CAN TECHNICAL TRADING BE PROFITABLE - EVIDENCE FROM VOLATILITY MARKETS

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Abstract. In this paper we research whether reversion trading strategies can be profitable in volatility markets over time. We examine these strategies on VIX time series and short term futures, which trade based on the VIX index. We compare the performance of the technical strategies with a benchmark strategy of buy and hold. We find out that technical strategies outperform the benchmark even after applying a power utility with a relative high risk aversion. We conclude that contrary to the equity markets, the profitability of technical reversion strategies, when applied on VIX series, is pronounced even after taking the friction costs into account.

1. Introduction

The subject of volatility behavior rose to prominence following the celebrated Black Scholes formula for option pricing. Black and Scholes (1973) were first to derive a formula for option prices. In the formula the only unobservable input was the volatility of the underlying asset.

In spite of its importance, measurement of volatility remains challenging because it is time varying and may depend on different economic regimes. Many researchers have attempted to build market models to capture the variability of the volatility process and explain its properties as observed in the market.

An example of such process that is used to improve option valuation was constructed by Heston (1993). He assumes the following stochastic dynamics for S_t:

$$dS_t = \mu S_t dt + \sqrt{\nu_t} S_t d{W_t}^S \tag{1}$$

where ν_t is the instantaneous variance and obeys the CIR process:

$$d\nu_t = \kappa\left(\theta - \nu_t\right) dt + \psi \sqrt{\nu_t} d{W_t}^{\nu} \tag{2}$$

$d{W_t}^S, dW_t, \nu$ are Wiener processes with correlation ρ. Under this dynamics the market is incomplete. By adding more instruments we estimate the additional parameters of the Heston model one of them is the expected future volatility. we apply this estimation to price illiquid exotic options that are not traded in the market. Despite its appeal, challenges remain in the implementation of the Heston model. The main hurdles are the necessity to estimate the additional parameters of the model, other than the volatility itself and parameter's inherent instability.

Another major way to estimate volatility, is to assume certain GARCH models. This direction of research was initiated by Bollerslev (1986) following Engle (1982). Bollerslev (1986) assumes dynamics with the following error term: $\epsilon_t = \sigma_t z_t$, z_t is a white noise process while the series σ_t^2 are modeled as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i {\varepsilon^2}_{t-i} + \sum_{i=1}^{p} \beta_i {\sigma^2}_{t-i} \tag{3}$$

According to recent literature, it seems that in this type of models, the difference between model estimated and realized variances may predict stock market returns and its direction in short term horizons. If true it has large potential applications, because it will be a powerful tool to estimate the direction of the stock market and various economic variables in general.

A similar direction is the application of Fractional Brownian Motions to model volatility. Bollersev,J.Marrone,Lai Xu and Hao Zhou(1993) considered these type of models to improve predictability of volatility model. They were the first to estimate its Hurst exponent and discovered that it is consistently below $\frac{1}{2}$. initiated by Comte, Coutin and Renault (2003) attempted to improve the Heston model and introduced Fractional Brownian Motion into the Heston model. They showed that their model fits the behavior of long dated options well while preserving other main properties of volatility. The general direction of these papers was to construct models that predict future volatility and apply them to improve option pricing and possibly predictability of the stock market. This line of research was continued recently by Gatheral, Jaisson and Rosenbaum (2014). In the paper they looked on the high frequency data of realized volatility for the last 4 years (2010-2014). Estimating the Hurst exponent of the log of the realized volatility they show that these series possess strong mean reversion properties. They arrive to the exponent of 0.1 for the high frequency data set they possess.

The general conclusions of the paper by Gatheral, Jaisson and Rosenbaum points to mean reversion in the volatility process. Intuitively this indicates that certain strategies that exploit mean reversion should be able to outperform other strategies like buy and hold.

We justify this intuition by considering a simple one period model that shows this. The model we consider doesn't rely on a specific presentation and applicable to any process that has a reasonable mean reversion (or momentum) properties. Conclude that these strategies should be palatable to it and out-perform simple buy and hold (for example for AR(1) process)

Reversion strategies are a recurring theme in equity and bond analysis research. Because technical trading isn't violates the efficient market hypothesis, many researchers attempted to determine whether technical trading of any kind produces abnormal returns in the stock market, for example, outperform buy and hold in a statistically significant way. Brock Lakonishok and Le Baron(1992) examine technical rules in sample for returns data of Dow Jones industrial and discover that technical rules outperform benchmark models of buy and hold. The rules they consider are moving average rules similar to reversion rules in this note. The analysis they carried was done in-sample and didn't address the question of data snooping. Sullivan, Timmerman and White(1999) expanded the research of technical trading rules. In their paper they design statistical test to address the data snooping bias that Brock, Lakinoshol and Le Baron (1992) raised in their study. Sullivan, Timmerman and White(1999) performed out of sample analysis of technical rules and found out that the reversion strategies didn't out-perform buy and hold in out of sample time frames. These papers don't explain how to construct a trading strategy that a real investor can implement. For example it may occur that multiple selling signals follow each other without a buy signal in between and the authors don't articulate investor's behavior in such a case. This type of analysis may be significant because in real world investments are made for a fixed time horizons and questions of this type may change the strategy performance. We explain in our paper one possible way to incorporate technical rules into a viable trading strategy that investor follows.

In another research direction reversion were applied to construction of market-neutral portfolios. These have a total market value if 0 and consist of long short equity investments.

Following Lo and Mckinlay (1990) who considered such strategies based on firm size, a recent paper by Raman Uppal, De Miguel and Nogales (2012) assumes a VAR (Vector AutoRegressive) model for stocks. The authors construct a portfolio that assigns positive weights to stocks whose expected conditional returns are higher than the return of an equally weighted portfolio and negative weights to the rest of the stocks. They show that such portfolio has a positive expected return. This strategy outperforms other strategies, for example, the equally weighted portfolios considered in the paper by Uppal, Garlappi and DeMiguel (2009).

As far as we are aware we are the first to research how technical rules perform on the VIX series and their futures. Ait Sahalia,Karaman and Mancini (2012) constructed a strategy resembling technical trading but which is model based: In their paper they conclude that the strategy they construct is profitable as evident from Sharpe ratios in table 7. Their work is essential because, the model they construct may provide a clue for the phenomena we discover in our paper.

The reversion strategy we employ is model free and relies on past VIX values and its futures and we are not attempting to construct a model of the VIX dynamics as done in Gatheral, Jaisson and Rosenbaum (2014).

At the beginning of the paper we define the FBM model and explain the method we use to measure the Hurst exponent of VIX time series and its futures. We discover that the Hurst exponent associated with the series is below $\frac{1}{2}$. For VIX futures we estimate different Hurst exponents based on their maturity horizon. We discovered that the Hurst exponent of returns (more precisely the *log* level of the future) of futures with short horizons seems to be less than $\frac{1}{2}$. This presents a certain puzzle as returns of the instruments that are actively traded in the market are supposed to be martingales and the Hurst exponent should be close to $\frac{1}{2}$. We use simulation to reject the hypothesis that $H = \frac{1}{2}$ for VIX and VIX futures using the P- value we determine.

In section 3 we derive a formula for the expected return of a one period look-back strategy for FBM process whose $H \leq \frac{1}{2}$, versus Buy-and-Hold. It follows that strategies we consider have a good chance of being profitable and beat the buy and hold benchmark. In the following sections we investigate whether this is true: We calculate some statistical metrics of our reversion strategies with different parameters. We compare these with buy-and-hold. All the tests are out of sample and rely on the methods introduced in the paper by Uppal, De Miguel and Nogales (2012). Out of sample is important, because an investor at a certain time t will not have any information about time $T \geq t$. It follows that in-sample analysis results assume the knowledge if the future at time $t \leq T$

To mimic actual trading environment we introduce leverage into our strategy. We discover that application of leverage with a reversion strategy can lead to significant losses and hence not advisable, however this doesn't explain the Hurst exponent puzzle of VIX futures because traders can fully fund their margin accounts as they trade the futures.

We finalize our paper with further discussions of our finding and outlining potential future work.

2. Fractional Brownian motion and VIX

Mandelbrot introduced Fractional Brownian to generalize the usual Gaussian motion. While the usual Brownian motion postulates that for an increment dt the corresponding stochastic process is normally distributed with a Variance being proportional to dt, Fractional Brownian motion assumes that the Variance of the stochastic process is proportional not to dt but to dt^{2H}. More formally:

Definition 2.1. A Gaussian process $B^H = \left\{ {B^H}_t, t \geq 0 \right\}$ is called Fractional Brownian Motion (FBM) of Hurst parameter $H \in (0,1)$ if it has mean 0 and covariance of

$$E\left(B_t^H B_s^H\right) = \frac{1}{2}\left(s^{2H} + t^{2H} - |t-s|^{2H}\right)$$

The Brownian motion is self similar and has the property that

$$E(B_t^H - B_s^H)^2 = |t-s|^{2H}.$$

Hence we see that the FBM is a generalization of the usual Brownian motion as the latter corresponds to $H = \frac{1}{2}$. Contrary to the usual Brownian motion, it follows that FBM doesn't have mutual independence between disjoint intervals in $[0,T]$. More precisely we have a mean reversion phenomenon (a down move is more probable after an up move) if $H < \frac{1}{2}$ and a momentum (an up move will be most likely to follow an up move) if $H > \frac{1}{2}$. Intuitively this implies that a strategy that uses this momentum should outperform a buy-and-hold strategy for processes that follow FBM. Our goal is to measure the Hurst exponents for different assets and explore whether our strategies (that are paper trades) can be more profitable than buy-and-hold strategies. We'll explain the methodology of measuring the Hurst exponents for time series and then produce a table with the results of our measurements.

2.1. **Measuring Hurst Exponent.** There are numerous representations of Hurst exponents measurement in the market. Usually the implementations of the algorithms are quite sophisticated and their precisions can vary. In order to determine which algorithm is the most suitable to measure Hurst exponents of potential time series we performed the following actions:

- We simulated FBM with different Hurst coefficients using R programming language for 1000 paths
- For each path we applied different measurements to calculate its Hurst exponent
- We selected the algorithm who had the closest expected value and minimal standard deviation of the Hurst exponent of the simulation
- We use this selected algorithm to measure Hurst exponents of different time series we investigate

Based on the methodology above we selected the DFA algorithm to measure Hurst exponents of different time series. De-trended Fluctuation Analysis attempts to measure the power proportionality of the variance versus the length of the interval in a direct manner. If a random walk $X_i, i = 1...t$ is given, we divide the sample of length t into sub samples Y_j of length L each. For each of the sub samples we calculate a linear trend $aj + b$ by minimizing the following expression:

$$E^2 = \sum_{j=1}^{L} (Y_j - aj - b)^2 \tag{4}$$

Define $F(L) = \left(\frac{1}{L}\sum_{j=1}^{L}(Y_j - aj - b)^2\right)^{\frac{1}{2}}$ (which is a deviation from the trend). We create a log log graph of L versus $F(L)$ considering different samples of length L. If the log log plot is a straight line we have that $F(L) \propto L^{\alpha}$. Then α equals to Hurst exponent. This algorithm was implemented in R. Using this implementation we measure the Hurst exponents for the time series of the following assets:

- SP 500 1957-1990 and 1990-2012
- VIX series 1957-1990 and 1990-2012
- Volatility time series 1957-1990 and 1990-2012

We chose the sub sample period of 1957-1990,since VIX was introduced in 1990 and we desired to verify that mean reversion of volatility and its Hurst exponent wasn't a result of regime change in the trading environment. We chose 30 day rolling window to measure the SP return time series at any given time. We annualized the monthly volatility we measured. We compared the Hurst exponent of the synthetic volatility with the Hurst exponent of the the SP 500 price levels, (based on its closing price during 1950-2012) . We summarize the results of the Hurst exponents for the time series and the periods we investigate in table (1): [2] The next table details the

Table 1. Hurst exponent results for Volatility and SP time series

Date	SP	Vol Series	VIX
1950-1990	0.45	0.35	N/A
1990-2012	0.47	0.44	0.33

Hurst exponents of VIX futures and their log for the entire period (2004-2015). The index for VIX futures that we used are the Bloomberg generic levels available for the different VIX futures contract with expiry of $1, 2, 3, 6$ month respectively. The first index represents the closest future to the current spot level of the VIX index. The second index represents the rolling price of the two months contract and similarly for other indices as well. The table below represents the log level of the different generic index levels. The first observation we have is that the term

Table 2. This table shows the Hurst exponent estimate of log levels of VIX futures

Date	1 Month	2 Month	3 Month	6 Month
2004-2014	0.39	0.42	0.46	0.51

structure of Hurst exponent is upward sloping. The mean reversion is the most eminent on the short end of expiry (close to the spot VIX index itself.) For longer expiry horizon we have processes that resemble a random walk.

Now assume P is the dynamics process of VIX futures. We have that

$$\frac{dP}{P} = \mu dt + \sigma(t) dW_t \tag{5}$$

Applying Ito's lemma we get that :

$$logP = \int_0^t (\mu - \sigma^2(t))dt + \int_0^t \sigma(t) dW_t \tag{6}$$

From this we see that the correct estimate of the Hurst exponent for returns time series should be done for the logarithm of price level that determines the return series. Because of the mean reversion of Hurst exponent the one month future

[2] Jim Gatheral communicated that the estimate of the Hurst exponent of VIX is consistent with his estimate

generic sequence reversion strategies should have a better performance than buy and hold strategy. We remark also that this implies that the generic time series of VIX futures doesn't belong as a martingale similar to the VIX index itself.

3. Hurst exponent confidence intervals and historical stability

In this section we examine the historical distribution of Hurst exponents for stock returns and the underlying rolling realized volatility. Our goal is to research whether the Hurst exponent of volatility is stable during different economic time periods and we like to contrast it with the behavior of the overall stock prices. We perform the following steps:

- We choose a rolling window of N days
- For series of length T we simulate k random uniform numbers between 1 to $T - N$
- for each simulated number t_i we calculate the Hurst exponent between x_{t_i} to x_{t_i+N}

As a result we obtain k Hurst exponents, which give a proxy of the distribution of Hurst exponents for our time series. If Hurst aren't stable, for example they fluctuate above and below 0.5, a strategy that assumes just mean reversion will fail. We investigate the possibility of the strategy being a stable one. Because of the nature of the DFA algorithm and its scaling, we are forced to choose large periods to measure the Hurst exponent reliably. We set $N = 5000$ and present Hurst exponents measurement during 1950-2012 for synthetic volatility series and SP 500 adjusted closing price series. We choose $k = 100$ and simulate uniform distribution within the interval $[1, 10000]$. We measure the Hurst exponent for SP 500 price series and VIX series for the time series $x_{t_k}...x_{t_k+N}$. The results of these time simulations are presented in the table (2): This table shows that over the entire

Table 3. Hurst exponent for 5000 days. Initial day is selected randomly

	Min	1 st Quantile	Median	Mean	3 rd Quantile	Max
Synthetic Volatililty	0.35	0.41	0.43	0.43	0.45	0.49
SPX	0.43	0.49	0.51	0.50	0.51	0.56

period (1950-2012) SP 500 Hurst exponent fluctuated between momentum and mean reverting strategy and most of the time the price dynamic of SP resembled a random walk. But the Hurst exponent of volatility time series has been consistently less than 0.5. We conclude that volatility is mean reverting during this period and furthermore the mean reversion property is invariant historically. In the next section we develop a simple model to show how an investor can possibly take advantage of this phenomenon using a simple look-back strategy.

3.1. **Testing for Random walk using the Hurst exponent.** It's possible that in spite of the Hurst exponent below or above 0.5 we can still have a random walk. For example we may have a Brownian motion but yet the probability of having a Hurst exponent consistently greater or smaller than 0.5 is high for certain time periods. This is similar to the following: Throw a fair coin large number of times. The probability of an event that consecutive throws having same outcome will be high. We proceed to exclude this possibility. We use a simulation based approach for that and perform the following steps:

(1) Simulate a normal random walk process for N times
(2) We measure the Hurst exponent in each path to obtain a Hurst exponent distribution
(3) We calculate the 95 percentile to obtain the probability that our Hurst exponents will be a measured Hurst exponent for a random walk process.

TABLE 4. Results of Hurst exponent measurement on a simulated random walk

Mean	Stdev	95% Percentile
0.50	0.02	0.45

Table (3) shows that although we have a random walk the measured Hurst exponent is close to 0.4 for the entire period is less than 5 percent according to the results of our simulation. Thus we for VIX series we can reject the hypothesis that the VIX series is a random walk with a confidence of 95%. Next section presents a simple model that justifies the expectation that a reversion strategy that a technical strategy applied to a process with Hurst exponent exponent consistently than $\frac{1}{2}$ produces positive returns versus a buy and hold benchmark.

4. Look back strategies for a mean reverting process

4.1. **General survey.** In this section we present a model that shows that for a mean reverting process we expect technical trading strategies to outperform a buy and hold strategy. The model we present includes more general processes than mean reverting processes. We conclude based on the Sharpe ratio we calculate that even if the Hurst exponent is not stable and time varying through different economic regimes we still expect technical trading to be profitable for such a process as long as the Hurst exponent is consistently less equal 0.5.

Consequently the stability we require in this case is a stability of Hurst exponent ranges, rather the number of itself. As we showed in previous section Hurst exponent of the VIX series is consistently below 0.5. This justifies theoretically our motivation to explore technical trading performance for VIX time series and our expectation of stable positive profit versus buy and hold strategies.

Additionally the model provides sensitivity analysis, and, expresses the Sharpe ratio of our strategy as a function of the Hurst exponent. This should be important for future theoretical investigations. The model we present is a one period model but the technical trading we employ is multi period. This makes a direct comparison of the strategy outcome with the model outcome challenging. Nevertheless we believe the model illuminates some of the quantitative features of the strategy and that's the reason we present it here.

4.2. **The model.** Let $x = (x_1, x_2)$ be a two dimensional normally distributed variable with mean 0, vector of standard deviations $\sigma = (\sigma_1, \sigma_2)$ and correlation $\rho \leq 0$. Assume that the state x_1 records the initial state of the asset at t_1 and x_2 records the return of asset between time t_1 and t_2. Employ the following trading strategy by looking at the state at t_1 :

(1) If $x_1 \geq 0$ then sell the asset
(2) If $x_1 \leq 0$ then buy the asset

The P&L of the asset will be the value of random variable x_2 returned at time t_2. Thus if we've sold the asset the return will be $-x_2$ and if we've bought the asset the return will be x_2. For the usual setup of conditional distributions we know that: x_2 is distributed normally given x_1 with the following parameters:

$$E(x_2|x_1) = \rho \frac{\sigma_{x_2}}{\sigma_{x_1}} x_1$$

and

$$Var\left(x_2|x_1\right) = \sigma_{x_2}^2\left(1-\rho^2\right).$$

But in our case because the sign flips if we sell the asset the distribution will have the same variance but the expected value will be:

$$E(x_2|x_1) = -\rho \frac{\sigma_{x_2}}{\sigma_{x_1}} x_1$$

when $x_1 > 0$. This is a random variable in x_1 and hence the expected return of the strategy will be:

$$-\frac{1}{\sqrt{2\pi}\sigma_{x_1}} \int_{x_1>0} \rho \frac{\sigma_{x_2}}{\sigma_{x_1}} x_1 \exp\left(-\frac{x_1}{2\sigma_{x_1}^2}\right) dx_1 + \frac{1}{\sqrt{2\pi}\sigma_{x_1}} \int_{x_1<0} \rho \frac{\sigma_{x_2}}{\sigma_{x_1}} x_1 \exp\left(-\frac{x_1}{2\sigma_{x_1}^2}\right) dx_1$$

Taking the common terms out of the parentheses we will get that:

$$\frac{1}{\sqrt{2\pi}} \rho \frac{\sigma_{x_2}}{\sigma_{x_1}^2} \left(-\int_{x_1>0} x_1 \exp\left(-\frac{x_1^2}{2\sigma_{x_1}^2}\right) dx_1 + \int_{x_1<0} x_1 \exp\left(-\frac{x_1^2}{2\sigma_{x_1}^2}\right) dx_1\right)$$

Thus we are reduced to evaluate :

$$\frac{1}{\sqrt{2\pi}} \rho \sigma_{x_2} \left(\left[\exp\left(-\frac{x_1^2}{2\sigma_{x_1}^2}\right)\right]_0^\infty - \left[\exp\left(-\frac{x_1^2}{2\sigma_{x_1}^2}\right)\right]_{-\infty}^0\right) = -\sqrt{\frac{2}{\pi}} \rho \sigma_{x_2} \geq 0$$

because $\rho < 0$.

Now it's turn to calculate the standard deviation. I will use the law of total variance that states that for variable X and Y we have that:

$$Var(Y) = E\left(Var\left[Y|X\right]\right) + Var\left(E\left[Y|X\right]\right).$$

Since $Var(x_2|x_1) = \sigma_{x_2}^2\left(1-\rho^2\right)$ is constant we will get that $E(\sigma_{x_2}^2\left(1-\rho^2\right)) = \sigma_{x_2}^2\left(1-\rho^2\right)$ and $Var\left(E\left[x_2|x_1\right]\right) = \rho^2\sigma_{x_2}^2$. It concludes from the total law of variance that the standard deviation is : σ_{x_2}. Thus the Sharpe ratio of the strategy is: $-\sqrt{\frac{2}{\pi}}\rho$. Now we apply this to the Hurst exponent case. Assume that the interval between consecutive trades is $\frac{1}{N}$. Hence we have:

(1) The variance corresponding to the trade at t_2 is: $\left(\frac{1}{n}\right)^{2H}$.
(2) The covariance between successive intervals t_{n-1} and t_n is: $\frac{1}{n^{2H}} \frac{\left(2^{2H-1}-1\right)}{2}$
(3) The Sharpe ratio is: $\frac{1}{\sqrt{2\pi}} | \left(2^{2H-1}-1\right) |$
(4) Note: It doesn't depend on the length of the interval
(5) The strategy remains profitable if $H \geq \frac{1}{2}$ or $H \leq \frac{1}{2}$ through the entire time period.

[3]Using this model as a motivating example we perform a simple look-back strategy on our VIX series.

5. Look back strategy and VIX

We saw in previous sections that the Hurst exponent is consistently below 0.5 for VIX process. Based on the theoretical model we presented we expect that it is profitable to apply look-back strategies on the VIX time series. We modify the strategy above to adept to level series (note that in the model we developed our series were return series). For VIX return series the power law structure is destroyed and hence no look-back strategy works for them. We apply the strategy on VIX level itself and not on its functional modification (like the logarithm of the time series) because we like to be consistent with the previous studies on technical trading. In those studies technical rules were applied directly on the level of the SP time series directly. To perform our look-back strategy select a fixed number of days N. Assume $v_1, ..., v_t, ..., v_T$ is the time series of the VIX. For each v_t calculate:

(1) $V_{max} = max(v_{t-1}, v_{t-2}, v_{t-3}...v_{t-N})$
(2) $V_{min} = min(v_{t-1}, v_{t-2}, v_{t-3}, ...v_{t-N})$

Compare v_t with V_{max} and V_{min} and perform the following trading actions:

(1) if $v_t > V_{max}$ and I *didn't short* the VIX then I either short it or sell my long shares. Otherwise do nothing.
(2) $v_t < V_{min}$ and I *didn't long* the VIX then I either buy shares or close my short position. Otherwise do nothing.

Our goal is to investigate the performance of this strategy during two historical periods:

(1) entire period 1990-2012
(2) 2004-2012

The reason for choosing this second period is the introduction of VIX future contracts and thus the ability to implement the strategy in real life using futures contracts. We would like to see whether the introduction of simple ways of trading the futures instrument on the VIX leading to removal or reduction of these trading opportunities. We calculated the daily return of this strategy assuming our investment in cash yields 0% gain. The returns are calculated daily (not compounded) hence they are what the trader will look at to decide whether or not to short or long the VIX index. We like to emphasize the following features of our paper trade:

(1) No doubling down
(2) Self financing

The first item implies that if we're already shorting the instrument because of a trading signal and more signals are coming through that imply further shorting we are not going to short more of it. Thus the next action after VIX shorting will be closing the short once a positive signal comes through. Similar comment applies to a signal to buy the instrument. We will avoid any further buying and our next action will be closing the position once a sell signal comes through. Our self financing strategy shows that we don't inject cash into positions in the middle of the trading strategy. Thus initial investment comes at the beginning of the period and **no**

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[3]We performed simulations to verify the formulas. Results of the simulations showing the derivations are correct are available upon request

further investments or cash contribution are allowed. If M_t is the money available for us after time t this money come from investing 1 dollar into the strategy and continuously trading with no cash contributions coming in. Our calculation of the return is in the usual manner. Thus if P_t is the price of the index at time t and P_{t+1} is the price of the index at $t+1$ the return is:

$$\frac{P_{t+1} - P_t}{P_t}$$

If we **short** the index the return we calculate is:

$$\frac{P_t - P_{t+1}}{P_t}$$

The next table presents results of the look-back strategy for the period 1990-2012 with different look back days:

Table 5. General Properties of Return Distribution 1990-2012 paper trades

	Maximal Daily Loss	1st Qu.	Median	Mean	3rd Qu.	Maximal Daily Gain
LookBack Days 3	-64.22%	-1.12%	0.00%	0.46%	2.32%	51.72%
LookBack Days 4	-64.22%	-1.33%	0.00%	0.51%	2.58%	51.72%
LookBack Days 5	-64.22%	-1.33%	0.00%	0.56%	2.72%	51.72%
LookBack Days 6	-64.22%	-1.54%	0.00%	0.51%	2.76%	51.72%
LookBack Days 7	-64.22%	-1.57%	0.00%	0.52%	2.85%	51.72%
LookBack Days 8	-64.22%	-1.68%	0.00%	0.48%	2.89%	51.72%
LookBack Days 9	-50.00%	-1.68%	0.00%	0.50%	2.88%	51.72%
LookBack Days 10	-50.00%	-1.83%	0.00%	0.44%	2.85%	51.72%
LookBack Days 11	-50.00%	-1.83%	0.00%	0.41%	2.80%	51.72%
LookBack Days 12	-50.00%	-1.86%	0.00%	0.41%	2.82%	51.72%
LookBack Days 13	-50.00%	-1.92%	0.00%	0.41%	2.84%	64.22%
LookBack Days 14	-50.00%	-1.94%	0.00%	0.42%	2.87%	64.22%
LookBack Days 15	-51.50%	-1.96%	0.00%	0.38%	2.84%	64.22%
LookBack Days 16	-51.50%	-2.07%	0.00%	0.38%	2.95%	64.22%

The table shows the general return distributions of the strategy for different look-back days. We can see that in general we can make around 40 basis points on a daily basis trading. These are substantial gains on an annual basis. However an investor who employs this strategy must be ready to withstand a potentially substantial daily loss. A daily swing of the VIX can be between in tens of percentages. Our next table presents the annual Sharpe return for the look-back strategy with different look-back days: We see that the returns of the strategy are highly volatile but the Sharpe ratios for our strategies are high.

6. Buy-and-Hold comparison

In this section we compare our strategies with a simple buy-and-hold strategy of investing and holding the VIX for the period from 1990 to 2012. The table describing the general properties of buy-and-hold strategy for the VIX is given below: The buy-and-hold strategy appears less risky than the corresponding look-back strategies. In particular we see that maximal loss we have on a daily basis

Table 6. Annualized Sharpe of Strategy Returns with different Look back window 1990-2012

	Annualized Mean	Standard Deviation	Sharpe Ratio Annualized
LookBack Days 3	116.05%	83.10%	1.40
LookBack Days 4	127.92%	84.21%	1.52
LookBack Days 5	140.57%	84.66%	1.66
LookBack Days 6	129.40%	86.57%	1.49
LookBack Days 7	131.80%	87.21%	1.51
LookBack Days 8	121.16%	87.41%	1.39
LookBack Days 9	124.84%	86.33%	1.45
LookBack Days 10	111.46%	86.87%	1.28
LookBack Days 11	103.17%	87.79%	1.18
LookBack Days 12	103.72%	88.16%	1.18
LookBack Days 13	102.77%	89.60%	1.15
LookBack Days 14	105.69%	89.72%	1.18
LookBack Days 15	95.05%	90.47%	1.05
LookBack Days 16	96.74%	90.93%	1.06

Table 7. VIX Buy and Hold Strategy

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
-29.57%	-3.49%	-0.31%	0.19%	3.16%	64.22%

is -29% versus -64% with the look-back strategies. According to the model we presented above we should expect that our look-back strategies perform better than the buy-and-hold. To quantify this we form a total return index for the strategies and observe the index at the end of the period. This tells us how much total return we will make if we follow the looking back strategy or the buy-and-hold strategy for the VIX. The results of this computations are given in the attached table: This

Table 8. Buy-and-Hold Comparison Versus a Look back strategy

	Cumulative Return
Look Back 3	455.95
Buy-and-Hold	0.96

table clearly shows that the simple look-back strategy we performed is much more profitable than any buy-and-hold strategy on the VIX. Furthermore, the strategy appears to be highly profitable prior to the crisis in 2007 and the total return index drops from 52 to 18 in a period of one day when the VIX loses sixty four percent of its value. But it takes the strategy less than a year to recover from this loss.

The profitability of the strategy implies the strong mean reversion property of the VIX series and it is supported by the simple one period model we presented. Table (8) in this section will be the comparison with the look-back of SP 500. The Hurst exponent we measured for the SP 500 is closer to 0.5 and hence we

Table 9. Strategies Sharpe ratios, VIX and S&P 1990-2012. The last column is the S&P Buy and Hold Strategy

LookBack Days	Sharpe VIX	Sharpe SP	Sharpe Ratio (VIX/SP)	Sharpe Ratio (VIX/ S&P B& H)
3	1.40	0.58	2.42	3.41
4	1.52	0.46	3.33	3.70
5	1.66	0.48	3.47	4.05
6	1.49	0.36	4.21	3.65
7	1.51	0.38	3.95	3.69
8	1.39	0.31	4.53	3.38
9	1.45	0.38	3.81	3.53
10	1.28	0.23	5.53	3.13
11	1.18	0.20	5.75	2.87
12	1.18	0.16	7.43	2.87
13	1.15	0.20	5.75	2.80
14	1.18	0.16	7.56	2.87
15	1.05	0.18	5.74	2.56
16	1.06	0.23	4.64	2.59
17	1.03	0.29	3.56	2.50
18	0.98	0.25	3.90	2.40
19	1.01	0.23	4.37	2.46
20	1.00	0.15	6.75	2.44
21	0.93	0.13	7.12	2.28
22	0.95	0.02	38.67	2.33

don't expect to achieve similar gains by using a look-back approach on it. The last column of the table quantifies the relationship between VIX Sharpe and SP 500 buy-and-hold strategies. We can see from the corresponding Sharpe ratios that the buy-and-hold strategy on the SP 500 is on par with the look-back strategies. This is contrary to the case of the VIX when the reversion strategies are outperforming the buy-and-hold strategy by a wide margin.

7. Reversion Strategies and VIX Futures

In this section we test our strategies on the actual instruments that are traded based on VIX, namely VIX futures. VIX futures started trading in 2004 the CBOE. One VIX futures contract has 1000 dollars of notional. Hence if the futures contract is 16.00 dollars and the final price is 17 dollars the buyer (or seller of the contract) will get on hand: $1000 \times (17-16) = 1000$. Futures contracts have margin requirements but we fully fund the contract when we short or long it. We incorporate bid ask information available in BB. We compare trading two time series:

(1) Generic short term series contract time formed by Bloomberg (Symbol UX1 Index)
(2) Futures contract monthly time series available at the CBOE website.

For reader's convenience we remind the reader of the Hurst exponent of the short term series: The mean reverting feature is present in the real time series trading and hence it shows that it might be possible to perform our technical strategy on

Table 10. Hurst Exponent Measurement for the monthly VIX and Generic Future Contract

Series	Hurst Exponent
One month Future	0.39
Generic Series	0.4

it. We incorporate transaction costs by introducing the bid ask spread. Namely the trading strategy will be as followed:

(1) Generate the signal using the Bid series for generic contracts
(2) Initiate a buy using the ask price (thus the higher price)
(3) Initiate a sell using the bid price (use the lower price)

The results of the strategy are presented in the table:

Table 11. General results for look-back strategy with transaction costs

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	Standard Deviation	Sharpe
LookBack Days 3	-25.00%	-1.38%	0.00%	0.26%	1.27%	30.31%	4.17%	1.01
LookBack Days 4	-25.00%	-1.62%	0.00%	0.22%	1.49%	30.31%	4.29%	0.81
LookBack Days 5	-25.00%	-1.67%	0.00%	0.20%	1.50%	30.31%	4.49%	0.72
LookBack Days 6	-28.77%	-1.80%	0.00%	0.14%	1.64%	30.31%	4.52%	0.49
LookBack Days 7	-25.00%	-1.75%	0.00%	0.22%	1.78%	30.31%	4.65%	0.76
LookBack Days 8	-25.00%	-1.76%	0.00%	0.22%	1.94%	30.31%	4.61%	0.76
LookBack Days 9	-25.00%	-1.79%	0.00%	0.25%	1.98%	30.31%	4.58%	0.87
LookBack Days 10	-25.00%	-1.94%	0.00%	0.17%	1.95%	30.31%	4.63%	0.57
LookBack Days 11	-25.00%	-1.96%	0.00%	0.14%	1.91%	30.31%	4.68%	0.48
LookBack Days 12	-25.00%	-2.02%	0.00%	0.13%	1.87%	30.31%	4.69%	0.44
LookBack Days 13	-25.00%	-2.03%	0.00%	0.14%	1.86%	30.31%	4.72%	0.46

To gauge the impact of transaction costs as manifested through the bid ask spread we adjust our algorithm eliminating buy high sell low resulting from transaction costs and assumed that we have just one series that we work with. The results of this strategy from are given below: Let us inspect both tables with and without transaction costs respectively. While the volatility of the daily return of both time series (with and without transaction costs) stays the same we can see that Sharpe ratios is reduced by around 30 percent if transaction costs are incorporated. However, with the presence of transaction costs the strategy still produces significant positive returns. The average return for different look-back days with the presence of transaction costs is around 26 basis points. This is translated roughly to 65% of annual gains. Investors should be able to withstand huge market swings if they like to invest in this type of strategy. A daily loss of 25 percentages in the monthly VIX series is plausible according to the analysis above. performance for investors in the time horizon of two years.

7.1. **Portfolio turn over.** We researched portfolio turnover for our strategies. Our definition of portfolio turnover is the average of the absolute value of trades

Table 12. General results of the strategy without transaction costs (one month futures) 2004-2012

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	Standard Deviation	Sharpe
LookBack Days 3	-25.43%	-1.08%	0.00%	0.37%	1.27%	31.04%	4.28%	1.36
LookBack Days 4	-25.43%	-1.28%	0.00%	0.31%	1.49%	31.04%	4.50%	1.09
LookBack Days 5	-25.43%	-1.36%	0.00%	0.28%	1.50%	31.04%	4.52%	0.99
LookBack Days 6	-29.45%	-1.50%	0.00%	0.22%	1.65%	31.04%	4.66%	0.75
LookBack Days 7	-25.43%	-1.46%	0.00%	0.30%	1.70%	31.04%	4.61%	1.02
LookBack Days 8	-25.43%	-1.47%	0.00%	0.28%	1.78%	31.04%	4.58%	0.98
LookBack Days 9	-25.43%	-1.52%	0.00%	0.31%	1.84%	31.04%	4.64%	1.07
LookBack Days 10	-25.43%	-1.69%	0.00%	0.25%	1.87%	31.04%	4.69%	0.84
LookBack Days 11	-25.43%	-1.73%	0.00%	0.21%	1.82%	31.04%	4.70%	0.70
LookBack Days 12	-25.43%	-1.77%	0.00%	0.20%	1.83%	31.04%	4.74%	0.68
LookBack Days 13	-25.43%	-1.78%	0.00%	0.22%	1.84%	31.04%	4.78%	0.72

across N available assets:

$$Turnover = \frac{1}{T-M} \sum_{i=1}^{T-M} \sum_{j=1}^{N} |\widehat{w}_{k,j,t+1} - \widehat{w}_{k,j,t}| \tag{7}$$

The results of the comparison of the turnover are given below:

Table 13. Turn over comparison between look-back and optimal portfolio

	365 days
LookBack Days 3	46.22%
LookBack Days 4	34.57%
LookBack Days 5	28.15%
LookBack Days 6	23.29%
LookBack Days 7	21.18%
LookBack Days 8	18.89%
LookBack Days 9	17.15%
LookBack Days 10	15.31%
LookBack Days 11	14.21%
LookBack Days 12	12.75%
LookBack Days 13	12.38%

Given the high turnover of the look-back strategy it is natural to ask whether this can be remedied. For that we introduce norm constrained portfolios. That is to say we are not going to invest our entire wealth into the strategy but just part of it while the rest will be invested in Treasury Bill. We display results of the strategy when we invest $0.75, 0.5$ of our money in the strategy respectively while the other portion is invested in Treasury Bill. The characteristics of the investments are given in tables (17),(18):

Table 14. Main characteristics for out of sample for 75% investing in VIX and the rest in treasury

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
lookback 3	-44.64%	19.85%	126.90%	160.50%	261.10%	980.20%
lookback 4	-21.85%	27.56%	74.04%	104.90%	157.70%	397.70%
lookback 5	-23.02%	17.97%	40.86%	72.86%	130.90%	323.60%
lookback 6	-50.54%	-6.19%	21.82%	44.21%	89.60%	262.30%
lookback 7	-58.58%	1.98%	59.59%	59.93%	105.20%	418.30%
lookback 8	-61.18%	12.21%	56.81%	65.05%	111.40%	527.20%
lookback 9	-66.48%	23.10%	59.85%	60.03%	103.90%	502.60%
lookback 10	-60.89%	21.08%	57.62%	58.89%	83.72%	666.60%
lookback 11	-67.32%	3.82%	32.28%	41.58%	61.90%	583.00%
lookback 12	-73.69%	-7.68%	25.12%	28.08%	59.71%	470.40%
lookback 13	-75.36%	-10.74%	29.98%	42.19%	75.35%	706.80%

Comparing both tables we see the following properties:

- The gains on the upside and downside for the 50% investment in VIX are roughly $\frac{3}{4}$ of that for the 75% investment in VIX
- The downside for the reduced investment is subdued. This potentially appeals to risk averse investors.

Table 15. Main characteristics for out of sample for 50% investing in VIX and the rest in treasury

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
lookback 3	-27.97%	18.62%	81.61%	91.51%	147.10%	424.00%
lookback 4	-11.73%	24.90%	53.12%	66.45%	99.14%	205.80%
lookback 5	-10.04%	19.31%	33.46%	49.62%	86.49%	176.40%
lookback 6	-32.66%	0.53%	21.73%	32.16%	62.35%	156.60%
lookback 7	-40.23%	6.81%	43.97%	41.47%	73.40%	224.80%
lookback 8	-42.85%	13.32%	43.21%	43.83%	74.32%	267.60%
lookback 9	-47.99%	20.58%	44.72%	41.60%	71.08%	259.40%
lookback 10	-42.30%	19.64%	43.35%	40.87%	58.78%	322.50%
lookback 11	-48.69%	7.81%	27.23%	30.32%	46.50%	291.10%
lookback 12	-55.68%	0.59%	22.92%	22.28%	44.58%	248.40%
lookback 13	-57.50%	-3.13%	26.91%	30.00%	54.45%	337.90%

7.2. **Turnover of Constrained Portfolios.** In the last subsection we present the average turnover of our constrained portfolios. They are given in the following table:

Table 16. Turn over of Normed Portfolio for 365 days

	weight =50%	weight = 75%
LookBack Days 3	23.1%	34%
LookBack Days 4	17%	25%
LookBack Days 5	14%	21%
LookBack Days 6	11.6%	17.4%
LookBack Days 7	10.6%	15.9%
LookBack Days 8	9.45%	14.17%
LookBack Days 9	8.575%	12.86%
LookBack Days 10	7.65%	11.475%
LookBack Days 11	7.10%	10.65%
LookBack Days 12	6.37%	9.5%
LookBack Days 13	6.19%	9.28%

The last table above shows that if we constrain the investment into our strategy that the average turnover drops significantly (approximately by a half in the case when only half of the portfolio is invested in the risky asset.) This is not surprising because our investment strategy is to invest only half (or three quarters respectively) of our wealth into the risky strategy while the other half is left as a reserve.

8. The cost of Leverage and Maximal Drawdown

We like to investigate the behavior of leveraged portfolios under our strategy. This is interesting because most market participants will trade VIX futures with margin accounts and it is instructive to verify whether a technical strategy will make money when the futures contract isn't fully funded. We first measure the maximal draw-downs of the strategies. This is defined as the maximal loss from the peak from the investment horizon. More formally if $X = X(t)$ is a random process we define:

$$D(T) = Max\left\{0, max_{t\in[0,T]}X(t) - X(T)\right\} \tag{8}$$

The maximum drawdown up to time T is:

$$MDD(T) = Max_{\tau\in(0,T)}\left\{0, max_{t\in[0,\tau]}X(t) - X(T)\right\} \tag{9}$$

We will use the draw down effect to determine the leverage a look-back strategy requires. Let us recall that if we introduce leverage the performance of the levered instrument will be $L\times r$, where r is the un-levered performance and L is the average ratio. Hence if $r \leq 0$ and we lever too much we will lose the entire investment.. We will use the average maximum drawdown to find the optimal levered ratio. (Note that we find this ratio in-sample.) Our levered ratio will be the minimal levered ratio required so that we will not lose the entire investment). The main properties of the MDD out of sample distribution are: From the table we see that there are

Table 17. Maximal Draw Down distribution properties

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
Lookback days 2	-28.00%	-43.00%	-51.00%	-51.59%	-58.00%	-77.00%
Lookback days 3	-26.00%	-40.00%	-45.00%	-45.09%	-49.00%	-58.00%
Lookback days 4	-32.00%	-40.00%	-49.00%	-46.58%	-53.00%	-60.00%
Lookback days 5	-30.00%	-48.00%	-51.00%	-50.81%	-58.00%	-67.00%
Lookback days 6	-31.00%	-42.00%	-58.00%	-54.45%	-61.00%	-72.00%
Lookback days 7	-29.00%	-35.00%	-54.00%	-51.89%	-57.00%	-75.00%
Lookback days 8	-29.00%	-33.00%	-54.00%	-52.00%	-57.00%	-77.00%
Lookback days 9	-27.00%	-29.00%	-54.00%	-51.85%	-57.00%	-79.00%
Lookback days 10	-23.00%	-37.00%	-57.00%	-53.78%	-59.00%	-79.00%
Lookback days 11	-21.00%	-37.00%	-58.00%	-55.56%	-67.00%	-79.00%
Lookback days 12	-21.00%	-37.00%	-57.00%	-57.45%	-79.00%	-83.00%
Lookback days 13	-21.00%	-37.00%	-57.00%	-57.15%	-77.00%	-84.00%

cases where the look-back strategy sustains heavy losses Hence it has a high risk of losing substantial amount of money. We implement the actual levered portfolio with a look-back strategy. Our implementation consists of the following steps:

(1) Fix an initial leverage amount - for example 1:2 means that 50% of our initial investment at the beginning of the period will be own capital and the rest being borrowed money.
(2) Fix a margin call limit - this is the limit that we liquidate if the total investment amount falls below this limit. For example if our margin call limit is 25% we sell the position once our original leverage investment drops to 25% of the entire position.

Once again we perform this strategy out of sample with a rolling window of one year. As an example we consider two cases:

(1) Initial margin of 50% and margin limit of 25%.

TABLE 18. Average return, Standard deviation, Sharpe ratio and certainty equivalence with $\rho = 4$

	Mean return	Standard Devation	Sharpe Ratio	Certainty Equivalence
look-back days 3	689%	963%	72%	0.23
look-back days 4	334%	629%	53%	0.37
look-back days 5	170%	362%	47%	0.16
look-back days 6	11%	143%	7%	0.10
look-back days 7	47%	137%	34%	0.04
look-back days 8	77%	227%	34%	0.04
look-back days 9	45%	129%	35%	0.05
look-back days 10	35%	124%	28%	0.06
look-back days 11	-23%	65%	-35%	0.06
look-back days 12	-16%	85%	-19%	0.04
look-back days 13	27%	186%	14%	0.04

The tables below describe the average return of the investment and its Sharpe ratios: We can see from the table that while the expected returns are high for the shorter look-back the Sharpe ratios are substantially lower than those when implying the strategy without any leverage (i.e. the initial contract is fully funded). This isn't a surprising result because the volatility of the look-back strategy is high and hence a levered investment has a high probabaility of being liquidated on the losing side. This is reiterated by the certainty equivalence calculated by us in this case. The certainty equivalent numbers are low because many of the 1 year rolling horizon sub-periods have a losing streak since they are forced to liquidate the investment in the middle because of a potential margin call.

9. Conclusion

In this note we considered reversion strategies on VIX and its futures. We researched this type of strategies for VIX because its Hurst exponent H displays stability over long period of time and is consistently less than half. Conclude that VIX time series are mean reverting and this mean reversion is stable over the historical time period we considered. A simple model we described shows that for such processes a reversion trading strategy should be profitable.

We started the paper by estimating the Hurst exponent of VIX on long time horizons. We constructed a rolling volatility index prior to the VIX and discovered that these series exhibit a remarkable stability of the Hurst exponent. The Hurst exponent is fluctuating between $0.35-0.45$ and never exceeds 0.5. This is in contrast with the underlying $S\&P$ index whose exponent is fluctuating around 0.5 between momentum and mean reversion regimes.

Another interesting finding is the behavior of the *log* of the VIX futures level. It turns out that short term contracts has a mean reversion property while the longer term contract display features of the usual martingale (The exponent approaches 0.5).

As a result reversion strategies should outperform buy and hold in an absolute and risk adjusted basis both in and out of sample. This is fully confirmed by simulation of reversion strategies both on VIX and VIX futures. We incorporated transaction costs through the bid ask spread and while they reduced gains they didn't eliminate the strategy out-performance. Trading futures using these strategies behooves us to fully fund the contract as otherwise the wild fluctuations of these strategies can cause a major loss to investors. On a risk adjusted basis we can see We show that the risk of the strategy considerably from Sharpe ratio and CE perspectives.

A use of normed constrained portfolios (where only part of the money is in the strategy while the rest sits in less risky investments like Treasury Bills. Our findings indicate that this might be an interesting way for investors to exploit the advantages of the technical trading strategy while limiting the downside risks. In this case potential losses of the strategy are diminished.

We are left with certain theoretical puzzles in our work: The major one is the Hurst exponent of the log level of the VIX futures. According to the accepted theories these must be martingales and hence their Hurst exponent must be close to $\frac{1}{2}$ and hence no strategy that is based on look-back prices should outperform a simple buy and hold strategy. Yet the analysis done in this paper shows that this isn't the case and technical trading strategies seem to outperform buy and hold even out-of-sample. A possible explanation is the nature of the futures trades which is done mostly on leveraged basis. As a result strategies that make money even on af fully funded basis are simply irrelevant to the current trading practiced by market participants. Another interesting finding is the long term stability of the Hurst exponent. During historical time periods Hurst exponent displays of volatility time series displays remarkable stability (below half). This is in contrast with the $S\&P$ that has a regime changing Hurst exponent and resembles a true martingale in its behavior. [4]

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