 3.70)</p>					
Calls	Puts	R_f (%)	σ (%)	Skewness (γ)	Kurtosis (DF)
March, 2008		4.68	14.38	0.54	10.62
	March, 2008	4.68	12.08	0.81	1.64
June, 2008		4.57	14.30	0.53	17.91
	June, 2008	4.57	12.99	0.74	2.02
Dec, 2008		4.19	14.38	0.81	3.30
	Dec, 2008	4.19	14.30	0.77	1.66
<p><i>Panel B.</i> Options maturing in 2009. Observation date is January 5, 2009. Physical historical S&P 500 pdf specifications for 2008 are standard deviation (σ) = 0.41 (σ_{MCMC} = 0.23), skewness = 0.19 (γ = 0.93), kurtosis = 3.84 (DF = 2.51)</p>					
Calls	Puts	R_f (%)	σ (%)	Skewness (γ)	Kurtosis (DF)
March, 2009		1.42	20.89	0.59	3.75
	March, 2009	1.42	24.73	0.72	2.62
June, 2009		1.79	24.84	0.65	6.37
	June, 2009	1.79	29.26	0.87	2.49
Dec, 2009		2.09	21.13	0.71	4.27
	Dec, 2009	2.09	24.53	1.0	1.17

Table 2. Cumulative Absolute Price Differences (CAD's). 'BS CAD' shows the sum of the absolute option price differences between the Black Scholes prices and market prices. 'Historical CAD' shows the difference for prices from the historical risk-neutral pdf and market prices. 'Forward-looking CAD' shows the price differences between the implied risk-neutral pdf and market prices.

<i>Panel A. Options maturing in 2008. Observation date is 1/2/2008.</i>					
Calls	Puts	No. of strikes	BS CAD(\$)	Historical CAD(\$)	Forward-looking CAD(\$)
March, 2008		10	145.80	313.61	0.86
	March, 2008	10	177.50	90.77	1.35
June, 2008		5	99.00	235.25	0.51
	June, 2008	5	155.10	84.36	0.75
Dec, 2008		8	154.70	438.25	26.49
	Dec, 2008	8	340.00	243.04	7.75
<i>Panel B. Options maturing in 2009. Observation date is 1/5/2009.</i>					
Calls	Puts	No. of strikes	BS CAD(\$)	Historical CAD(\$)	Forward-looking CAD(\$)
March, 2009		13	119.20	84.07	9.85
	March, 2009	13	80.90	75.91	6.00
June, 2009		5	93.80	71.70	8.23
	June, 2009	5	47.20	36.67	1.66
Dec, 2009		8	240.80	262.66	46.80
	Dec, 2009	8	60.60	188.95	27.72

Table 3.

Panel A: Price Comparison for March, 2008 S&P 500 Index Option Series Using the Call Implied Risk-Neutral Specification ($R_f = 4.68\%$, $\sigma = 14.38\%$, $\gamma = 0.54$, $DF = 10.6$).						
Strike Price *	Call Mkt. Price	Call Model Price	Put Mkt. Price	Put Model Price	Mkt. Put-Call Parity	Model Put-Call Parity
\$1400	\$98.9	\$99.3	\$40.7	\$30.2	(\$3.36)	\$7.58
1450	66.2	66.2	57.5	46.7	(\$3.36)	\$7.58
1455	63.3	63.2	59.5	48.6	(\$3.31)	\$7.58
1460	60.3	60.3	61.5	50.7	(\$3.36)	\$7.58
1465	57.4	57.4	63.6	52.8	(\$3.41)	\$7.58
1470	54.6	54.6	65.7	54.9	(\$3.36)	\$7.58
1475	51.9	51.9	67.9	57.1	(\$3.31)	\$7.58
1480	49.2	49.3	70.2	59.4	(\$3.36)	\$7.58
1500	39.3	39.3	80.1	69.2	(\$3.36)	\$7.58
1550	19.8	19.7	110.2	99.1	(\$3.46)	\$7.58
Panel B: Price Comparison for March, 2008 S&P 500 Index Option Series Using the Put Implied Risk-Neutral Specification ($R_f = 4.68\%$, $\sigma = 12.08\%$, $\gamma = 0.81$, $DF = 1.64$).						
Strike Price *	Call Mkt. Price	Call Model Price	Put Mkt. Price	Put Model Price	Mkt. Put-Call Parity	Model Put-Call Parity
\$1400	\$98.9	\$124.1	\$40.7	\$41.2	(\$3.36)	21.46
1450	66.2	90.7	57.5	57.3	(\$3.36)	21.46
1455	63.3	87.8	59.5	59.3	(\$3.31)	21.46
1460	60.3	84.8	61.5	61.3	(\$3.36)	21.46
1465	57.4	82.0	63.6	63.4	(\$3.41)	21.46
1470	54.6	79.1	65.7	65.5	(\$3.36)	21.46
1475	51.9	76.3	67.9	67.7	(\$3.31)	21.46
1480	49.2	73.6	70.2	69.9	(\$3.36)	21.46
1500	39.3	63.7	80.1	79.8	(\$3.36)	21.46
1550	19.8	44.7	110.2	110.3	(\$3.46)	21.46

*Index level= 1447.16

Table 4.

Panel A: Price Comparison for March, 2009 S&P 500 Index Option Series Using the Call Implied Risk-Neutral Specification ($R_f = 1.42\%$, $\sigma = 20.89\%$, $\gamma = 0.59$, $DF = 3.75$).						
Strike Price *	Call Mkt. Price	Call Model Price	Put Mkt. Price	Put Model Price	Mkt. Put-Call Parity	Model Put-Call Parity
\$600	\$327.9	\$330.0	\$2.9	\$2.4	(\$4.19)	(\$1.60)
750	189.1	189.7	14.1	11.6	(\$4.62)	(\$1.60)
800	147.0	146.9	21.9	18.7	(\$4.66)	(\$1.60)
850	108.4	107.5	33.1	29.2	(\$4.61)	(\$1.60)
900	74.4	73.3	48.8	44.9	(\$4.45)	(\$1.60)
925	59.5	58.8	59.0	55.2	(\$4.63)	(\$1.60)
950	46.5	46.0	70.9	67.4	(\$4.60)	(\$1.60)
975	35.2	34.7	84.4	81.0	(\$4.47)	(\$1.60)
1000	25.5	25.0	99.7	96.3	(\$4.54)	(\$1.60)
1050	11.8	11.6	136.0	132.7	(\$4.69)	(\$1.60)
1100	4.3	5.2	178.3	176.1	(\$4.63)	(\$1.60)
1150	1.4	2.5	225.2	223.3	(\$4.58)	(\$1.60)
1200	0.5	1.1	274.1	271.8	(\$4.52)	(\$1.60)
Panel B: Price Comparison for March, 2009 S&P 500 Index Option Series Using the Put Implied Risk-Neutral Specification ($R_f = 1.42\%$, $\sigma = 24.73\%$, $\gamma = 0.72$, $DF = 2.62$).						
Strike Price *	Call Mkt. Price	Call Model Price	Put Mkt. Price	Put Model Price	Mkt. Put-Call Parity	Model Put-Call Parity
\$600	\$327.9	\$336.8	\$2.9	\$3.4	(\$4.19)	\$4.26
750	189.1	197.8	14.1	13.9	(\$4.62)	\$4.26
800	147.0	155.6	21.9	21.6	(\$4.66)	\$4.26
850	108.4	116.8	33.1	32.6	(\$4.61)	\$4.26
900	74.4	83.1	48.8	48.8	(\$4.45)	\$4.26
925	59.5	68.6	59.0	59.2	(\$4.63)	\$4.26
950	46.5	55.5	70.9	71.0	(\$4.60)	\$4.26
975	35.2	44.1	84.4	84.5	(\$4.47)	\$4.26
1000	25.5	34.3	99.7	99.6	(\$4.54)	\$4.26
1050	11.8	20.7	136.0	135.9	(\$4.69)	\$4.26
1100	4.3	13.3	178.3	178.3	(\$4.63)	\$4.26
1150	1.4	9.1	225.2	224.1	(\$4.58)	\$4.26
1200	0.5	6.5	274.1	271.3	(\$4.52)	\$4.26

*Index level= 927.45

Table 5. The Implied Risk-Neutral Distribution for March, 2008 S&P 500 Index Options
That Precludes Violation of Put-Call Parity (PCP) and Pricing Implications

<i>Panel A. The Implied Risk-Neutral Distributions</i>				
Specification	Calls	Puts	PCP = 0	
R_f (%)	4.68	4.68	4.68	
σ (%)	14.38	12.08	11.3	
γ (skewness)	0.54	0.81	0.71	
DF (kurtosis)	10.62	1.64	20.84	
<i>Panel B. Price Comparisons</i>				
Strike Price* (\$)	Market Calls (\$)	PCP=0 Calls (\$)	Market Puts (\$)	PCP = 0 Puts (\$)
1400	98.9	75.0	40.7	13.5
1450	66.2	41.1	57.5	29.2
1455	63.3	38.3	59.5	31.3
1460	60.3	35.6	61.5	33.5
1465	57.4	33.0	63.6	35.9
1470	54.6	30.5	65.7	38.3
1475	51.9	28.1	67.9	40.9
1480	49.2	25.9	70.2	43.6
1500	39.3	18.0	80.1	55.5
1550	19.8	5.7	110.2	92.7

*Index level= 1447.16

Table 6. The Implied Risk-Neutral Distribution for March, 2009 S&P 500 Index Options
That Precludes Violation of Put-Call Parity (PCP) and Pricing Implications

<i>Panel A. The Implied Risk-Neutral Distributions</i>				
Specification	Calls	Puts	PCP = 0	
R _f (%)	1.42	1.42	1.42	
σ (%)	20.89	24.73	22.10	
γ (skewness)	0.59	0.72	0.79	
DF (kurtosis)	3.75	2.62	4.71	
<i>Panel B. Price Comparisons</i>				
Strike Price* (\$)	Market Calls (\$)	PCP=0 Calls (\$)	Market Puts (\$)	PCP = 0 Puts (\$)
600	327.9	329.3	2.9	0.1
750	189.1	183.6	14.1	3.9
800	147.0	138.8	21.9	9.0
850	108.4	97.3	33.1	17.4
900	74.4	62.3	48.8	32.2
925	59.5	47.8	59.0	42.7
950	46.5	35.7	70.9	55.5
975	35.2	26.1	84.4	70.8
1000	25.5	18.6	99.7	88.2
1050	11.8	9.3	136.0	128.7
1100	4.3	4.9	178.3	174.2
1150	1.4	3.0	225.2	222.2
1200	0.5	2.1	274.1	271.1

*Index level= 927.45

Figure 1. 2008 Implied Volatilities. S&P 500 index = 1447.16.

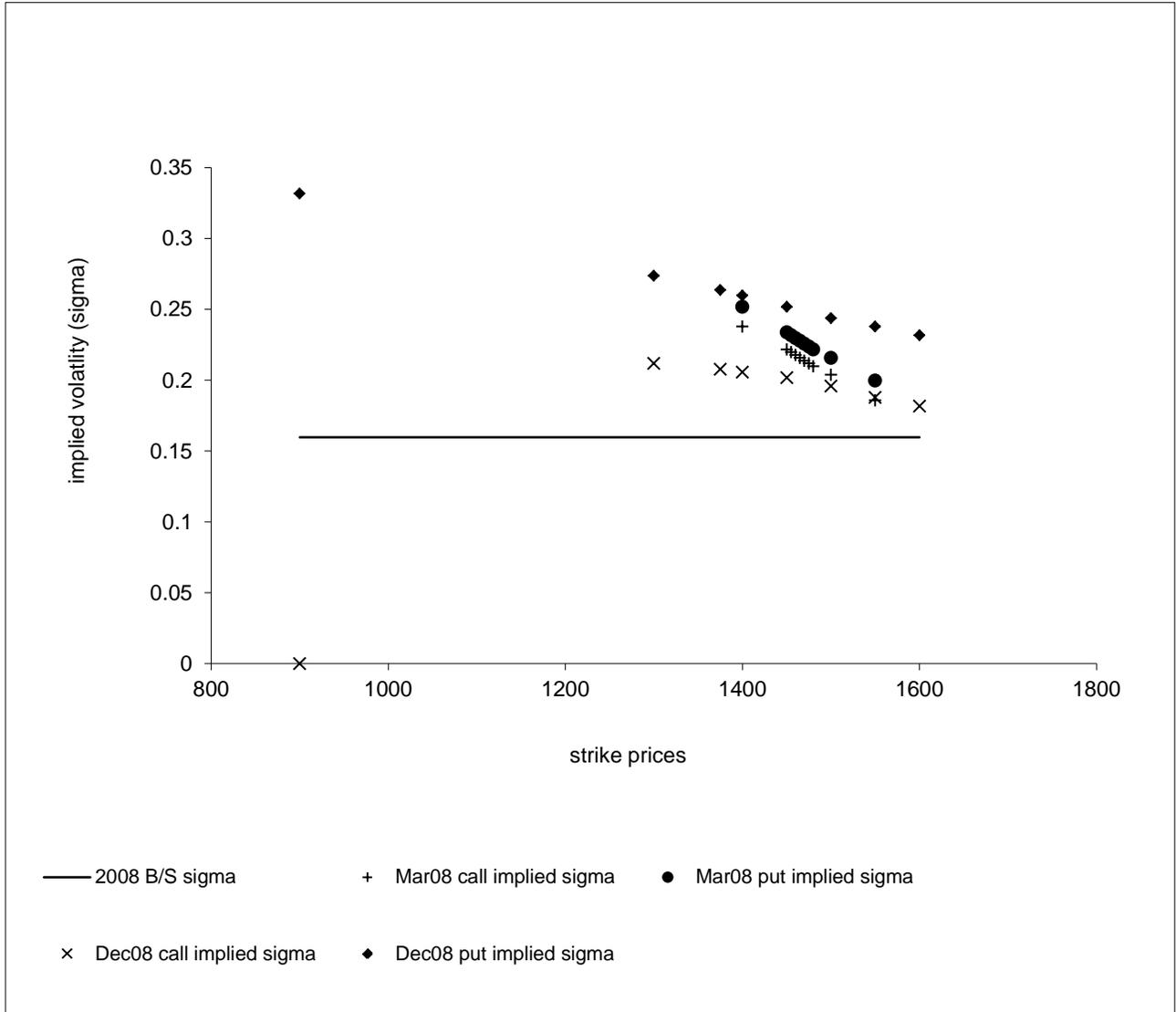


Figure 2. 2009 Implied Volatilities. S&P 500 index = 927.45.

