The Level, Slope and Curve Factor Model for Stocks

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Abstract

I develop a method to extract only the priced factors from stock returns. First, I use multiple regression on anomaly characteristics to predict expected returns. Next, I form portfolios of stocks sorted by their expected returns. Then, I extract statistical factors from these sorts using principal components. The procedure isolates and emphasizes the comovement across assets that is related to expected returns as opposed to firm characteristics. The procedure produces level, slope and curve factors for stock returns. The factors perform better than the Fama and French (1993, 2014) three and five factor models and comparably to the four factor models of Carhart (1997), Novy-Marx (2013) and Hou, Xue, and Zhang (2012). Horse races show that other factors add little to the Level, Slope and Curve factors.

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1 Introduction

The number of potential new asset pricing factors is exploding. Some of these factors are priced and some are not. A priced factor exposes the investor to systematic risk and comes with a risk premium. An unpriced factor represents common movement across stocks unaccompanied by systematic risk or a risk premium. The standard approach is to identify a common factor, and then to test whether this factor is priced.

In contrast, I describe a method to search specifically for the priced common factors in stock returns. First, sort stocks by expected returns using several different return predictors. Second, use principal components to extract the common factors. The first stage produces portfolios sorted in one dimension on expected returns. The second stage isolates comovement that is priced in the cross-section of stocks by searching for common factors in portfolios already sorted by expected returns.

In order to separate priced from unpriced common factors, the method relies on sorting expected returns using many different predictors. Sorting stocks by only book to market in step one produces a sort on expected returns, but it is unlikely to isolate priced risk factors, because portfolios sorted on book to market have both priced and unpriced common movement (Daniel and Titman (1997) and Gerakos and Linnainmaa (2014)). To overcome this obstacle, I sort on expected returns using multiple regressions to combine signals from many different predictor variables. The procedure focuses only on factors with nonzero prices of risk and separates these factors from their unpriced counterparts.

Using this method, I show that extracting common factors from these portfolios extracts three factors with the familiar level, slope and curve pattern that can be extracted from returns of bond portfolios sorted by maturity (Litterman and Scheinkman, 1991). The level factor is highly correlated with the market factor, but the slope and curvature factors are notably distinct from commonly used factors.

I find that the Level, Slope and Curve model outperforms the Fama and French three factor and five factor model model and performs similarly to three leading models: a four factor model with momentum of Carhart (1997), the four factor model of Hou, Xue, Zhang (2012), and the four factor model of Novy-Marx (2013).

In addition, I use "horse races" to test whether adding a factor to the Level, Slope and Curve model adds any explanatory power. I find that additional factors add very little to the Level, Slope and Curve model.

Finally, I test whether the Arbitrage Pricing Model motivation to uncover factors relates to deeper models. I find evidence that the three factors proxy for priced risk in accordance with the Intertemporal Capital Asset Pricing Model. Factor returns correspond to changes in the investment opportunity set. I also find that at an annual frequency, the cross-sectional dispersion across stocks can largely be explained by the portfolio or factor's exposure to changes in consumption growth.

This procedure to extract priced factors makes an important contribution to the literature on the cross-section of stocks. First, Cochrane (2011) calls for a reorganization of the factor structure of stock returns. Which factors are the most important? Which factors should we be writing consumption based asset pricing models to explain? The procedure searches for the most economically important risk factors that are *priced* in the cross-section, and finds the cross-section can be summarized by three important factors. This result stands in stark contrast to Green et al. (2014) who find staggering multidimensionality in stock returns and suggest a factor model with twenty-four factors. Their approach follows the more traditional procedure of grouping portfolios by characteristics, but this may allow unpriced factors to creep into the results.

Second, this paper bridges a gap between empirically generated factor models and theoretically generated factor models. The models of Fama and French (1993), Carhart (1997), and Novy-Marx (2013) are all empirically motivated factor models. The prediction of Arbitrage Pricing Theory is expected returns should be accompanied by common factors. Novy-Marx (2013) explains the motivation, "While I remain agnostic here with respect to whether these factors are associated with priced risks, they do appear to be useful in identifying underlying commonalities in seemingly disparate anomalies." Unfortunately, the general procedure is prone to concerns about data mining. It is not clear precisely when, after observing alphas on sorted portfolios in time-series, we should add another return factor.

The traditional response to data mining concerns is to lean more heavily on theoretical motivations. The five factor model of Fama and French (2014) and the Q-factor model of Hou, Xue and Zhang (2014) are both advocated on theoretical grounds. But these theoretical motivations are different and contentious. Hou et al. (2015) criticizes the Fama and French (2014a) on theoretical grounds and Novy-Marx (2015) criticizes the Q-factor model on theoretical grounds. More importantly, models based on these specific theories offer little use to researchers developing distinct theoretical insights to deepen our understanding of the cross-section of returns. Cochrane (2011) calls not for a perfect theory to end all debate, but rather a synthesis of data, a parsimonious description of the important factors in the cross-section that theory is meant to explain.

This paper offers a theoretically motivated search for empirical factors. This procedure offers a bridge between theory and empirics to identify the salient facts that deeper models should be trying to explain. The Arbitrage Pricing Theory *predicts* that expected returns should be associated with factor structure. Kozak et al. (2015) reminds us that empirical factor models don't have the ability to distinguish between rational and behavioral explanations of the cross-section of returns. This highlights a great strength of this approach. By using only the law of one price and the absence of arbitrage, the paper can synthesize the most important factors for both rational and behavioral models to explain without taking a stand on the explanations. Theoretical work will continue to be rich and insightful, but also contentious. This paper argues that we can summarize the important empirical facts without answering these difficult theoretical questions.

Third, a strength of this paper is offering a description of the factor structure of returns that is not centered around firm characteristics. While much has been learned by characteristic based sorts and described by characteristic based factors, it is at least conceivable that the common movement across stocks is not principally caused or described by firm characteristics. For instance, Adrian et al. (2014) develop a factor model driven by financial intermediaries. Barberis et al. (1998) consider a model in which the cross-section is driven by investor sentiment. In this paper, characteristics are just useful signals for identifying latent factors. The Level, Slope and Curve Model offers a lens through which to view the factor structure of the cross-section without appealing to characteristics.

There are limitations to this approach. The procedure may not find all priced factors. There may be important priced factors that are only important to a small number of stocks or there may be factors that affect many stocks, but have very small prices of risk. The procedure is also limited by the predictors used in the first stage. A strong enough predictor could change the factors found in this paper. But the method has great resilience to these changes, since a new predictor would have to be strong *in the presence of other predictors* (Fama and French, 2014b). Thus, identifying priced factors using this procedure makes it less likely that future researchers will write consumption based asset pricing models explaining false factors. The method also provides some protection against the datamining concerns of Kogan and Tian (2013). There may be datamining in the first stage, but it need not be associated with the strong factor structure produced in the second stage.

The benefit of the approach in this paper is narrowing the factor space once again. A primary goal of reducing a set of portfolio returns to a much smaller set of common factors is data reduction (Cochrane, 2011). After all, we do not require a theory that explains all of the comovement of the hundreds of assets used in this paper; one that explains the common factors that price them would be very useful. Since this method shows some robustness to new anomalies, theoretical work to explain the statistically extracted factors is less likely to go off course. The underlying factor structure is more stable. A disciplined empirical approach to generating priced factors can help narrow the focus of new theoretical work from many disparate characteristics to a few priced factors. While unpriced common factors may be interesting in their own right, they are unlikely to be central components connecting asset price movements to business cycle movements. They are not likely to be central puzzles in the intersection of macroeconomics and finance.

2 Literature Review

I draw on the large literature of firm characteristics that predict returns (often called anomalies). There are several papers that have used multiple regression on many predictive characteristics. Haugen and Baker (1996) use regressions on many variables to sort stocks by predicted next period returns. Fama and French (2008) show that many characteristic variables contain separate and distinct information that varies across size groups. Lewellen (2011) shows that these variables also predict returns out of sample.

Each predictor individually has information about expected returns, and portfolios sorted on each individual predictor likely have unpriced common variation. But as long as the unpriced variation is not perfectly correlated across predictors, the common signal of expected return will be reinforced and the noise will be averaged out. Zhang (2009) demonstrates this logic using principal components analysis to find factors similar to the Fama and French (1993) small minus big (size) and high minus low (value). When using principal components to extract common factors from individual stocks, he finds no evidence of common variation due to differences in size and book to market (Connor and Korajczyk, 1986, 1988), but when using principal components analysis on a 10 by 10 portfolio sort on size and book to market, he does recover the common variation due to the size and book to market factors. Sorting on a common characteristic reinforces the common signal. The difference in my approach is that I use expected return as the common signal rather than a firm characteristic. By sorting on expected returns, I reinforce the priced factors and average out the unpriced factors allowing me to uncover the priced factors through principal components.

The method can both grow with and show robustness against the growth and inclusion of new anomaly variables. Including an additional anomaly with predictive power will help generate an even sharper estimate of the true underlying factors, but it would take a very strong new anomaly to dramatically alter the factors. The anomaly must have large explanatory power even after controlling for several other strong anomalies.

Often, authors choose factors in response to the observation that there is a common comovement across stocks. The factor captures the common movement, and the authors test to see if the comovement is priced in the cross-section. Fama and French (1993) saw persistent differences in average returns created by a double sort on size and book to market, and reasoned that the spread in value versus growth stocks holding size constant and the spread in small minus large stocks holding book to market constant could explain returns by proxying for latent priced factors. Fama and French (1996) show the model successfully prices other anomalies such as sales growth and long-term reversal. In a general sense, the method and goal of this paper are the same. I start with anomaly variables that generate large spreads in return. Then identify comovement across the portfolios caused by a factor structure. Finally, I determine whether these factors can price other assets as the Arbitrage Pricing Theory predicts. The difference is that I am looking only for common factors that explain expected returns. I am not looking for common factors related to firm characteristics that may be priced or may have a priced component.

While the three factor model of Fama and French (1993) provided a workable replacement for the Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965), the number of anomalies has continued to grow. The response has traditionally been to add additional factors to describe portfolio spreads unexplained by the three factor model, such as the momentum factor of Carhart

(1997), the liquidity factors of Pastor and Stambaugh (2003) and Sadka (2006), and the volatility factor of Ang et al. (2006). More recently, authors have questioned the three factor model more fundamentally and produced factor models combining different anomalies, such as the four factor models of Novy-Marx (2013) and Hou et al. (2012), and later the five factor model of Fama and French (2014a). All of these factors are formed from sorts on firm characteristics or firm betas. This paper is the first to extract factors from portfolios sorted by expected returns using many characteristics.

3 Factor Structure of Anomalies

The Arbitrage Pricing Theory of Ross (1976) posits that returns are generated by an asset's loadings on common factors and an idiosyncratic term. If the APT holds, the return on a security, X_i, is the sum of its expected return and its loadings multiplied by priced and unpriced factors:

$$X_i = E_i + \beta_{1,i}F_1 + \beta_{2,i}F_2 + ... + \beta_{N,i}F_N + \phi_{1,i}G_1 + \phi_{2,i}G_2 + ... + \phi_{N,i}G_N + \epsilon_i$$

Without loss of generality, I separate the priced factors (F) from the unpriced factors (G). An unpriced factor has a zero risk premium, for example industry factors.

The loadings on the unpriced factors (ϕ) do not enter into the expected return of the asset.

$$E[X_i] = E_i = \lambda_1\beta_{1,i} + \lambda_2\beta_{2,i} + ... + \lambda_N\beta_{N,i}$$

Only the risk premiums of the priced factors (λ) and the asset's loadings on those factors (β) determine its expected return.

Using this as motivation, I sort stocks by their expected returns, in order to reinforce the priced comovement across stocks and wash out the unpriced comovement. The one dimensional sort strengthens patterns created by the comovement of the portfolios related to expected returns, and weakens the patterns created by common factors with zero risk premiums. If there is only one predictor of expected returns, such as book to market, then our ability to isolate priced movements from unpriced movements is minimal. But by utilizing the expansive anomaly literature, we can sort on expected returns from several different sources. This creates large spreads in expected returns both in and out of sample.

4 Data and Variables

The sample runs from July 1963 until December 2012. The variable definitions are identical to Fama and French (2008) with two exceptions. Reacting to Novy-Marx (2013), Fama and French (2014a) argue that operating profit is a more robust predictor of average returns in the cross-section than return on book equity, and Aharoni et al. (2013) show that asset growth at the firm level is a better and more theoretically motivated predictor than asset growth per share. Thus, I slightly alter the Fama and French (2008) regressions to reflect these insights and to match the definition used in Fama and French (2014a).

Returns are monthly holding period returns obtained from the Center for Research in Security Prices (CRSP) and adjusted for delisting return when available. The accounting data is from Compustat. The sample includes only common equity securities (share code 10 and 11) for firms traded on NYSE, NASDAQ or AMEX. I drop financial firms (Standard Industry Classification codes of 6000 to 6999). All anomaly variables are measured at the end of June using the last fiscal year's accounting data, except for momentum, which is defined monthly. I choose anomaly variables and their definitions to match Fama and French (2008). The precise variable definitions can be found in Fama and French (2008) and include: size, book to market, momentum, net stock issues, accruals, investment, and profitability.[1]

5 One Dimensional Portfolio Sorting Procedure

In order to sort stocks by expected returns, I use a procedure that forms portfolios using many firm characteristics as predictors. Fama and French (2006) provide a logical three step procedure to do this. First, run Fama-MacBeth cross-sectional regressions of one month ahead firm-level returns on current values of the anomaly variables. Second, use the coefficient estimates from the regressions

[1] Size is attributable to Banz (1981), book to market to Rosenberg et al. (1985), Chan et al. (1991), and Fama and French (1992), momentum to Jegadeesh and Titman (1993), and net stock issues to Daniel and Titman (2006) and Pontiff and Woodgate (2008) following earlier work by Ikenberry et al. (1995) and Loughran and Ritter (1995). Accruals is attributable to (Sloan, 1996), profitability to Haugen and Baker (1996), Cohen et al. (2002) and Novy-Marx (2013), and investment to Fairfield et al. (2003), Titman et al. (2004) and Cooper et al. (2008).

to predict the one month ahead return for each stock. Third, sort stocks into portfolios based on the predicted returns.

The goal of the procedure is to yield a portfolio sort that creates a wide spread in average returns using only information in the investor's opportunity set. An economically significant predictor will account for a relatively large portion of the spread. Clearly, I must be explicit when I define an investor's information set. Fama and French (2006) use parameter estimates from the full sample in order to sort into portfolios. A rationale for this approach is that the whole time series best reflects the contribution of each anomaly to returns. Alternatively, I could use regressions only on past data to form sorts or rolling regressions that capture time varying betas as in Haugen and Baker (1996) and Lewellen (2011). Since my goal is as identifying the factors, rather than trading on them, I use the full sample for my main tests. In robustness checks, I show that the level, slope and curve factors are not very sensitive to the choice of information set.

Each cross-sectional regression takes the following form:

$$\begin{aligned} Ret_{i,t+1} = \beta_0 &+ \beta_1 Size_{i,t} + \beta_2 BtM_{i,t} + \beta_3 Mom_{i,t} + \beta_4 zeroNS_{i,t} + \beta_5 NS_{i,t} \\ &+ \beta_6 negACC_{i,t} + \beta_7 posACC_{i,t} + \beta_8 dAtA_{i,t} + \beta_9 posOP_{i,t} + \beta_{10} negOP + e_{i,t+1} \end{aligned} \tag{1}$$

The stock return in excess of the risk free rate for each stock in the following month is regressed on firm size, book to market, momentum, a dummy if no stock was issued, net stock issues, negative accruals, positive accruals, asset growth, positive operating profit and negative operating profit. Fama and French (2008) find that stocks of different size groups (micro, small and large) have different exposures to characteristic predictors. Thus, I run the regression above separately for each size group allowing the parameter estimates to differ across these groups.

These sorts are very effective at generating a spread in portfolio returns. Figure 1 shows the results of the sort for each portfolio. Predicted returns, represented by the line, are produced from the fitted values of the regressions for each stock combined into a value-weighted portfolio. Average returns, represented by dots, are the average value-weighted returns for each portfolio. The S shape of the predicted and average returns is a natural result if expected returns are linear in the characteristics and the characteristics are normally distributed.

Table 1 displays the summary statistics of the twenty-five sorted portfolios. Each portfolio characteristic is formed by the value-weighted average (using beginning of the month market equity) of each stock in the portfolio. Thus, the portfolios are not dominated by the plentiful, but tiny micro cap stocks. All the characteristics, except for size, show monotonically increasing or decreasing patterns in expected returns with the sign predicted by previous research. Momentum and investment show especially strong patterns, the difference between the extreme low return and extreme high return portfolios are two or more standard deviations. Net stock issues also shows a strong trend, but it is concentrated in the low return, high net issue portfolios. Accruals and book-to-market both create spreads of less than one standard deviation between the high and low return portfolios. Lastly, size has a somewhat curved sort. The June month end market equity increases to a max at portfolio four and then decreases from portfolio four to the highest return portfolio twenty-five consistent with the size effect. The extreme low return portfolios aren't especially dominated by small stocks. The value-weighted June market equity is still larger than over half the portfolios.

6 Factor Structure of One Dimensional Sorts

The next step is to extract common factors from these portfolios. I use principal components analysis (PCA), which uses an eigenvalue decomposition to identify common factors across portfolios. By construction, the method extracts linear combinations of the test asset returns that explain the structure of the covariance matrix (Tsay, 2005). This approach translates the comovement between the test assets from a covariance matrix to uncorrelated factors. Each factor is formed as a set of weights on the test portfolios. The first factor explains the largest amount of the covariance between the portfolios. The second factor explains the next largest amount that is not captured by the first factor and so on. In total, the factors describe the entire covariance structure between test assets.

When used on a large sample of individual stocks, as in Connor and Korajczyk (1986, 1988), PCA has little power to extract useful factors from stock returns, but Zhang (2009) uses portfolios to recover the underlying comovement across stocks related to their characteristics. His insight is that portfolios sorted on firm-level characteristics strengthen the patterns in stock returns related to the firm-specific pattern. Patterns in returns unrelated to the characteristics cancel out. This paper extends that insight by sorting the portfolios on expected returns, rather than the firm-level characteristics. Using PCA on these portfolios isolates the common factors that determine expected returns.

I use PCA on the twenty-five portfolios sorted from low to high by expected returns using the anomaly regressions. Table 2 shows the results of the principal components analysis. The table presents the first ten components, the respective eigenvalues and variance explained. The first three components explain 85% of the variance of the portfolios. The first component explains 73% of returns, the second explains 9% of returns, and the third explains 4% of returns. In Figure 2, I present the weightings of the first three components.

The first factor resembles a general market portfolio because it approximately equally weights all 25 portfolios. The factor has a correlation of .95 with the CRSP value weighted market index used in all popular factor models. This is a "level" factor as it represents comovement with the level of the market. Stocks tend to rise and fall together. The second factor is long low return stocks and short high return stocks. Weights decrease monotonically from long to short. This "slope" factor captures the feature that the high return stocks often move opposite from the low return stocks. Since the factor is going long low return stocks and short high return stocks, on average it has a negative realization.

Most factors already identified in the finance literature are slope factors. The hml factor captures the tendency of growth stocks to move opposite of value stocks, while the smb factor captures the tendency for small stocks to move opposite of large stocks. Other examples include slope factors for momentum, profitability, investment, volatility, and liquidity.

My slope factor is different in that it captures common movement using all of the characteristics at once. The underlying characteristic of interest is expected returns and not a firm-level proxy for expected returns. While each firm-level characteristic offers some information about expected returns, portfolios built on characteristics alone may share a large degree of common movement that isn't related to expected returns.

The last "curve" factor is short the extreme low and high return portfolios and long the middle portfolios. The curvature factor shows that extreme stocks tend to move together. If the curvature factor has a positive realization, both very high and very low expected return stocks will have relatively low returns and the stocks with moderate expected returns will have relatively high returns. Altogether, the factors bear a striking resemblance to the bond factors found by Litterman and

Scheinkman (1991). [2] Lord and Pelsser (2007) show that level, slope and curvature characterizes a robust fact about the variance-covariance matrix. Since any factor model can by written as a one factor model, the important point is not the number of factors that principal components produces, but that the factors yield a stable description of the variance-covariance matrix (Roll (1977) and Hansen and Richard (1987)).

Principal components are identified down to a scalar transformation of the factors. To reinforce a portfolio interpretation, Campbell et al. (1997) suggest dividing by the sum of the loadings on each factor, so that the weights sum to one. That works well for the first factor, but creates a very unintuitive hedge fund for the slope factor. Since the factors are excess returns, the slope factor would represent borrowing $1 at the risk-free rate investing over $7 long and over $6 short. Since the slope factor is negative, the hedge portfolio is long low return stocks and short high return stocks and losing money at a very rapid pace. Instead I adopt a different definition of the slope and curve factors by limiting the factors to 100% short. This choice is made in the spirit of Fama and French (1993) who define their high low factors by investing $1 short and $1 long. The choice of scalar is somewhat arbitrary and made only to aide the interpretation of the factors.

In order to ascertain if the principal components analysis uncovers a true common signal, I also use the same method for sorts on ten portfolios and 100 portfolios. The results also show the level, slope and curve patterns. In Table 3, I show the correlations of the first five components using sorts on 10, 25 and 100 portfolios. The results show a very strong correlation among the first three components, regardless of the number of portfolios used. The lowest correlation is always between the component extracted from 100 portfolios and the component extracted from 10 portfolios and for the first three components is .994, .956, and .846, respectively. The fourth and fifth components are not nearly as correlated across sorts. For the fourth, the 100 and 10 portfolio sorts only share a correlation of .403. For the fifth component, the 100 and 25 sorts share the smallest correlation of -.009. I exclude the fourth and higher factors from this study. While I make no attempt to rule out the possibility that the fourth and fifth factors represent some form of priced risk, there is at a minimum an issue with measuring that signal precisely. I only include the first three factors in this study in order to get strong common signals that are not dependent on the sorting procedure.

The factors are very stable across subsamples. Table 4 shows the results of splitting the sample

[2] Lustig et al. (2011) find level and slope factors in portfolios formed on the carry trade.

into two halves and conducting principal components on each subsample. The first sample runs from July 1963 to March 1988, while the second sample runs from April 1988 to December 2012. Each subsample shows the level, slope and curve factors, and even though the weights are formed on entirely different subsamples the resulting factors are very highly correlated. In the first half of the sample, when the end of sample components are out-of-sample, the in and out of sample level, slope and curve factors have correlations of 1.00, -0.87, and 0.72. In the second half of the sample, the correlations are 1.00, -0.96, and 0.70.

These factors differ from Fama and French's three factors. Table 5 shows the correlation of the level, slope, and curvature factors with several other proposed factors. None of the three extracted factors has a correlation above 0.30 with HML. SMB is correlated with the level factor at a moderate level of 0.51, but has correlations below 0.40 with slope and curve. The slope factor has a correlation of 0.66 with momentum, the strongest correlation on the table. The profitability factors, RMW, ROE and PMU have low correlations with the slope factor, but typically small correlations with the curve factor. Investment shares a weak correlation with both slope and curve. None of the three factors are very correlated with liquidity. The slope and curve factors are different than the factors already represented in these leading models.

7 Time-series Asset Pricing Tests

The Arbitrage Pricing Theory predicts that the wide spread of excess returns created by sorting stocks into portfolios based on their expected returns will be explained by each portfolio's loadings on common factors. Table 6 shows the results of time series regressions of the portfolio returns in excess of the risk free rate on the first one, two, three and four principal components. Since the factors are uncorrelated, the pattern in betas are captured by the loadings shown in Figure 2. The table shows the alphas, t-statistics and R-squared from each of the four time series regressions on each of the twenty-five portfolios.

The third column α_1 shows that the large spread in returns is not captured by the first factor. This regression is almost identical to the traditional CAPM, so while it is not surprising that the alphas are not captured, it is interesting that almost all of the alpha shifts to the short leg. Over 60% of the alpha on the high return portfolio is explained by the first factor. The level loadings in Figure

2 actually mask a somewhat significant variation in market betas across porftolios. The extreme portfolios have loadings of 0.26 and the middle portfolios of 0.17, which while a barely perceptible curved pattern in the figure represents a 50% increase from the low beta middle portfolios to the high beta extremes. This curved pattern in the level beta exacerbates the result in column three, helping the first factor capture the alpha on the high beta, high return portfolio and increasing the alpha on the extreme low return portfolio.

The fourth column shows that the second factor explains a large portion of the one factor alpha. The alpha on the extreme high portfolio is slightly negative and insignificant, while the alpha on the extreme low portfolio has fallen 50%. The average R-squared of the twenty-five regressions rises from 74% to 82%, while the average alpha falls from 0.37% per month to 0.19% per month. Column five shows adding the curve factor increases the average R-squared to 85% and decreases the alphas to 0.18% per month. The fourth factor adds little additional R-squared, and while it seems to decrease some alphas, it follows a somewhat suspect zig-zag pattern that as already shown appears somewhat unstable.

The GRS Tests show that all four specifications are strongly rejected, not unlike Fama and French (1993). Importantly, the three factor model captures a large portion of the spread in average returns, and a large portion of the variance of the twenty-five portfolios. Unsurprisingly, the low return portfolio proves much more difficult to price than the high return portfolio given that an arbitrager must take a short position to profit of these portfolios.

8 Cross-sectional Asset Pricing Tests

If the APT holds and this method succeeds at extracting priced factors, the model predicts a relationship between expected returns and factor loadings. In this section, I perform a number of cross-sectional asset pricing tests in order to compare the Level, Slope and Curve Model to leading factor models. I compare the model to the Fama and French three factor, four and five factor models, as well as the four factor models of Novy-Marx (2013) and Hou et al. (2012) using a variety of test portfolios.[3] Lewellen et al. (2010) point out a number of problems with cross-sectional asset

[3] I would like to give special thanks to each author for generously sharing their factors. I obtained the Fama and Frech factors SMB, HML and MOM from Ken French's data library. I obtained the Novy-Marx four factor model from his data library. Ken French shared the Fama and French five factor model through email correspondence. Chen Xue shared the Q-Factor model through email correspondence.

pricing tests, especially when only twenty-five portfolios of size and book to market are used for test assets. If the test portfolios have a strong factor structure, the cross-sectional asset pricing tests may not be informative. They show that including many diverse test assets relaxes the factor structure and creates more informative asset pricing tests.

In my tests, I include a diverse set of portfolios in order to relax the factor structure and attain more informative results. These tests have three testable implications. The R-squared of the cross-sectional regression should be close to 1, as the assets should be priced by the factors. The constant term should be close to zero, as the constant return represents the zero-beta rate, which should be near the risk free rate. The coefficients of the cross-sectional regressions should be near the average return on the factors, as the coefficient should equal the cross-sectional risk premium.

8.1 Factors and Test Assets

The Fama and French three factor model (Fama and French, 1993) uses the market portfolio and two hedge portfolios, one long high book to market stocks and short low book to market stocks (HML) and the other long small stocks and short large stocks (SMB). The Fama and French four factor model, also called the Carhart (1997) model, adds a momentum factor (MOM), long stocks that have risen over the last 12 months and short stocks that have fallen over the last 12 months. The Fama and French (2014a) five factor model excludes momentum and includes a factor long low investment stocks and short high investment stocks (CMA) and a factor long high profit stocks and short low profit stocks (RMW). Since both the five factor models and three factor models use multidimensional sorts to form factors, the SMB and HML differ across the two models. I use the appropriate version for each. The Novy-Marx four factor model uses the market portfolio combined with hedge portfolios of industry adjusted value (HML), momentum (UMD) and gross profitability (PMU). The Hou, Xue and Zhang four factor model uses the market portfolio combined with hedge portfolios on size (SIZE), investment (INV) and profitability measured by return on equity (ROE).

For test assets, I use two groups, one consisting of 112 portfolios of stocks, bonds and asset pricing factors, and the other consisting of 140 decile portfolios formed on stock characteristics. For the 112 portfolios, I use ten portfolios formed by the results of the dissecting anomalies regressions in Section 4, twenty-five portfolios sorted on size and book-to-market, 10 portfolios sorted on momentum, returns on five treasury bonds, forty-nine industry portfolios and thirteen factor portfolios.[4] The included factors are the excess return on the market, the five factor model's hml, smb, rmw, cma, Carhart's momentum factor, Novy-marx's profitability factor, HXZ's profitability and investment factors, as well as the slope and curve factors.

The second group of test assets consists of 140 anomaly portfolios formed by decile sorts on 14 anomalies by Novy-Marx and Velikov (2013)[5]. Since primarily the spread across test assets in the first set of portfolios is created by sorts on value, size and momentum. I pick an assortment of different anomalies for the second set of test portfolios. The fourteen anomaly variables are gross profitability, accruals, net stock issues, asset growth, asset turnover, gross margin, O-score, failure probability, idiosyncratic volatility, earnings surprise, long run reversal, return on market equity, beta arbitrage, and short run reversal.[6]

8.2 The Level, Slope and Curve Model vs. The Fama and French Three Factor Model

I follow the Black et al. (1972) two-step approach. First, I estimate the full-sample betas of each test asset on the level, slope and curvature factors using time-series regressions, then I regress the average returns on the estimated betas. The regression estimates the risk premium associated with each factor. If the risk premium is significantly different from zero, the factor is priced. A high R-squared indicates the spread in average returns is explained by the spread in the betas of the test assets on the common factors. I estimate the model with an intercept term. The model predicts the intercept term should be close to zero as the zero-beta rate should be close to the risk free rate. Since an arbitrager would have to borrow at the risk-free rate and buy a zero beta asset to profit from a spread in the risk-free and zero beta rate, the two rates will only be equal if the arbitrager can borrow at the risk free lending rate (Brennan, 1971). Because the error terms may be cross-sectionally correlated, I report coefficients and t-statistics using Fama and MacBeth (1973) regressions. I report the R-squared statistics from the ordinary least squares regressions.

[4] I obtain the twenty-five size and book to market sorted portfolios, as well as the industry portfolios, momentum portfolios and the Fama and French factors from Ken French's website. I obtain the investment and gross profitability portfolios from Robert Novy-Marx's website. I obtain the bond portfolio returns from CRSP which include the 1 month, 1 year, 5 year, 10 year, 20 year.

[5] I obtain these portfolios from Robert Novy-Marx's data library

[6] Novy-Marx and Velikov (2013) provides 320 test portfolios on 32 anomalies, but many are quite similar with large return spreads created by variation on momentum, which the LSC model prices very well. This subset of portfolios offers a distinct set of alternative test assets.

The first two columns of Table 7 show the results of the two step procedure for the Level, Slope and Curve Model relative to the Fama and French models. For the LSC model, each factor risk premium is large and statistically significant. The first factor, which is highly correlated with the market portfolio, generates a factor risk premium of .58% monthly, reasonably close to the historical excess return of the level factor over the time period of .80%. The second factor generates a risk premium of -0.98% per month close to the factors average return of -1.26%. The third factor, curvature, is associated with a risk premium of 0.59% monthly equal to the factor's historical average excess return of 0.59%. Since the test assets are excess returns, the APT implies that the constant term should be close to zero. The constant term estimate is 0.24% and also statistically significant. This implies a difference in the zero-beta rate and the risk free rate of 24 basis points monthly or 2.92% annually. The 68 percent R-squared implies that the model captures a large amount of the cross-sectional spread in risk.

Table 7 displays the results of the Fama and French three factor model on the same test assets. The coefficient estimate on the market factor is positive but not significant. The coefficient estimate on HML is 0.33% is significant at the 10% level and near it's average return of 0.50%, and the coefficent estimate on SMB is 0.21% and not significant, but neither is its sample counterpart with an average return of 0.05% over the sample period. The Level, Slope, Curve factor model shows much more explanatory power than the Fama and French model, with an R-square of 0.68 compared to 0.20.

Figures 3 and 4 show the R-squared result graphically. For each model, I graph the predicted return on the X-axis and the average realized return on the Y-axis. Thus, the vertical distance from a point on the graph to the X-axis is data. The horizontal distance from each point to the 45 degree line is the model fit. If the portfolios don't have much vertical spread, there isn't much for the model to price (such as industries). If the vertical spread in portfolios isn't producing horizontal spread, the model is failing. The Level, Slope and Curve model in Figure 3 shows a strong pattern along the forty-five degree line. Predicted returns are strongly associated with average returns. While the Fama and French model in Figure 4 less association between predicted return and realized return. The negative return portfolio is the slope factor, which has an average return of -1.26% a month.

Figures 5 through 10 show each group of assets presented in Figures 3 and 4 separately. Displaying each group of test assets individually highlights how well each model explains the cross-sectional dispersion of each group of test assets. Figure 5 shows each model applied to the ten dissecting anomaly sorted portfolios. The LSC factor model does extremely well at pricing portfolios one through nine, only the highest return portfolio is problematic. The predicted return is not as high as the average return. The Fama and French three factor model does poorly on the dissecting anomaly portfolios. The horizontal distance between the low return DA1 and the high return DA10 and the 45 degree lines are very large. In fact, the three factor model hardly generates a spread in expected returns at all, as the points show little horizontal spread.

Figure 6 shows how well the two models explain the twenty-five size and book to market portfolios. The points are labeled for Size and Book to Market quintiles (SB), so that SB11 is the smallest size quintile and lowest book to market quintile (small growth). The Fama and French three factor model generates a slightly larger cross-sectional dispersion than the Level, Slope and Curve model. The small growth portfolio stands out as difficult to explain with factor loadings for both models, though the Fama and French Factor models does better at the next to smallest growth portfolio (SB21). The other portfolios are priced similarly across models.

Figure 7 shows the ten momentum sorted portfolios. These portfolios are extremely well explained by the Level, Slope, Curve factor model, but extremely troublesome for the Fama and French three factor model. On the left side, the Level, Slope and Curve model generates a nice horizontal spread along the 45 degree line, while on the right side of the panel the Fama and French three factor model generates a reverse spread. The model predicts the high return tenth portfolio to have lower returns than the low return first portfolio.

Figure 8 presents the predicted returns of each model for the forty-nine industry portfolios. The Level, Slope, Curve model performs better. The industry portfolios don't generate a large spread in average returns for the models to price, but there appears to be some relation between the predicted returns and actual returns in the Level, Slope and Curve model, while there is none in the Fama and French three factor model.

Figure 9 shows each model applied to the five bond portfolios. The Level, Slope and Curve model shows some ability to price long term bonds with relatively small pricing errors on the five, ten and twenty year bonds; but the model fails at pricing shorter term bonds graphically demonstrating the positive and significant zero-beta rate. The Fama and French three factor model has very little ability to price any bond returns. The model predicts that long term bonds should have slightly lower returns than short term bonds, opposite from what we see in the data.

Lastly, Figure 10 shows the two models against the factor portfolios. Do these risk factors price themselves and each other? The Level, Slope and Curve model does well, creating a horizontal spread in factors with the factors average returns close to their predicted returns. Both the slope factor and the momentum factor have predicted returns a bit too high to match the data. The Fama and French three factor model prices less of the dispersion across factors, and does especially poorly on the slope factor.

8.3 Level, Slope and Curve Model Vs. Leading Factor Models

The Level, Slope and Curve Model performs very well versus the Fama and French three factor model. The comparison is important, because of the preeminence of the model's status as the default method for risk adjustment over the last two decades, and also, because the two models have the same number of factors. The poor performance of the Fama and French three factor model is evidence that the choice of test portfolios have alleviated many of the concerns of Lewellen et al. (2010). I also test the LSC model versus more recent models that use additional factors to price assets. I find that the LSC model, despite having fewer factors, performs comparably and often better than other leading models.

Table 7 shows the cross-sectional tests using the 112 test portfolios of stocks, bonds and factors. The other candidate models are the Fama and French three, four (Carhart), and five factor models. All three models have small, but strongly significant zero-beta rates. Despite having fewer factors, the Level, Slope and Curve model has a higher R-squared than all the models except Novy-Marx's four factor model, which has an R-squared of 69%, one percent higher than the LSC's 68%. None of the other five models have every factor priced. The PMU is not priced in the RNM model, while the investment factor is not quite priced in the HXZ model. The Fama and French three and five factor models, along with the HXZ model, all fail to price the market.

The Fama and French five factor model, which adds investment (CMA) and profitability (RMW) to the three factor model, performs much better than the three factor model. The 50% R-squared is a big improvement over the 20% of the three factor model, but is still lower than the 68% of the LSC model.

Table 8 presents cross-sectional asset pricing tests using 140 portfolios built by decile sorts on

14 characteristics. The Fama and French three factor model does worst again. The zero-beta rate measured by the constant term is significantly different from zero and economically large, 51 basis points or over 6% annually. The R-squared is low at 15%. The addition of momentum in the four factor model improves the performance quite dramatically. The R-squared jumps to 29% and the zero-beta rate is not significantly different from zero with a point estimate of 14 basis points. The momentum and value factors again drive the results as both have prices of risk significantly different from zero. The market factor is not significantly different from zero. The Fama and French five factor model does worse than the four factor model. The R-squared is slightly more at 30%, but the constant term is an .37 and significantly different than zero at the 10% level. Only the profitability and investment factors are all priced in the cross-section.

The Level, Slope and Curve model does better than the Fama and French factor models. The R-squared is 35%, and the zero-beta rate is only 15 basis points, insignificantly different from zero. The slope and curve factors are highly significant and the level factor is marginally significant. Again all three coefficients are close to the average returns on the factors, an important prediction of the model. When compared to the Fama and French models, with both sets of test assets, the Level, Slope and Curve model has the highest R-squared and is the only model where all of the factors have a significant positive price of risk. Only once in the six tests of the Fama and French style models is the market risk premium ever significantly different from zero.

The RNM model has the highest R-squared at 41% followed by HXZ at 38% and the LSC at 35%. The RNM model has a negative zero-beta rate of 13 basis points that is not significantly different than zero, while the HXZ model has a zero-beta rate of 33 basis points that is marginally significant. The LSC and RNM model are the only two to have a market price of risk for the market factor significantly different from zero and both models achieve this in both tests. The value (HML) and momentum (UMD) factors are again both priced, but the profitability factor is not, which is perhaps surprising given that many of the test assets are sorts in whole or in part on a profitability measure. The market risk factor is not priced, but size, investment and profitability proxied by ROE all have market prices of risk significantly different from zero.

8.4 Horse Races of All Factors

A central question remains, which factors are important in explaining the cross-section of returns? Which factors provide marginal explanatory power in the presence of other factors? The goal of this paper is to organize the many disparate characteristics and factors into a parsimonious factor model of expected returns. If all or many of the factors in the literature can be boiled down to a much more parsimonious representation, the space left to explain with theory is dramatically reduced. If Level, Slope, and Curve is a better representation of the latent factor structure in returns, then it should drive out other factors. Other factors may just be some combination of level, slope and curve and potentially several unpriced factors.

I follow the procedure in Cochrane (2005) to conduct factor horse races. I run ordinary least squares regressions with returns on each individual asset pricing factor. When the estimated coefficient is added to a cross-sectional asset pricing test with other factors, the resulting coefficient estimate yields the marginal significance of the factor. If a factor is insignificant, it adds little explanatory power to the model. I start with the level, slope and curve factors. Since these factors are orthogonal, there is no need to run a horse race with them. Tables 9 and 10 present the results starting with the three factor LSC model.

Table 9 shows the resulting horse race using the 112 test portfolios. The leftmost column shows the results with the LSC model. All three factors are significant, demonstrating that each has important explanatory power for the cross-section not captured by the other factors. The addition to these three factors of either momentum or HML, presented in columns two and three, has little effect on the results. Momentum and HML are not significant and the LSC factors remain significant. The addition of SMB has some effect as the curve factor becomes insignificant, but the SMB factor remains insignificant.

The next three columns add the three profitability factors, the PMU of the RNM model, the RMW of the Fama and French five factor model and the ROE of the HXZ model. None of the three factors are significant. The RMW factor again causes the curve factor to become insignificant. Together this suggests that the curve factor may be related to size and profitability. The last column shows that in the presence of all the factors there is relatively little increase in explanatory power relative to just the LSC model. The R-squared rises to 79%. The SMB factor becomes significant with a negative coefficient, while the PMU become significant with the appropirate sign. Together the factors jointly drive out the curve factor, but if anything strengthen the slope factor.

Table 10 shows the same horse race using the 140 test assets of characteristic decile sorts. Again the LSC model in the leftmost column shows that each factor brings distinct information useful to explaining returns. The addition of the momentum and HML factors again change the results very little as all three factors remain significant and neither momentum nor HML is significant. The addition of the SMB factor absorbs the significance of the market factor and the slope factor, but is not itself significant. None of the three profitability factor nor investment is significant. When all the factors are used simultaneously, momentum, SMB and RMW all become significant with the wrong sign.

Across both tests, the Level, Slope and Curve model performs very well. The three factors are consistently priced in both tests. Only the RMW factor shows some ability to drive out the curve factor across both tests, suggesting that the two may capture similar information about the cross section. Little explanatory power is added with the additional factors. In the first horse race in Table 9 the R-squared increases from 68% to 79% with the addition of all factors. In the second horse race in Table 10, the R-squared increases from 35% to 44% with the addition of all the factors.

An important goal of summarizing the cross-section is finding strong proxies for the latent factors that price assets. That the Level, Slope and Curve Model survives in the presence of these other factors, while the other factors do not shows that Level, Slope and Curve are the stronger signals. Slope and momentum share some common movements as represented by their correlation, but it is slope which is the stronger signal that empirically drives out momentum in both horse races. Not only that, but level and slope drive out all of the other factors. It is more likely that momentum or HML are noisey sorts of slope than the converse.

9 Robustness Checks

In the main results, I use Level, Slope and Curve factors constructed from the entire sample. Since the main motivation is to find the true underlying factors predicted by the APT, so that future research can link these factors back to deeper models of the macroeconomy and investor behavior, this choice seems natural. Use all the data to identify the factors as they are. But if these factors are very sensitive to in sample vs. out of sample formation, that would raise serious concerns about their formation. Further, it would be very surprising, since Lewellen (2011) shows that these first stage dissecting anomaly regressions work well out of sample, and as reported in this paper, principal components analysis on return sorted portfolios is very stable.

In this section, I explore the robustness of the level, slope and curve model to alternative specifications of information sets. I find that out of sample factors are quite highly correlated with in sample factors and price portfolios similarly. The first specification I choose is a "No Peeking" formation of the factors. In the first stage I form all of the portfolios using only data available at the end of the previous month. Thus, rather than running a full sample regression, I only use data before the portfolio formation date, and roll forward each month increasing the data set. I use the same rules for forming the factors. I perform principal components analysis only on the past data and use the weights from past data to form the current month's factors. Again rolling forward to extend the sample each month.

The second robustness check I perform is a "simple" formation procedure for level, slope and curve. Most factors in this literature are formed using simple high minus low sorts on decile portfolios, while the principal components approach is a very natural way to capture the spirit of the APT, it is decidedly less popular. I also form the level, slope and curve factors in a simple way, such that principal components analysis is not necessary. If a researcher had the intuition that level, slope and curve factors might be the result of a one dimensional portfolio sort on expected returns, they might form factors like this. For this simple version, the level factor is just the excess return on the value weighted market portfolio from CRSP. The slope factor is formed as one third of a dollar invested long in the first three low return portfolios and one third of a dollar invested short in the three high return portfolios. Thus, it is very similar to the popular high minus low decile sort. The simple curve factor is one fourth of a dollar invested long in the four middle portfolios (eleven through fourteen), and one fourth of a dollar invested short in the four extreme portfolios, the two highest return and the two lowest return.

Table 11 shows the correlation table with the factors and the two no peeking versions of the factors. The correlation of the slope factor for the full sample and No Peeking model is .92, while the simple no peeking factor has a correlation of .88. The slope factor is not very sensitive to how it is constructed. The curve factor for the No Peeking model as a .70 correlation with the full sample curve factor. As we've shown earlier, as a factor explains less of the overall variance we also tend to measure it less precisely. The simple no peeking factor has a correlation of .58 with the full sample curve factor. This partly reflects that the curve factor recovered from PCA isn't quite symmetrical. There is a lot more weight on the twenty-fifth (high return) portfolio than the twenty-fourth, while the difference is less stark in the weights on the first and second portfolios.

Table 12 shows that all of these formations do well at pricing the test assets. In the first column, for comparison, is the full sample model, while the next two columns show the no peeking and simple no peeking construction of the factors. I scale the No Peeking factors with the same scalar as in the original construction. This scalar isn't known in advance, but it is also irrelevant to how well the model prices assets. The scalar simply makes the coefficients similarly scaled. In the first panel, with the 112 test assets, the R-squared of 66% for the no peeking and 65% for the simple no peeking are nearly as high as the 68% for the full sample factors. The coefficients are all similarly significant and also close to the expected returns on the factors. Lastly, the alphas are similarly sized and actually smaller for the no peeking versions of the model.

In the second panel, the R-sqaured is 29% for each of the no peeking models, which is smaller than the 35% for the full sample factors, but still almost twice as large as the Fama and French three factor model's R-squared of 15%. The slope and curve coefficients are similarly significant for the two no peeking versions. Only the level factor from the simple no peeking goes from marginally significant to slightly insignificant. The simple curve factor is much more highly correlated with the market than the other constructions, so it is not surprising to see the factor subsume some of its explanatory power. Taken altogether, the evidence shows that these factors are quite robust to how the information set with which they are constructed.

10 ICAPM Interpretation

While the methodology in this paper is general enough to find pricing factors consistent with a wide range of pricing models, the asset pricing literature has stressed Merton (1973) Intertemporal Capital Asset Pricing Model interpretations of empirical factor models, at least since Fama and French (1996) suggest this interpretation for their three factor model. While I don't stress an ICAPM interpretation above any other, the relation between the Level, Slope and Curve factors to well established state variables is still of great interest. Petkova (2006) shows a simple way to embed pricing factors into an ICAPM in the style of Campbell (1996). First, she sets up a Vector Autoregression Model to capture the relationship between the state variables and the market return, as well as the predictability of each state variable. Then, she tests whether changes in the pricing factors proxy for innovations in the economic state variables.

I specify the following VAR model:

$$\begin{bmatrix} R_{M,t} \\ DIV_t \\ TERM_t \\ DEF_t \\ RF_t \\ SVAR_t \\ R_{Lev,t} \\ R_{Slp,t} \\ R_{Cur,t} \end{bmatrix} = A \begin{bmatrix} R_{M,t-1} \\ DIV_{t-1} \\ TERM_{t-1} \\ DEF_{t-1} \\ RF_{t-1} \\ SVAR_{t-1} \\ R_{Lev,t-1} \\ R_{Slp,t-1} \\ R_{Cur,t-1} \end{bmatrix} + u_t$$

The first term $R_{M,t}$ is the excess return on the market defined as the value-weighted return on the CRSP index less the risk-free rate. Both the excess market return and the risk free rate, the one month U.S. treasury return, are obtained from Ken French's data library. The remaining state variables, dividend to price, the term spread, the default yield and stock variance. All these state variables are obtained from the Goyal and Welch data library on Amit Goyal's website.[7]

The dividend to price is defined as the log of the trailing sum of the 12 month dividends minus the log month end value of the CRSP index. The term spread is the *U.S. Yield on Long-term United States Bonds* series from NBER's Macrohistory database minus the *3-Month Treasury Bill: Secondary Market Rate* from the research database at the Federal Reserve Bank at St.Louis (FRED). The default spread is the difference between the yield on BAA- from FRED and the long-term U.S. Yield (defined identically to the term spread). The stock variance variable is the sum of squared daily returns on the S&P 500.[8] Additionally, the excess returns on the Level, Slope

[7] Special thanks to Amit Goyal and Ivo Welch for making this data available and keeping it updated. The data is available at [http://www.hec.unil.ch/agoyal/].

[8] Detailed definitions are available at Amit Goyal's website [http://www.hec.unil.ch/agoyal/docs/AllTables2013.pdf].

and Curve factors are included in the VAR system as potential state variables. The error term u_t is a vector of innovations, unpredicted changes in state variables. The question is whether the Level, Slope and Curve factors are good proxies for these unexpected innovations in state variables.

Following Petkova (2006), I orthogonalize each innovation to the excess return on the market, and scale the innovation so the variance is equal to the market. The Table 13 shows the results of the innovations in predictive variables regressed on the Level, Slope and Curve factors. The t-statistics are corrected for heteroskedasticity and autocorrelation with Newey-West regression using five lags. Table 13 shows that Level, Slope and Curve are all significantly correlated with innovations in dividend yield. The slope factor and curve factors are strongly negatively associated with innovations in dividend yield. Since an decrease in the dividend yield is associated with higher future returns, the slope factor does best (other things equal) when expected future returns are high. Recall the slope factor is long low return stocks and short high return stocks, so that an increase in the slope factor means low return stocks are doing better versus high return stocks. This suggests a good beta, bad beta interpretation for the slope factor. Low return stocks do well, when future expected returns are relatively low (good beta). High return stocks do relatively better, when future expected returns are relatively high (bad beta).

None of the factors seem to be associated with large moves in the term spread or risk-free rate. The Curve factor is associated with an increase in the default spread. The curve factor has greater returns when the default spread increases. The curve factor is long larger, often more profitable and less volatile stocks. These stocks do relatively better, when the default spread is higher. Thus, this finding is similar to Petkova (2006) that SMB has a negative association with the default spread.

Lastly, increases in the slope factor are associated with increases in the monthly stock variance. Low return stocks do relatively well when future stock variance is higher than expected. Since high stock variance is relatively bad for the investment opportunity set, low return stocks again act as a hedge for negative shocks to investors. Taken together, the slope and curve factors seem to capture risk relative to the marginal investor's opportunity set consistent with an ICAPM interpretation.

11 Consumption Based Asset Pricing

A considerable body of theoretical work argues that returns across assets should be explained by an asset's covariance with investor consumption. Since assets that covary positively with consumption increase the investor's consumption volatility and thus have lower prices and higher returns than assets covarying negatively with consumption that act as insurance for the investor.[9] Empirical research has found little support for the Consumption Based Approach. [10] In contrast to the thrust of the literature, Jagannathan and Wang (2007) find that annual returns matched with annual consumption growth measured in the fourth quarter provides surprisingly successful evidence in favor of the linearized consumption based model (CCAPM). The Fama and French 25 size and book to market portfolio show a spread in average returns largely explained by each portfolio's covariance with consumption.

I follow their methodology for the ten Dissecting Anomaly portfolios and the Level, Slope and Curve factors. Figure 11 displays the results. Average returns, on the y-axis, are regressed on covariance with consumption growth on the x-axis. The resulting regression line is displayed, with most of the portfolios and factors fitting tightly around the line. The adjusted R^2 is 78%, matching the smooth linear pattern in the graph.

The x-intercept is a statistically insignificant -3.0% (p-value .11). The equity premium puzzle remains as the coefficient on consumption growth is 112. But the important point is that the line passing near the origin and near the level factor (a proxy for the market), also crosses through nearly all the other test assets. The cross-sectional premium is mostly explained by the covariance with consumption.

The two assets that standout are the curve factor and the lowest return Dissecting Anomaly portfolio. Since the curve factor has a heavy short position in low return stocks, these assets are closely related. These low return stocks are least well-explained by the Level, Slope and Curve factor and share many anomalies related with low expected returns. The APT and CCAPM both fail on these low return stocks. Capitalizing on this mispricing requires a short position in these stocks, which may be impossible or expensive.

Much of the failure of the CCAPM may be related to a small number of stocks. The one dimen-

[9] Rubinstein (1976), Lucas Jr (1978) and Breeden (1979)

[10] Hansen and Singleton (1982), Hansen and Singleton (1983) and Hansen and Jagannathan (1997)

sional characteristic sorts pre-dominantly used in the literature may be poor proxies of expected returns with much less stable covariance with consumption. The success of the CCAPM suggests that the Level, Slope and Curve model not only provides factors consistent with Arbitrage Pricing Theory, the resulting factors show a strong relationship between risk and return as measured by macroeconomic data.

12 Conclusion

This paper develops a new method for extracting the priced factors in the cross-section of stock returns. The first step is using cross-sectional regressions on many predictive variables to sort stocks into portfolios from high return to low return. The second step is using principal components to extract factors from these portfolios. The goal of this approach is to sort portfolios on expected returns and then extract factors related to expected returns. The resulting factors are level, slope and curve as the loadings resemble the level, slope and curvature factors found when principal components are extracted from bond returns. I perform asset pricing tests using the Level, Slope and Curve model compared to several leading models. I find that the model performs very well, despite having only three factors. Horse races show that the factors generally retain their explanatory power even in the presence of other factors. The factors have compelling relationships with the ICAPM and CCAPM, suggesting a deeper relationship with the Level, Slope and Curve Model and systematic risk.

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Table 1: 25 Portfolios Sorted by Expected Returns

The table shows the result of first running cross-sectional Fama-MacBeth regressions with each firm's return in excess of the risk-free rate on size, book to market, momentum, net stock issues (and a dummy for 0), accruals split into positive and negative, asset growth and operating profit. The regression is run each month, but separately for big, small and mirco cap stocks defined with size breakpoints of 50% and 20% of NYSE market equity. Firms are then sorted on the predicted return for each month using firm date from the previous month. Firms are sorted into twenty-five value weighted portfolios based on the predicted returns. The table shows the excess returns, predicted returns and characteristics, all value weighted.

Portfolios	Ret	$\widehat{\text{Ret}}$	Size	BtM	Mom	dAtA	AtBE	NS	OP
1	-1.10	-0.74	8774	0.43	-0.12	0.63	0.09	0.27	-0.12
2	-0.17	-0.25	12029	0.45	-0.04	0.41	0.03	0.17	0.13
3	0.18	-0.03	13875	0.46	-0.02	0.27	0.03	0.09	0.25
4	0.40	0.12	14908	0.49	0.02	0.21	0.03	0.05	0.30
5	0.37	0.22	14579	0.53	0.06	0.17	0.02	0.04	0.31
6	0.56	0.30	14519	0.56	0.10	0.14	0.02	0.02	0.32
7	0.65	0.37	13801	0.61	0.13	0.12	0.01	0.01	0.32
8	0.73	0.44	13225	0.63	0.18	0.11	0.01	0.01	0.32
9	0.65	0.49	12226	0.66	0.21	0.10	0.01	0.01	0.33
10	0.82	0.55	11104	0.69	0.25	0.09	0.01	0.01	0.33
11	0.78	0.60	10177	0.72	0.29	0.08	0.01	0.00	0.36
12	0.74	0.65	8717	0.75	0.32	0.08	0.01	0.00	0.33
13	0.93	0.70	7455	0.78	0.36	0.08	0.00	0.00	0.37
14	0.95	0.75	6953	0.79	0.39	0.07	0.00	0.00	0.39
15	0.97	0.81	6049	0.81	0.42	0.07	0.00	0.00	0.35
16	0.98	0.86	5520	0.83	0.46	0.07	-0.01	0.00	0.36
17	1.06	0.92	4859	0.85	0.50	0.07	-0.01	0.00	0.36
18	1.25	0.99	4152	0.87	0.54	0.06	-0.01	0.00	0.36
19	1.11	1.06	3507	0.88	0.58	0.06	-0.02	0.00	0.39
20	1.33	1.13	2319	0.91	0.62	0.05	-0.02	-0.01	0.41
21	1.18	1.22	1968	0.97	0.67	0.04	-0.03	-0.01	0.43
22	1.26	1.32	1732	1.00	0.76	0.03	-0.05	-0.01	0.52
23	1.58	1.44	1639	1.05	0.86	0.01	-0.08	-0.01	0.50
24	1.49	1.62	1336	1.15	1.01	-0.02	-0.10	-0.01	0.43
25	1.76	1.96	653	1.24	1.32	-0.08	-0.26	-0.01	0.86

Table 2: Principal Components Analysis on Twenty-Five Anomaly Sorted Portfolios

The table shows principal components analysis of 25 anomaly portfolios. I form anomaly portfolios using Fama-MacBeth regression on seven anomaly variables with separate regressions for each size group.

Component	Eigenvalue	Variance Explained	Cumulative
Component 1	667.18	72.59%	72.59 %
Component 2	78.68	8.56%	81.15 %
Component 3	36.79	4.00%	85.15 %
Component 4	16.88	1.84%	86.98 %
Component 5	13.85	1.51%	88.49 %
Component 6	11.92	1.30%	90.88 %
Component 7	10.07	1.10%	91.85 %
Component 8	8.87	0.96%	92.69 %
Component 9	7.77	0.84%	93.48 %
Component 10	7.27	0.79%	94.13 %

Table 3: Cross-Correlation Table for the First Five Components

In each panel, the table shows the correlation of each of the first five principal components with the identical principal component formed using a different number of sorted portfolios.

First Component			
	10 Portfolios	25 Portfolios	100 Portfolios
10 Portfolios	1.000		
25 Portfolios	0.998	1.000	
100 Portfolios	0.994	0.997	1.000
Second Component			
	10 Portfolios	25 Portfolios	100 Portfolios
10 Portfolios	1.000		
25 Portfolios	0.977	1.000	
100 Portfolios	0.956	0.978	1.000
Third Component			
	10 Portfolios	25 Portfolios	100 Portfolios
10 Portfolios	1.000		
25 Portfolios	0.858	1.000	
100 Portfolios	0.846	0.922	1.000
Fourth Component			
	10 Portfolios	25 Portfolios	100 Portfolios
10 Portfolios	1.000		
25 Portfolios	0.542	1.000	
100 Portfolios	0.403	0.548	1.000
Fifth Component			
	10 Portfolios	25 Portfolios	100 Portfolios
10 Portfolios	1.000		
25 Portfolios	0.014	1.000	
100 Portfolios	0.126	-0.009	1.000

Table 4: Cross-Correlation Table Separating Beginning and End of Sample

In each panel, the table shows the correlation of each of the first three principal components formed in the first half of the sample and in the second half of the sample. The first panel shows the beginning of the sample, thus the beginning sample principal components are formed in sample and compared with the out of sample principal components formed using principal components on the second half of the sample. The second panel shows the correlations of components in the latter half of the sample. The End components are in sample and compared with the out of sample Beg components that were formed using only data from the first half of the sample.

Beginning of Sample						
	Beg 1	Beg 2	Beg 3	End 1	End 2	End 3
Beg 1	1.00					
Beg 2	0.00	1.00				
Beg 3	0.00	-0.00	1.00			
End 1	1.00	0.02	-0.04	1.00		
End 2	0.45	-0.87	0.19	0.42	1.00	
End 3	0.63	0.24	0.72	0.61	0.20	1.00
End of Sample						
	Beg 1	Beg 2	Beg 3	End 1	End 2	End 3
Beg 1	1.00					
Beg 2	0.16	1.00				
Beg 3	-0.55	-0.36	1.00			
End 1	1.00	0.21	-0.60	1.00		
End 2	0.05	-0.96	0.36	0.00	1.00	
End 3	0.04	0.14	0.70	-0.00	-0.00	1.00

Table 5: Cross-correlation table

The table shows cross-correlation of the Level, Slope and Curve factor to the market factor, HML, SMB, Momentum, Profitability (PMU), and Liquidity.

Variables	PC 1	PC 2	PC 3	PC 4	PC 5
Mkt-RF	0.95	0.23	0.10	0.09	0.01
SMB	0.51	-0.40	-0.33	-0.32	-0.07
HML	-0.30	-0.18	0.15	0.14	0.14
RMW	-0.28	0.02	0.31	0.13	0.12
CMA	-0.38	-0.26	0.11	0.13	0.05
MOM	-0.02	-0.66	0.26	0.08	-0.03
HML*	-0.12	-0.24	0.07	0.07	0.05
UMD*	-0.09	-0.59	0.22	0.08	-0.04
PMU*	-0.34	-0.07	0.23	0.03	0.04
ROE	-0.22	-0.21	0.38	0.14	0.09
INV	-0.36	-0.28	0.19	0.19	0.11
Liq-T	-0.05	0.03	0.05	0.07	-0.05

Table 6: Time Series Regressions of 25 Expected Return Sorted Portfolios on the Extracted Principal Components

The table shows regression results for the 25 portfolios sorted by expected returns on the extracted principal components. Each portfolio is regressed on the first one, two, three and four components. The alphas, t-statistics and R-squared for each regression are displayed. Portfolios are formed based on each stock's expected return based on regressions on seven anomaly variables.

Port	Ret	α_1	α_2	α_3	α_4	t_1	t_2	t_3	t_4	R_1^2	R_2^2	R_3^2	R_4^2
1	-1.10	-2.18	-1.03	-0.79	-0.56	-9.91	-7.15	-6.99	-5.75	0.62	0.85	0.91	0.93
2	-0.17	-1.02	-0.20	-0.06	0.07	-5.90	-1.56	-0.52	0.60	0.62	0.81	0.84	0.85
3	0.18	-0.60	0.26	0.35	0.29	-3.54	2.22	3.09	2.55	0.59	0.82	0.84	0.84
4	0.40	-0.30	0.42	0.41	0.34	-2.06	4.08	3.93	3.23	0.60	0.82	0.82	0.82
5	0.37	-0.30	0.24	0.19	0.13	-2.46	2.56	2.08	1.39	0.68	0.82	0.82	0.83
6	0.56	-0.13	0.41	0.35	0.23	-1.06	4.41	3.83	2.69	0.69	0.83	0.84	0.86
7	0.65	-0.04	0.27	0.17	0.06	-0.37	2.84	1.98	0.76	0.76	0.81	0.83	0.85
8	0.73	0.05	0.30	0.19	0.05	0.51	3.09	2.15	0.63	0.76	0.79	0.83	0.85
9	0.65	-0.02	0.16	0.03	-0.09	-0.25	1.76	0.42	-1.15	0.77	0.79	0.85	0.87
10	0.82	0.13	0.26	0.12	0.01	1.42	2.67	1.46	0.11	0.78	0.79	0.85	0.87
11	0.78	0.09	0.07	-0.06	-0.16	0.99	0.80	-0.88	-2.35	0.82	0.82	0.88	0.89
12	0.74	0.03	-0.08	-0.21	-0.28	0.31	-0.87	-2.72	-3.59	0.81	0.81	0.87	0.87
13	0.93	0.21	0.09	-0.03	-0.08	2.17	0.97	-0.36	-0.89	0.80	0.81	0.85	0.85
14	0.95	0.17	0.04	-0.07	-0.14	1.77	0.41	-0.87	-1.59	0.83	0.83	0.87	0.87
15	0.97	0.19	-0.02	-0.11	-0.16	1.93	-0.18	-1.19	-1.70	0.81	0.83	0.85	0.85
16	0.98	0.19	-0.04	-0.12	-0.20	1.81	-0.35	-1.24	-2.08	0.80	0.82	0.84	0.84
17	1.06	0.24	0.01	-0.07	-0.06	2.57	0.12	-0.87	-0.67	0.84	0.86	0.88	0.88
18	1.25	0.39	0.09	0.05	0.11	3.57	0.89	0.44	1.08	0.81	0.84	0.84	0.85
19	1.11	0.23	-0.13	-0.20	-0.15	2.14	-1.30	-2.03	-1.50	0.81	0.86	0.87	0.87
20	1.33	0.41	-0.02	-0.06	0.08	3.41	-0.23	-0.60	0.81	0.79	0.85	0.86	0.87
21	1.18	0.26	-0.20	-0.17	0.01	2.05	-1.77	-1.50	0.07	0.78	0.84	0.85	0.87
22	1.26	0.30	-0.15	-0.15	0.03	1.97	-1.07	-1.09	0.25	0.74	0.79	0.79	0.81
23	1.58	0.62	0.19	0.21	0.38	4.36	1.44	1.61	2.96	0.76	0.81	0.81	0.83
24	1.49	0.47	-0.04	0.04	0.21	2.99	-0.31	0.30	1.54	0.74	0.80	0.81	0.83
25	1.76	0.67	-0.14	0.35	-0.04	2.86	-0.67	3.42	-2.09	0.59	0.70	0.93	1.00
$\|\alpha\|$		0.37	0.19	0.18	0.16					0.74	0.82	0.85	0.86
GRS		5.29	3.30	2.99	2.34								

Table 7: Level, Slope and Curve Model and Other Models Using 112 Test Portfolios

I regress monthly excess returns of 112 test assets on one of four factor models in time series regressions from July 1973 to December 2012. I then regress the average excess return on each test asset on the estimated beta from the time series regression in a cross-sectional regression with Fama-MacBeth standard errors. The four models include the Level, Slope and Curve model, the Fama and French three factor model, the Carhart model, and the Fama and French five factor model.

Factor	LSC Model	FF3 Model	Carhart	FF5 Model	RNM Model	HXZ Model
	b/t	b/t	b/t	b/t	b/t	b/t
β Level	0.58**					
	(2.32)					
β Slope	-0.98***					
	(-4.09)					
β Curve	0.59**					
	(2.44)					
β Market		0.24	0.43**	0.31	0.52**	0.31
		(1.07)	(1.97)	(1.42)	(2.23)	(1.40)
β HML		0.33*	0.41**	0.05	0.21	
		(1.77)	(2.21)	(0.31)	(2.28)**	
β SMB		0.21	-0.06	0.33**		0.42***
		(0.98)	(-0.26)	(2.23)		(2.71)
β MOM			0.87***		0.61***	
			(3.97)		(4.20)	
β INV				0.65***		0.19
				(4.50)		(1.60)
β PROF				0.02	0.02	0.42***
				(-0.16)	(0.28)	(2.80)
Cons	0.24***	0.30***	0.19***	0.21***	0.14***	0.20***
	(6.94)	(6.50)	(4.90)	(6.15)	(4.31)	(5.89)
R^2	0.68	0.20	0.66	0.50	0.69	0.55
Avg $\|\alpha\|$	0.17	0.23	0.16	0.23	0.21	0.19

t statistics in parentheses

* p<0.10, ** p<0.05, *** p<0.01

Table 8: Level, Slope and Curve Model and Other Models Using 140 Test Portfolios

I regress monthly excess returns of 140 test assets on one of four factor models in time series regressions from July 1973 to December 2012. I then regress the average excess return on each test asset on the estimated beta from the time series regression in a cross-sectional regression with Fama-MacBeth standard errors. The four models include the Level, Slope and Curve model, the Fama and French three factor model, the Carhart model, and the Fama and French five factor model.

Factor	LSC Model	FF3 Model	Carhart	FF5 Model	RNM Model	HXZ Model
	b/t	b/t	b/t	b/t	b/t	b/t
β Level	0.55*					
	(1.79)					
β Slope	-0.96***					
	(-3.39)					
β Curve	1.12***					
	(3.39)					
β Market		-0.08	0.52	0.15	0.65**	0.19
		(0.27)	(1.74)	(0.50)	(2.20)	(0.66)
β HML		0.28*	0.43***	0.14	0.34	
		(1.73)	(2.61)	(0.87)	(3.57)***	
β SMB		-0.16	-0.01	0.17		0.37**
		(-0.92)	(-0.07)	(1.06)		(2.26)
β MOM			0.99***		0.49***	
			(3.71)		(3.07)	
β INV				0.24**		0.25
				(2.25)		(2.12)
β PROF				0.32***	0.10	0.50***
				(2.62)	(1.26)	(3.53)
Cons	0.15	0.63**	0.03	0.37*	-0.09	0.33*
	(0.84)	(3.18)	(0.17)	(1.88)	(-0.50)	(1.67)
R^2	0.35	0.13	0.32	0.30	0.41	0.38
Avg $\|\alpha\|$	0.13	0.15	0.12	0.12	0.15	0.13

t statistics in parentheses

* p<0.10, ** p<0.05, *** p<0.01

Table 9: Horse Race Using 112 Portfolios

I regress monthly excess returns of 112 test assets on each factor in time series regressions from July 1973 to December 2012. I then regress the average excess return on each test asset on the estimated betas from the time series regressions in a cross-sectional regression with Fama-MacBeth standard errors.

Factor	LSC	+Mom	+HML	+SMB	+PMU	+RMW	+ROE	+CMA	All
	b/t	b/t	b/t	b/t	b/t	b/t	b/t	b/t	b/t
β Market	0.58**	0.55**	0.68***	0.83**	0.65**	0.73***	0.58**	0.77**	1.89***
	(2.32)	(2.19)	(2.48)	(2.53)	(2.23)	(2.58)	(2.20)	(2.52)	(4.84)
β Slope	-0.98***	-1.26***	-0.96***	-1.17***	-0.96***	-1.02***	-0.97***	-0.87***	-2.12***
	(-4.09)	(-3.24)	(-3.96)	(-3.82)	(-3.98)	(-4.30)	(-3.95)	(-3.23)	(-3.24)
β Curve	0.59***	0.69**	0.51***	0.36	0.52**	0.31	0.58**	0.52**	-0.02
	(2.44)	(2.47)	(2.12)	(1.15)	(1.90)	(1.21)	(2.02)	(2.18)	(-0.08)
β MOM		-0.32							-0.93
		(-0.82)							(-1.64)
β HML			0.17						-0.10
			(1.01)						(-0.28)
β SMB				-0.26					-0.86***
				(-1.03)					(-3.09)
β PMU					0.04				0.36*
					(0.44)				(1.95)
β RMW						0.21			-0.24
						(1.24)			(-0.64)
β ROE							0.00		-0.17
							(0.00)		(0.44)
β CMA								0.18	0.35
								(1.16)	(1.46)
Cons	0.24***	0.24***	0.22***	0.23***	0.23***	0.23***	0.24***	0.21***	0.20***
	(6.94)	(6.92)	(6.76)	(6.95)	(6.80)	(6.82)	(6.85)	(6.46)	(6.85)
R^2	0.68	0.69	0.70	0.69	0.68	0.69	0.68	0.70	0.79

t statistics in parentheses

* p<0.10, ** p<0.05, *** p<0.01

Table 10: Horse Race Using 140 Portfolios

I regress monthly excess returns of 140 test assets on each factor in time series regressions from July 1973 to December 2012. I then regress the average excess return on each test asset on the estimated betas from the time series regressions in a cross-sectional regression with Fama-MacBeth standard errors.

Factor	LSC	+Mom	+HML	+SMB	+PMU	+RMW	+ROE	+CMA	All
	b/t	b/t	b/t	b/t	b/t	b/t	b/t	b/t	b/t
β Market	0.55*	0.50	0.72**	0.21	0.55	0.69**	0.53*	0.76**	1.78***
	(1.79)	(1.63)	(2.17)	(0.39)	(1.58)	(1.95)	(1.68)	(2.12)	(3.00)
β Slope	-0.96***	-1.36***	-0.89***	-0.67	-0.96***	-1.02***	-0.97***	-0.85***	-2.47***
	(-3.39)	(-3.28)	(-3.11)	(-1.49)	(-3.38)	(-3.63)	(-3.36)	(-2.80)	(-3.18)
β Curve	1.12***	1.37***	1.10***	1.44***	1.12***	0.66*	1.22**	1.07***	0.31
	(3.39)	(3.81)	(3.29)	(3.00)	(3.06)	(1.70)	(2.58)	(3.14)	(0.67)
β MOM		-0.54							-2.15***
		(-1.31)							(-2.70)
β HML			0.22						0.06
			(1.20)						(0.16)
β SMB				0.33					-0.78**
				(0.79)					(-2.09)
β PMU					0.00				0.31
					(0.00)				(1.43)
β RMW						0.23			-0.82**
						(1.37)			(-4.38)
β ROE							-0.05		1.00***
							(-0.36)		(2.61)
β CMA								0.16	0.22
								(1.06)	(1.04)
Cons	0.15	0.15	0.09	0.20	0.15	0.21	0.14	0.09	0.07
	(0.84)	(0.81)	(0.50)	(0.99)	(0.85)	(1.17)	(0.75)	(0.46)	(1.04)
R^2	0.35	0.39	0.39	0.36	0.35	0.37	0.35	0.37	0.44

t statistics in parentheses

* p<0.10, ** p<0.05, *** p<0.01

Table 11: Cross-correlation table

The table shows cross-correlation of the Level, Slope and Curve factor to the same factors formed out of sample. The first three rows are the No Peeking Level, Slope and Curve factors, which uses only Fama-MacBeth regressions on past data and predicts one month forward out of sample. Additionally, the principal components analysis is only done on portfolios using past data and the in-sample weights are used to form the out of sample factor one month forward. The next through rows are a simple version of the no peeking factor, which rather than using PCA uses the market return as the level factor, a high minus low portfolio of the two highest return and two lowest return portfolios, and a curve factor as the four middle portfolios minus the four extreme portfolios.

Factors	Level	Slope	Curve
No Peeking Level	0.99	0.10	0.06
No Peeking Slope	0.17	0.92	-0.18
No Peeking Curve	-0.23	-0.03	0.70
Simple Level	0.95	0.27	0.11
Simple Slope	-0.07	0.88	-0.20
Simple Curve	-0.56	-0.24	0.58

Table 12: Robustness Checks With No Peeking Factors

I regress monthly excess returns of 112 test assets in the first panel and 140 assets in the second panel on one of the level, slope and curve factor models in time series regressions from July 1973 to December 2012. I then regress the average excess return on each test asset on the estimated beta from the time series regression in a cross-sectional regression with Fama-MacBeth standard errors. The first column is the full sample LSC Model. The second column is a No Peeking LSC Model, which uses only Fama-MacBeth regressions on past data and predicts one month forward out of sample. Additionally, the principal components analysis is only done on portfolios using past data and the in-sample weights are used to form the out of sample factor one month forward. The last column is a simple version of the no peeking factor, which rather than using PCA uses the market return as the level factor, a high minus low portfolio of the two highest return and two lowest return portfolios, and a curve factor as the four middle portfolios minus the four extreme portfolios.

Panel 1: Robustness Checks Using 112 Test Portfolios

Factor	LSC Model	NP LSC	NP Simple
	b/t	b/t	b/t
β Level	0.58**	0.62**	0.44
	(2.32)	(2.48)	(1.95)
β Slope	-0.98***	-0.98	-1.45***
	(-4.09)	(-3.96)	(-4.56)
β Curve	0.59**	0.45**	0.30
	(2.44)	(2.07)	(1.30)
Cons	0.24***	0.20***	0.21***
	(6.94)	(6.73)	(6.94)
R^2	0.68	0.66	0.65
Avg $\|\alpha\|$	0.17	0.16	0.19

Panel 2: Robustness Checks Using 140 Test Portfolios

Factor	LSC Model	NP LSC	NP Simple
	b/t	b/t	b/t
β Level	0.55*	0.57*	0.48
	(1.79)	(1.85)	(1.62)
β Slope	-0.96***	-0.89***	-1.17***
	(-3.39)	(-2.85)	(2.87)
β Curve	1.12***	0.82***	0.65**
	(3.39)	(3.02)	(2.45)
Cons	0.15	0.11	0.10
	(0.84)	(0.59)	(0.49)
R^2	0.35	0.29	0.29
Avg $\|\alpha\|$	0.13	0.12	0.13

t statistics in parentheses

* p<0.10, ** p<0.05, *** p<0.01

Table 13: Innovations in State Variables Regressed on Level, Slope and Curve

I regress the innovation from each state variable in the VAR model on the Level, Slope and Curve factors. The state variables are dividend to price, term spread, default spread, the risk-free rate and one month stock variance.

Dep. Variable	a_0	Level	Slope	Curve
u_{DIV}	-0.32	0.11	-0.24	-0.18
	-1.03	2.68	-5.36	-3.36
u_{TERM}	0.00	0.02	-0.01	-0.07
	-0.02	0.38	-0.22	-1.04
u_{DEF}	-0.01	-0.05	0.03	0.19
	-0.04	-0.10	0.85	3.75
u_{RF}	0.05	-0.01	0.03	-0.02
	0.29	-0.11	0.69	-0.33
u_{SVAR}	0.10	-0.03	0.08	0.07
	0.35	-0.24	1.77	1.14

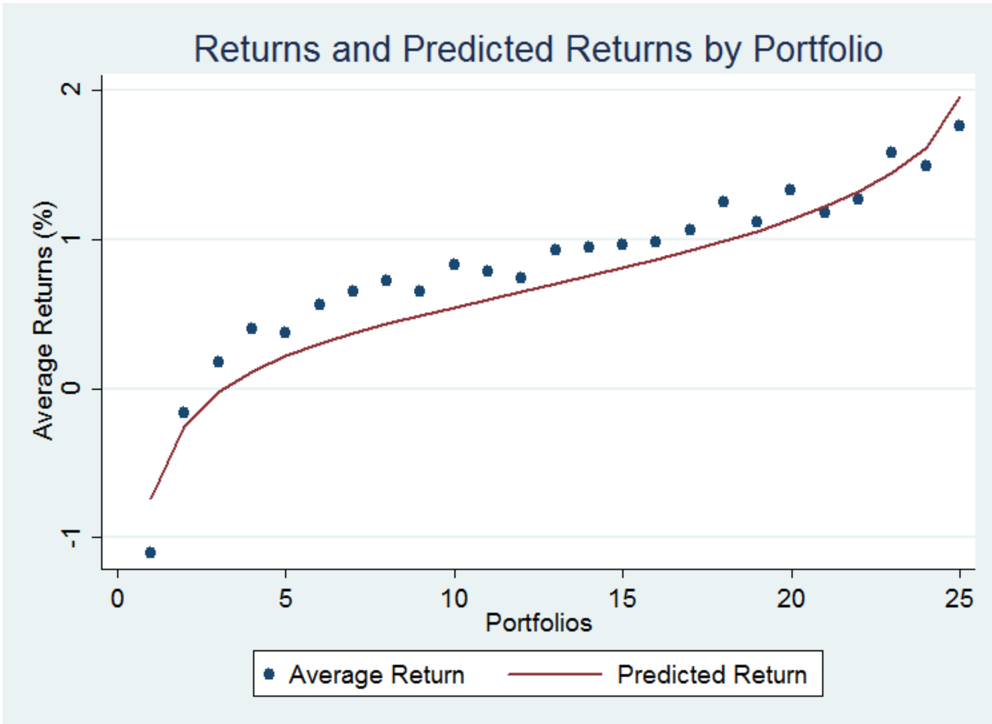


Figure 1: Twenty-Five Dissecting Anomaly Portfolios

The figure shows average returns and predicted returns for twenty-five portfolios built on seven asset pricing anomalies. I form anomaly portfolios using Fama-MacBeth regression on seven anomaly variables with separate cross sectional regressions for each size group. Stocks are sorted into portfolios based on the fitted value from each cross sectional regression. Predicted return is the value-weighted average fitted value in each regression. Average return is the value-weighted average return on each portfolio.

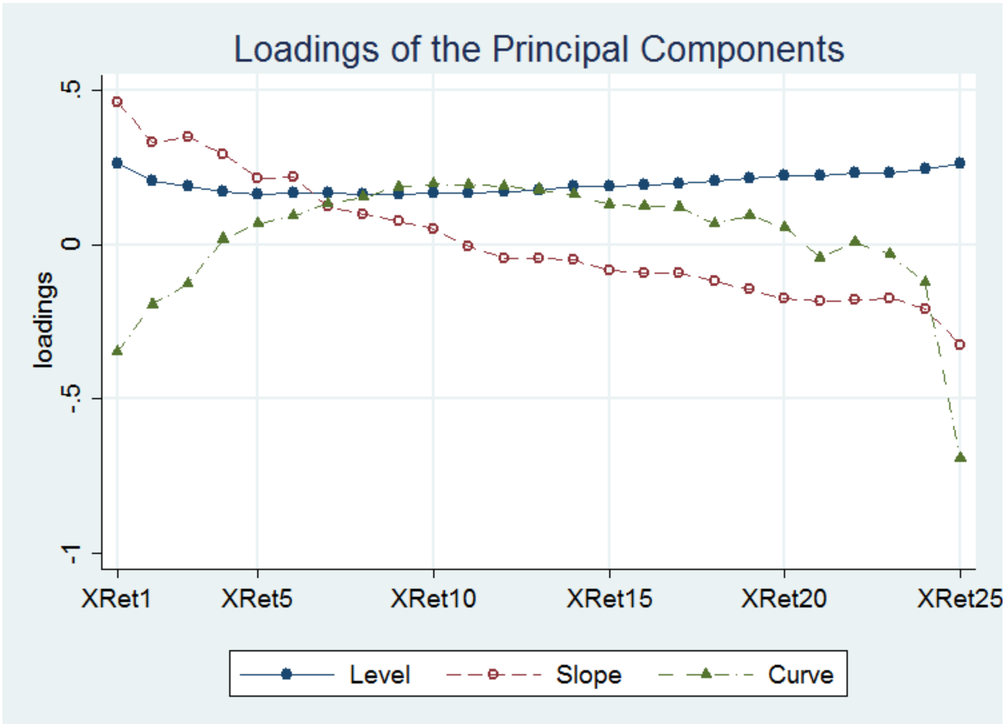


Figure 2: PCA Weights

The figure shows the loadings of each of the first three principal components of twenty-five anomaly portfolios.

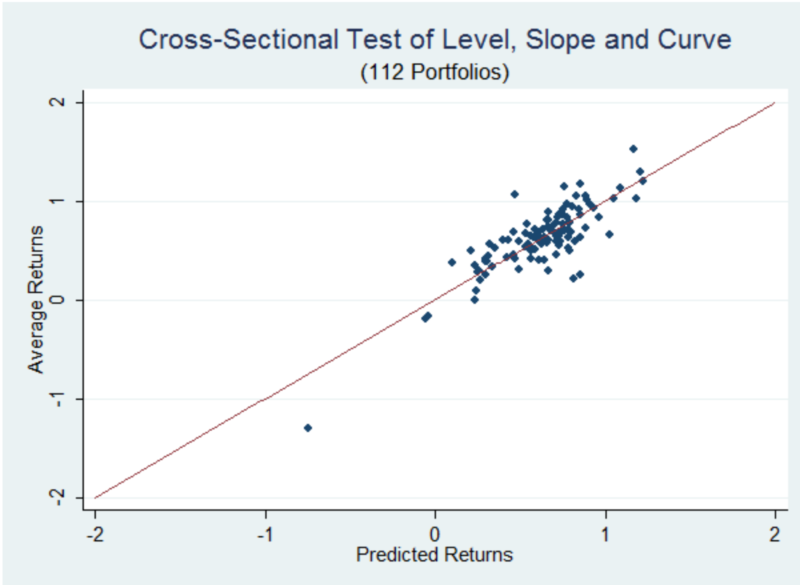


Figure 3: Level, Slope and Curve Model vs. 112 Portfolios

The figure shows the results of the cross-sectional regressions of the Level, Slope and Curve model on 112 portfolios. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample.

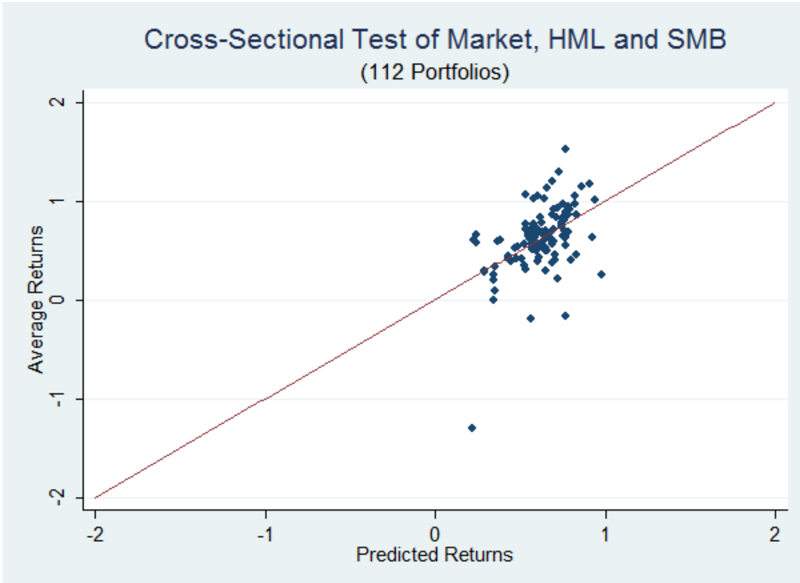
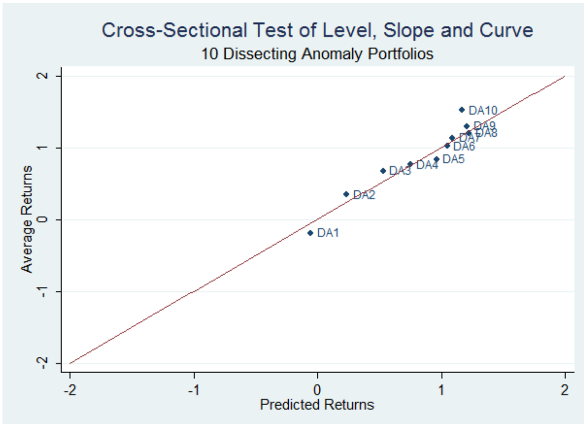
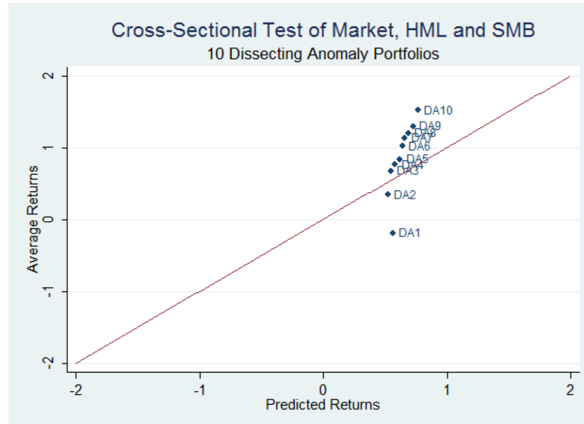


Figure 4: Fama French Three Factor Model vs. 112 Portfolios

The figure shows the results of the cross-sectional regressions of the Fama and French three factor model on 112 portfolios. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample.



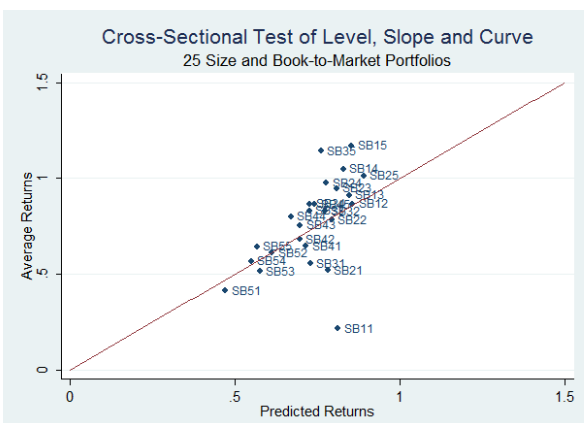
(a) LSC



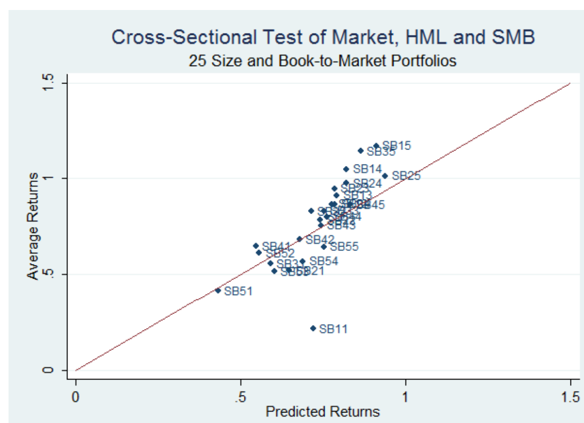
(b) FF3

Figure 5: Models vs. 10 Dissecting Anomaly Portfolios

The figure shows the results of the cross-sectional regressions of the Level, Slope and Curve model and the Fama and French three factor model on 10 portfolios formed on seven anomaly variables. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample.



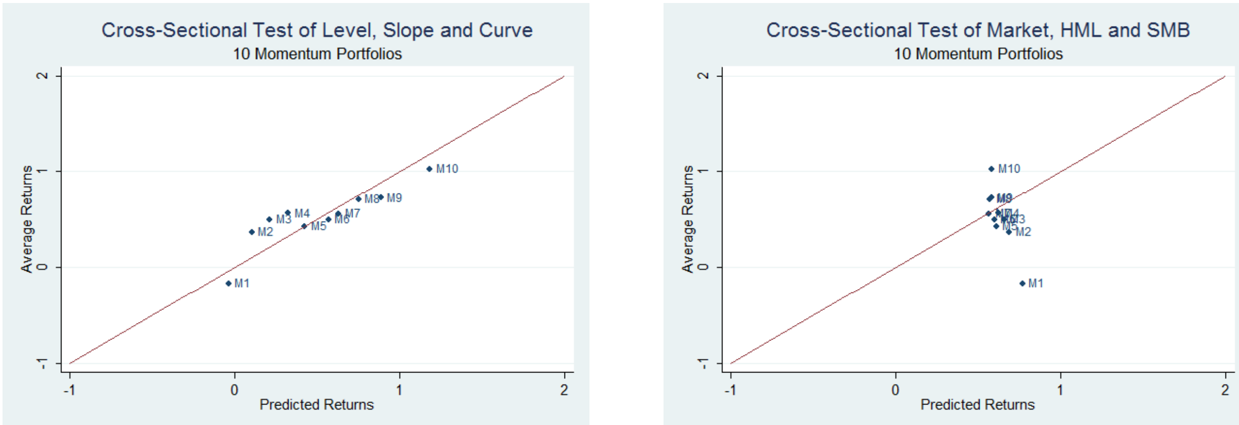
(a) LSC



(b) FF3

Figure 6: Models vs. 25 Size and Book to Market

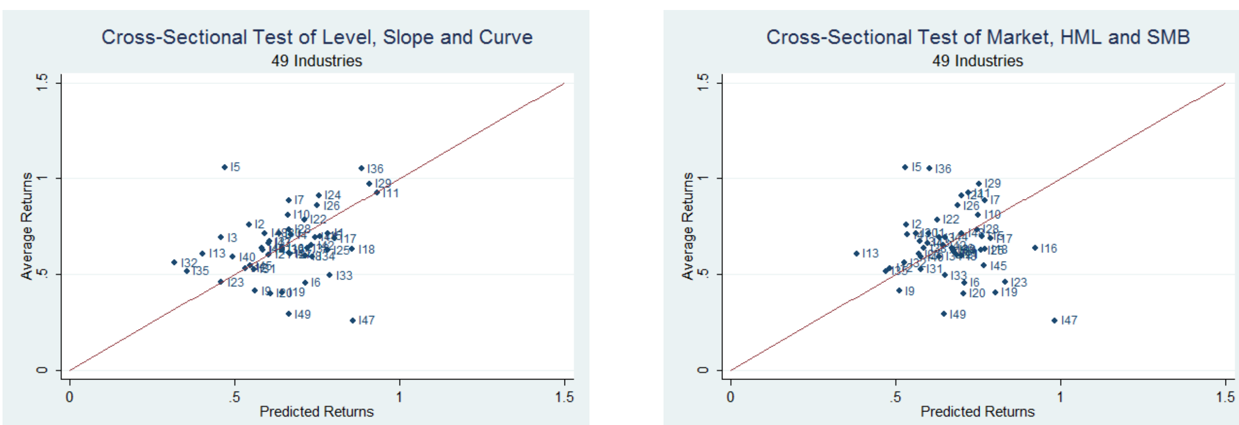
The figure shows the results of the cross-sectional regressions of the Level, Slope and Curve model and the Fama and French three factor model on twenty-five portfolios formed on size and book to market. The X axis is the model predicted excess return. The Y axis is the average excess return of the portfolio over the sample.



(a) LSC (b) FF3

Figure 7: Models vs. 10 Momentum Portfolios

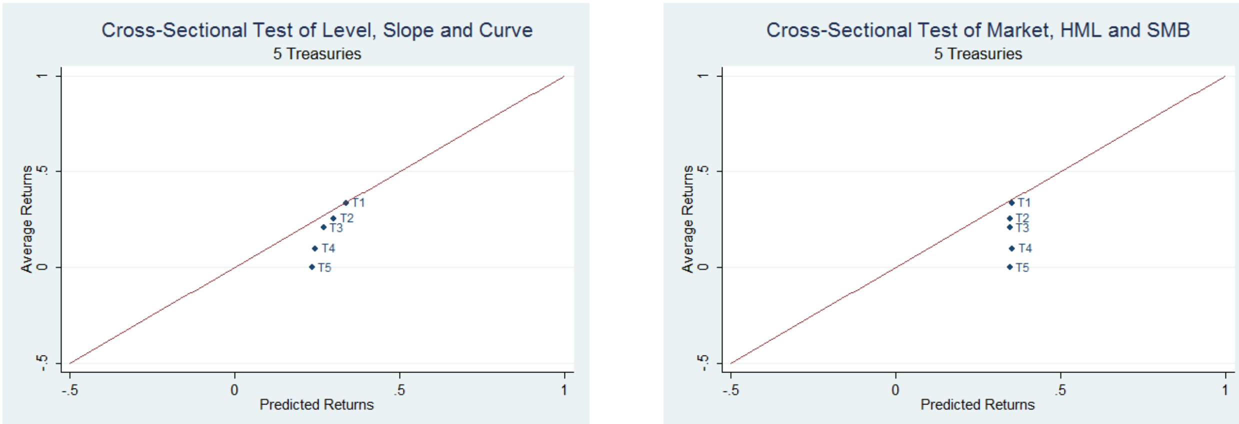
The figure shows the results of the cross-sectional regressions of the Level, Slope and Curve model and the Fama and French three factor model on ten portfolios formed on momentum. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample.



(a) LSC (b) FF3

Figure 8: Models vs. 49 Industry Portfolios

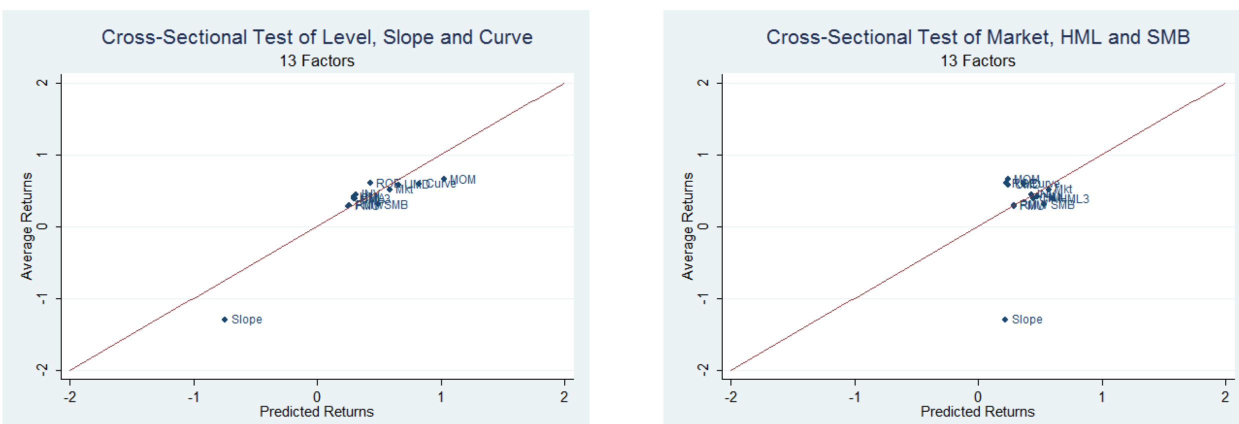
The figure shows the results of the cross-sectional regressions of the Level, Slope and Curve model and the Fama and French three factor model on 49 portfolios formed on industry. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample.



(a) LSC (b) FF3

Figure 9: Models vs. 5 Bond Portfolios

The figure shows the results of the cross-sectional regressions of the Level, Slope and Curve model and the Fama and French three factor model on five treasury bond returns of different maturities. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample.



(a) LSC (b) FF3

Figure 10: Models vs. 12 Pricing Factors

The figure shows the results of the cross-sectional regressions of the Level, Slope and Curve model and the Fama and French three factor model on five treasury bond returns of different maturities. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample.

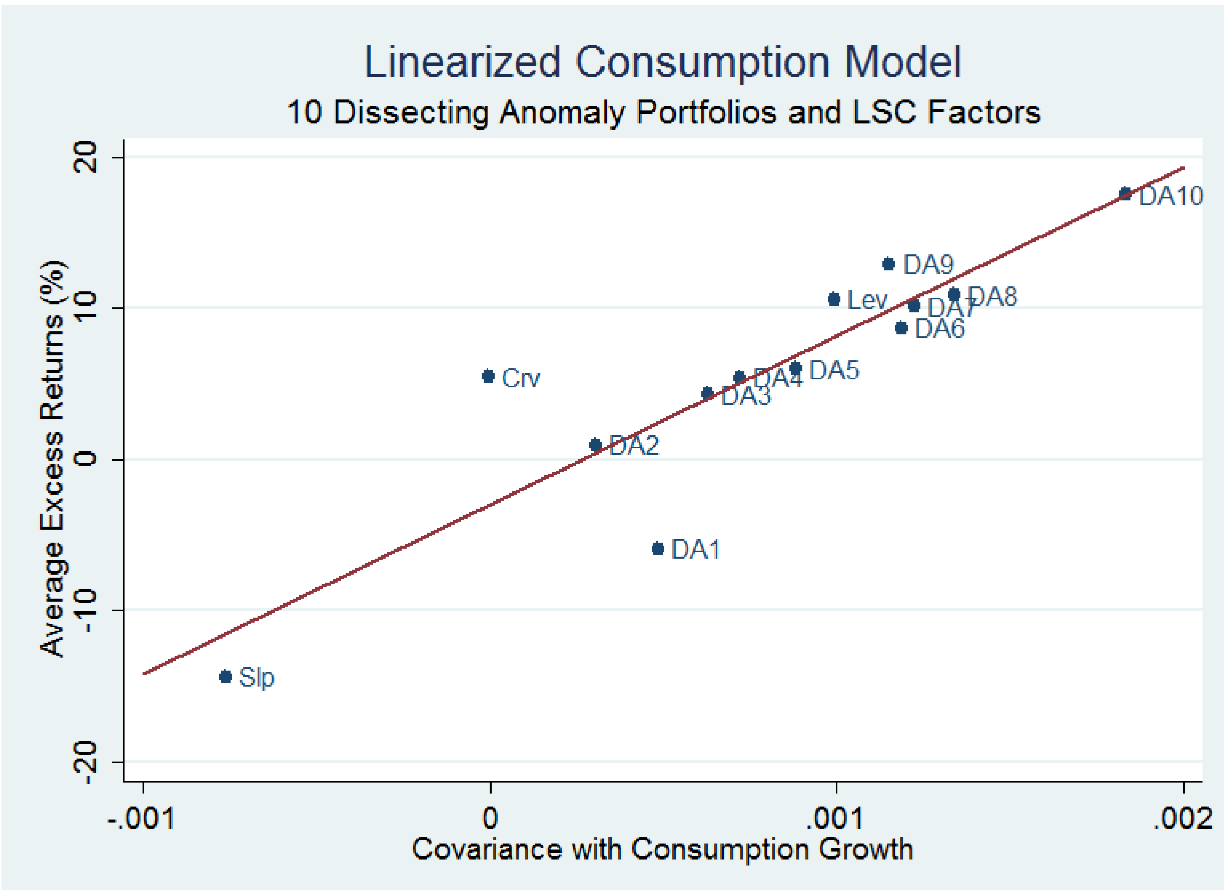


Figure 11: Consumption Based Asset Pricing

The figure shows the Linearized Consumption Model. The test assets are 10 portfolios of annual excess returns sorted from high to low returns using the Dissecting Anomalies predictors as well as annual returns on the Level, Slope and Curve Factors. Consumption is measured as nondurable consumption plus services, and consumption growth is the change from Q4 to Q4 of the calendar year, which matches the January to December annual returns.